Diffusion Slip for a Binary Mixture of Hard–Sphere Molecular Gases: Numerical Analysis Based on the Linearized Boltzmann Equation

Shigeru Takata

Abstract. The diffusion-slip problem for a binary mixture of gases is investigated on the basis of the linearized Boltzmann equation for hard-sphere molecules with the diffuse reflection boundary condition. The problem is analyzed numerically by the finite-difference method, where the collision integrals are computed by the numerical kernel method first introduced by Sone, Ohwada and Aoki for single-component gases [Sone et. al., Phys. Fluids A, 1, 363 (1989)]. This is the first report in which the method is extended and applied to the case of mixtures. The analysis is carried out for several combinations of the component gases and the behavior of the mixture is clarified at the level of the velocity distribution functions. As a result, the coefficient of the diffusion-slip and the associated Knudsen-layer functions are obtained.

INTRODUCTION

As is well known, if there is a concentration gradient of a component gas in a binary mixture, the diffusion takes place. It is a relative flow of a component gas to the other, and is not in itself related to whether or not the total mixture flows. On the other hand, if the gradient is established along the surface of a body in a mixture of slightly rarefied gases, a flow of the total mixture is induced along the surface. This phenomenon is called the diffusion slip (creep) and the induced flow is called the diffusion-slip flow. Since its physical mechanism was first given in Ref. [1], the phenomenon has attracted much interest of researchers in the field of kinetic theory [2–6].

Theoretically, the study of the phenomenon is reduced to that of a half-space boundary-value problem of the linearized Boltzmann equation, the “diffusion-slip problem”. Various approaches have been so far taken for the analysis of the problem. They are, however, limited to those based on model equations, which are not as successful as the BGK model for single-component gases, or those based on rough approximations such as variational and moment methods.

In the meantime, we have recently shown [7] that the diffusion slip is a source of the ghost effect [8,9] in gas mixtures. Thus, it can cause the failure of the classical fluid dynamics for the description of gas mixtures even in the continuum limit. This fact gives a new importance to the diffusion-slip problem and stimulates us to study it in detail. In the present study, in order to understand the behavior of the mixture comprehensively, we carry out an accurate numerical analysis of the linearized Boltzmann equation for a binary mixture of hard-sphere molecular gases. The numerical method is the combination of the finite-difference and numerical kernel methods, the latter of which was introduced in Ref. [10] for single-component gases. The extension of the method to the case of mixtures is the other aspect of the present work.

PROBLEM AND NOTATION

Consider a semi-infinite expanse \((X_1 > 0)\) of a binary mixture of gases, gas A and gas B, over a plane wall \((X_1 = 0)\), where \(X_i\) is the rectangular coordinate system. The wall is at rest and is kept at a uniform temperature \(T_0\). Far from the wall, the mixture is at the same temperature \(T_0\) and has a uniform molecular
number density $n_0$, but there is a uniform gradient of the concentration $X^A$ or $X^B (= 1 - X^A)$ of the component gas $A$ (or gas $B$) in the $X^A$ direction. We will investigate the steady behavior of the mixture under the following three assumptions. (i) The behavior of the mixture is described by the Boltzmann equation for hard-sphere molecules. (ii) The gas molecules are diffusively reflected on the wall. (iii) The magnitude of the concentration gradient of each component gas is so small that the equations and the boundary conditions can be linearized around a reference state. The reference state is the absolute equilibrium state at rest characterized by temperature $T_0$, molecular number density $n_0$ of the mixture, and concentrations $X^A$ and $X^B (= 1 - X^A)$ of the individual component gases.

We here introduce the notation of basic quantities: $m^A$ (or $m^B$) and $d^A$ (or $d^B$) are the mass and diameter of a molecule of gas $A$ (or gas $B$); $\ell_0 = (\sqrt{2}\pi (d^A)^2 n_0)^{-1}$ is the mean free path of the molecules of gas $A$ at the equilibrium state at rest with the molecular number density $n_0$; $x_i = X_i \ell_0^{-1}(\sqrt{\pi}/2)^{-1}$ is the nondimensional space rectangular coordinate system; $(2kT_0/m^A)^{1/2} \zeta_i$ (or $(2kT_0/m^A)^{1/2} \zeta_i$) is the molecular velocity, where $k$ is the Boltzmann constant. With $\alpha = A, B$, $\hat{n}^\alpha = m^\alpha/m^A$, $d^\alpha = d^\alpha/d^A$, $\zeta = |\zeta|$, $E^\alpha = (\hat{n}^\alpha/\pi)^{3/2} \exp(-\hat{n}^\alpha \zeta^2)$, and $n_0(2kT_0/m^A)^{-3/2} E^A(X_0^A + \phi^A)$ is the velocity distribution function of the molecules of gas $A$.

