INERTIA AND TEMPERATURE EFFECTS IN VOID GROWTH

L. Seaman and D. R. Curran

SRI International, 333 Ravenswood Avenue, Menlo Park, California 94025-3493, U.S.A.

Abstract. The usual solution for void growth in a ductile metal is based on the assumption that the velocity of growth is proportional to the current void size and inversely proportional to the material viscosity. In this study we examine the additional effects of inertia and temperature on void growth. The solution of Poritsky (1) includes the large effect of inertia on the growth rate of large voids. As the voids rapidly expand during high-rate fracture, the material near the void surface becomes hot and its strength and viscosity may both reduce because of this heating. We performed finite-element simulations to study the growth of a single spherical void under tensile loading in a viscous material, thereby accounting for viscous, thermal, and inertia effects. In aluminum the thermal strength reduction effect allows small voids to expand about 35% more rapidly than without the effect. We propose a simple approximate method for treating the viscous, thermal, and inertia effects in void growth.

INTRODUCTION

This study began with an effort to determine the importance of the thermal effects which must occur near the surface of a void growing at high rate in a ductile metal. Johnson (2) suggested that the thermal effect is unimportant, yet Wang (3) concluded that the thermal effect is dominant. In addition we wanted to pursue the inertial effect which arises for large voids, according to the equation of Poritsky (1) for a void growing in an infinite block of viscous material:

$$\frac{d^2R}{dt^2} + \frac{3}{2R