The macroscopic variables of interest are expressed in terms of $\phi^\alpha$ as follows: for each component gas $\alpha$ ($\alpha = A, B$), the molecular number density $n_0(X_0^\alpha + \phi^\alpha)$, flow velocity $(2kT_0/m^A)^{1/2} u_0^\alpha$, temperature $T_0(1 + \tau^\alpha)$, pressure $p_0(X_0^\alpha + \phi^\alpha)$, stress tensor $p_0(X_0^\alpha + \phi^\alpha)$, and heat-flow vector $p_0(2kT_0/m^A)^{1/2}Q^\alpha_i$ are written as

$$\begin{align*}
N^\alpha &= \int \phi^\alpha E^\alpha d\zeta, \\
u_0^\alpha &= X_0^\alpha \int \zeta_i \phi^\alpha E^\alpha d\zeta, \\
\tau^\alpha &= \frac{2}{3} X_0^\alpha \frac{1}{2} \int (\hat{n}^\alpha \zeta^2 \phi^\alpha E^\alpha d\zeta, \\
P^\alpha &= \frac{2}{3} \hat{n}^\alpha \int \zeta_i \phi^\alpha E^\alpha d\zeta, \\
Q^\alpha_i &= \frac{2}{3} \int \zeta_i \phi^\alpha E^\alpha d\zeta.
\end{align*}$$

and, for the total mixture, the molecular number density $n_0(1 + N)$, density $n_0 m^A(\sum_{\beta=A,B} \hat{n}^\beta X_0^\beta + \omega)$, flow velocity $(2kT_0/m^A)^{1/2} u_0$, temperature $T_0(1 + \tau)$, pressure $p_0(1 + P)$, stress tensor $p_0(\delta_{ij} + P_{ij})$, and heat-flow vector $p_0(2kT_0/m^A)^{1/2}Q_i$ are written as

$$\begin{align*}
N &= \int \sum_{\alpha=A,B} \phi^\alpha E^\alpha d\zeta, \\
u &= \int \sum_{\alpha=A,B} \hat{n}^\alpha \phi^\alpha E^\alpha d\zeta, \\
\tau &= \frac{2}{3} \sum_{\alpha=A,B} (\hat{n}^\alpha \zeta^2 \phi^\alpha E^\alpha d\zeta, \\
P &= \frac{2}{3} \sum_{\alpha=A,B} \frac{1}{2} (\hat{n}^\alpha \phi^\alpha E^\alpha d\zeta, \\
Q_i &= \frac{2}{3} \sum_{\alpha=A,B} \frac{1}{2} \hat{n}^\alpha (\zeta^2 - \frac{5}{2} \phi^\alpha E^\alpha d\zeta, \\
\text{where } d\zeta &= d\zeta_1 d\zeta_2 d\zeta_3 \text{ and the integrations are performed for the whole space of } \zeta.
\end{align*}$$

**FORMULATION**

Let us first introduce the following function $\phi^\alpha_F$ ($\alpha = A, B$) which is the solution of the linearized Boltzmann equation expressing the state of the component gas $\alpha$ far from the wall:

$$\phi^\alpha_F = (\delta_{\alpha A} - \delta_{\alpha B}) \epsilon_2 - \zeta_2 |D^{(A)\alpha}(\zeta) - D^{(B)\alpha}(\zeta)| X_0^\alpha + 2 \hat{n}^\alpha b \zeta_2 X_0^\alpha (\partial X^\alpha/\partial x_2)_{x_1=\infty}, \quad (\alpha = A, B),$$

where $\delta_{AA} = \delta_{BB} = 1$ and $\delta_{AB} = \delta_{BA} = 0$, $b$ is an undetermined constant, and the set of the functions $D^{(B)\alpha}$ is the solution of the following simultaneous integral equations [11]:

$$\sum_{\beta=A,B} \left( \frac{\hat{n}^\alpha + \hat{n}^\beta}{2} \right)^2 X_0^\alpha X_0^\beta \tilde{L}^{\beta \alpha} (\zeta_i D^{(\gamma)\beta}(\zeta_i D^{(\gamma)\alpha}) = -\zeta_i (\delta_{\gamma \alpha} - \frac{\hat{n}^\alpha X_0^\alpha}{\sum_{\beta=A,B} \hat{n}^\beta X_0^\beta}), \quad (\alpha, \gamma = A, B),$$

subsidiary condition:

$$\sum_{\beta=A,B} \hat{n}^\beta X_0^\beta \int_0^\infty \zeta^4 D^{(\alpha)\beta} d\zeta = 0.$$
Then, if we put the function \( \phi^a (a = A, B) \) in the form \( \phi^a = \phi^a_F + (\partial X^A / \partial x_2)_{x_1 = \infty} \Phi^a_K \), we can reduce the problem to the following one-dimensional boundary-value problem for \( \Phi^a_K \):

\[
\zeta^1 \frac{\partial \Phi^a_K}{\partial x_1} = \sum_{\beta = A, B} \left( \frac{\dot{a}^\beta + \ddot{a}^\beta}{2} \right)^2 \tilde{L}^\beta \Phi^\beta_K, \quad (\alpha = A, B),
\]

\[
\Phi^a_K = -2m^a b \zeta_2 X^a_0 + \zeta_2 [D^{(A)a}(\zeta) - D^{(B)a}(\zeta)], X^a_0 - 2(\pi m^a)^{1/2} \int_{\zeta_1 < 0} \zeta_1 \Phi^a_K \rho \, d\zeta, \quad \zeta_1 > 0, \ x_1 = 0, \ (6)
\]

\[
\Phi^a_K \to 0, \quad x_1 \to \infty, \quad (7)
\]

where

\[
\tilde{L}^\beta(f, g) = \frac{1}{4\sqrt{2\pi}} \int E^\beta(\zeta_*)[f(\zeta_*) - f(\zeta)] \rho(\zeta - \zeta_*) \cdot e \, d\Omega(e) \, d\zeta_*,
\]

\[
\zeta_1 = \zeta + \hat{\mu}^\beta \frac{1}{m^\beta} \rho(\zeta - \zeta_*) \cdot e, \quad \zeta_2 = \zeta - \hat{\mu}^\beta \frac{1}{m^\beta} \rho(\zeta - \zeta_*) \cdot e, \quad \hat{\mu}^\beta = \frac{2m^a m^\beta}{m^a + m^\beta}.
\]

Here \( e \) is a unit vector, \( \zeta \) is the variable of integration corresponding to \( \zeta \), \( d\zeta_* = d\zeta_1 d\zeta_2 d\zeta_3 \), and \( d\Omega(e) \) is the solid angle element in the direction of \( e \). The integration in Eq. (8) is carried out for the whole space of \( \zeta_* \) and for all directions of \( e \). The \((\partial X^A / \partial x_2)_{x_1 = \infty} \Phi^a_K \) is called the Knudsen-layer part of the solution of the problem. When the boundary-value problem (5)-(7) is solved, the undetermined constant \( b \) in Eq. (6) is simultaneously determined with the solution \( \Phi^a_K \).

**BASIC PROPERTIES OF MACROSCOPIC VARIABLES**

Corresponding to the decomposition of \( \phi^a \), we can split each macroscopic variable into the fluid-dynamic part, denoted by a subscripted \( F \), and the Knudsen-layer part, denoted by a subscripted \( K \). The former is defined by Eqs. (1) and (2) with \( \phi^a \) replaced by \( \phi^a_F \), and the latter by those with \( \phi^a \) replaced by \( \phi^a_K \). Then, from Eq. (3) we have the following expression of the fluid-dynamic parts: for individual component gases

\[
N^a_F = P^a_F = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty}, \quad \tau^a_F = 0, \quad u^a_{1F} = u^a_{2F} = Q^a_{1F} = Q^a_{2F} = 0, \quad P^a_{ijF} = P^a_{F} \delta_{ij},
\]

\[
u^a_{2F} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} (b - \Delta a A + \Delta a B), \quad Q^a_{2F} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} X^a_0 \left( \Gamma^{(B)a} - \Gamma^{(A)a} \right),
\]

and for the total mixture

\[
N_F = P_F = \tau_F = 0, \quad u_{1F} = u_{3F} = Q_F = 0, \quad P_{ijF} = 0, \quad \omega_F = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} (m^A - m^B), \quad P_{ijF} = 0, \quad P_{ijF} = P^a_{ijF} \delta_{ij},
\]

\[
u_{2F} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty}, \quad Q_{2F} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} [(\hat{\zeta}_T A - \hat{\zeta}_T B) + 5 \sum_{\alpha = A, B} X^\alpha_0 (\Delta a A - \Delta a B)],
\]

where \( \hat{\Delta}_{a\beta}, \hat{\Gamma}_D^{(a\beta)}, \) and \( \hat{\Delta}_T (a, \beta = A, B) \) are moments of the function \( D^{(a\beta)} \) (see the Appendix A in Ref. [11] for their definitions). They are related to the transport coefficients of gas mixtures.

On the other hand, we can seek the solution \( \Phi^a_K \) of the boundary-value problem (5)-(7) in the form

\[
\Phi^a_K (x_1, \zeta_1) = \zeta_2 (\zeta_2^2 + \zeta_3^2)^{-1/2} \Phi^a_K (x_1, \zeta_1 / \zeta_2).
\]

It follows from this similarity that the Knudsen-layer part can appear only in \( u^a_2, u_2, P^a_{12} \) (or \( P^a_{21} \)), \( P_{12} \) (or \( P_{21} \)), \( Q^a_2 \), and \( Q_2 \). That is,

\[
u^a_{2K} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} \hat{U}^a(x_1), \quad u^a_{2K} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} U(x_1),
\]

\[
P^a_{12K} = P^a_{21K} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} \hat{S}^a(x_1), \quad P_{12K} = P_{21K} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} S(x_1),
\]

\[
Q^a_2 = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} \hat{H}^a(x_1), \quad Q_{2K} = \left( \frac{\partial X^A}{\partial x_2} \right)_{x_1 = \infty} H(x_1),
\]
where \( U^\alpha, U, S^\alpha, S, H^\alpha, \) and \( H \) are called the Knudsen-layer functions and are defined as follows:

\[
U^\alpha(x_1) = \frac{1}{X_0} \int \zeta_2 \Phi_k^\alpha E^\alpha d\zeta, \quad U(x_1) = \left( \sum_{\beta=A,B} \hat{n}^\beta X_0^\beta U^\beta(x_1) \right) / \left( \sum_{\beta=A,B} \hat{n}^\beta X_0^\beta \right),
\]

\[
S^\alpha(x_1) = 2\hat{n}^\alpha \int \zeta_1 \zeta_2 \Phi_k^\alpha E^\alpha d\zeta, \quad S(x_1) = S^A(x_1) + S^B(x_1),
\]

\[
H^\alpha(x_1) = \int \zeta_2 (\hat{n}^\alpha \zeta_2 - \frac{5}{2}) \Phi_k^\alpha E^\alpha d\zeta, \quad H(x_1) = \sum_{\beta=A,B} \left( H^\beta(x_1) + \frac{5}{2} X_0^\beta [U^\beta(x_1) - U(x_1)] \right).
\]

In the meantime, from the conservation law of the momentum, it is easily found that the stress tensor of the total mixture \((P_{12} \text{ or } P_{21})\) should be constant in the whole space. Therefore the Knudsen-layer function \( S \) should be equivalent to zero because of the condition at infinity [see Eq. (7)]. This property, the uniformity of the stress tensor (or \( S = 0 \)), is used as a measure of accuracy of the numerical computation.

To summarize, the Knudsen-layer part appears only in the flow velocities, stress tensors, and heat-flow vectors except the stress tensor of the total mixture. The other macroscopic variables can be expressed by the fluid-dynamic part of the solution. Among them, the molecular number density, pressure, temperature, and stress tensor of the total mixture as well as the temperatures of the individual gases are in particular uniform in the whole space.

Finally the expression of \( u_{2F} \) in Eq. (10), if considered at \( x_1 = 0 \), is no other than the slip condition for the flow velocity of the mixture on the wall. We call the constant \( b \) the coefficient of diffusion slip in the present paper.

### NUMERICAL COMPUTATION

The adopted numerical method is the combination of the finite-difference and numerical kernel methods. For the latter, we first transform the expression (8) of \( \tilde{L}^\alpha \) into that in terms of integral kernels, and then, taking into account the similarity (11), make the database of the kernels numerically before the computation of the problem itself. The database is also available for analyses of other problems with the same similarity.

In the computation, the original region \((0 < x_1 < \infty, 0 < \zeta < \infty, -1 < \zeta_1/\zeta \leq 1)\) is truncated to a finite region \(0 < x_1 < d(\leq \infty), 0 < \zeta \leq Z(\leq \infty), \) and \(-1 < \zeta_1/\zeta \leq 1, \) where the parameters \( d \) and \( Z \) are chosen as \( d = 20.70 \) (or 26.40) and \( Z = 4.5/\sqrt{\hat{m}^\alpha} \) for gas \( \alpha (\alpha = A, B). \) It is made sure that the error by this truncation is negligibly small. The number of lattice points used in the present computation is 221 (or 241) for \( d = 20.70 \) (or 26.40), of which 201 points are in the region \(0 < x_1 \leq 15, \) in \( x_1 \)-space, 49 in \( \zeta \)-space, and 97 in \( \zeta_1/\zeta \)-space, respectively. The lattice system is nonuniform in \(0 < x_1 \leq 15 \) and in \( \zeta \)-space with a monotonically increasing interval. It is uniform in \( 15 < x_1 \leq d \) (with the same interval as the largest in \(0 < x_1 \leq 15 \)) and in \( \zeta_1/\zeta \)-space.

The boundary-value problem (5)–(7) is characterized by the three parameters: \( \hat{m}^B \) (or \( m^B/m^A \)), \( d^B \) (or \( d^B/d^A \)), and \( X_0^A \). We carried out numerical computations for \( d^B = 1 \) and various values of \( m^B/m^A \) and \( X_0^A \).

### RESULTS AND DISCUSSIONS

The coefficient \( b \) of the diffusion slip is tabulated in Table 1. Although \( b \) is non-zero for \( X_0^A = 0 \) and \( X_0^A = 1 \), the diffusion slip itself does not take place for these cases because no concentration gradient is possible. Since

<table>
<thead>
<tr>
<th>( X_0^A )</th>
<th>( m^B/m^A )</th>
<th>( X_0^B )</th>
<th>( m^B/m^A )</th>
</tr>
</thead>
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</tr>
</tbody>
</table>

* The data are analytically obtained.
\*b\* is positive, the slip flow is induced in the direction of the higher concentration of the lighter component gas (gas A).

The profiles of the Knudsen-layer functions for flow velocities are shown in Figure 1. Figure 1a shows the profiles of \( U \) for various values of \( X_0^A \) in the case \( m_B^A/m_A^A = 2 \) and \( d_B^A/d_A^A = 1 \). Figure 1b shows those of \( U \), \( U^A \), and \( U^B \) for various values of mass ratio \( m_B^A/m_A^A \) in the case \( X_0^A = 0.5 \) and \( d_B^A/d_A^A = 1 \).

Figure 2a shows the profiles of \( U \) in the case \( X_0^A = 0.5 \). Figure 2b shows those of \( S^A \) in the case \( X_0^A = 0.5 \). Figure 2c shows those of \( H^A \) in the case \( X_0^A = 0.5 \). Figure 2d shows those of \( H^B \) in the case \( X_0^A = 0.5 \). From Figures 1b and 2, the contributions of \( U \), \( U^A \) and \( U^B \) to \( H \) are found to be larger than those of \( H^A \) and \( H^B \) [see the last equation in Eq. (13)].
remains almost unchanged. Going away from the wall, the $\Phi^A \zeta^A$ becomes milder and decays with keeping the difference of the features between the two regions. The Knudsen-layer part of the other component $-\Phi^B E^B$ shows the similar behavior except the higher peak and the narrower width of its form. These differences are due to the larger mass of a molecule B ($m_B/m_A > 1$).

In Figure 4 the present result of the coefficient $b$ is compared with those in Refs. [5,6], where the data are given by the (dimensional) coefficient divided by the diffusion coefficient. In the figure, according to the description in Refs. [5,6], comparison is made using the first approximation of the diffusion coefficient. The relative error of Loyalka’s formula [5], which is obtained by the variational method, is larger for smaller $X_0^A$ and becomes 10% for the worst case ($m_B/m_A = 10$ and $X_0^A = 0$). With more reliable data of the diffusion coefficient, however, Loyalka’s formula gives better agreement with the present result. The relative error of the formula using our numerical data of the diffusion coefficient [12] is less than 4.1% for $m_B/m_A = 2$ and 2.2% for the other cases.

Finally, we give some pieces of information about the accuracy of the present computation. The values of $b$ with different lattice systems in $\zeta$- and $\zeta_1/\zeta$-spaces are tabulated in Table 2. The values are computed using a coarser system (101 lattice points) in $x_1$-space with $d = 15$ in the case $m_B/m_A = 2$ and $X_0^A = 0.5$. Table 3 shows the values of $b$ in the same case using different lattice systems in $x_1$-space with different $d$ and the common systems in $\zeta$- and $\zeta_1/\zeta$-spaces (49 points for the former and 97 points for the latter). Another measure of accuracy is, as stated before, the computed value of the Knudsen-layer function $S$. The computed $|S|$, which should theoretically be zero, is less than $1.7 \times 10^{-4}$ for $X_0^A = 0.9$ and $m_B/m_A = 10$, less than $2.6 \times 10^{-4}$ for $X_0^A = 1$ and $m_B/m_A = 10$, and less than $1.1 \times 10^{-4}$ for the other cases.

The present computation was carried out on Fujitsu VPP800/12 computer at the Institute of Space and Aeronautical Science and on VT-Alpha 533, 600 and VT-Alpha6 500 workstations in our laboratory.

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FIGURE 3. Knudsen-layer part $\tilde{\Phi}_K^A E^A$ of the velocity distribution function of gas A and its contour plot in the case $m_B/m_A = 2$, $d_B/d_A = 1$, and $X_A^0 = 0.5$ at (a) $x_1 = 0$, (b) $x_1 = 0.0375$, (c) $x_1 = 0.1904$, (d) $x_1 = 0.5649$, and (e) $x_1 = 1.6731$. Here $\zeta_r = (\zeta^2 - \zeta_1^2)^{1/2}$. In the contour plots the curves are drawn with the interval 0.01 in (a)–(d) and 0.005 in (e); the outermost curves in (a)–(d) indicate 0.01.
present results

$X_0^A = 1$

$X_0^A = 0.999$

$X_0^A = 0.5$

4-moment method [6]

- variational method [5]

FIGURE 4. Coefficient $b$ of the diffusion slip for $d^B/d^A = 1$: Comparison with previous works. Dashed lines indicate the spline curves connecting the present results for the same $X_0^A$. The others are explained in the figure.

TABLE 2. Accuracy test I. Dependence of the value of $b$ on the lattice systems in $\zeta$- and $\zeta_1/\zeta$-spaces. The values are computed using a system with 101 points in $x_1$-space with $d = 15$ in the case $m^B/m^A = 2$, $d^B/d^A = 1$, and $X_0^A = 0.5$.

<table>
<thead>
<tr>
<th>$N_{\zeta_1/\zeta}$</th>
<th>$N_{\zeta}$</th>
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<th>37</th>
<th>49</th>
<th>61</th>
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<tr>
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<td>–</td>
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<td>–</td>
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</tr>
</tbody>
</table>

* The $N_\zeta$ and $N_{\zeta_1/\zeta}$ are the numbers of lattice points in $\zeta$- and $\zeta_1/\zeta$-spaces, respectively.

TABLE 3. Accuracy test II. Dependence of the value of $b$ on $d$ and the lattice system in $x_1$-space. The values are computed using a system with 49 points in $\zeta$-space and 97 points in $\zeta_1/\zeta$-space in the case $m^B/m^A = 2$, $d^B/d^A = 1$, and $X_0^A = 0.5$.

<table>
<thead>
<tr>
<th>$d$</th>
<th>S-I</th>
<th>S-II</th>
<th>S-III</th>
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<tbody>
<tr>
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<tr>
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<tr>
<td>26.40</td>
<td>0.163664</td>
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</tbody>
</table>

* S-I: The system with 101 points for $d = 15$, 111 points for $d = 20.70$, and 121 points for $d = 26.40$.

S-II: The system with 201 points for $d = 15$ and 221 points for $d = 20.70$.

S-III: The system with 331 points for $d = 20.70$.

REFERENCES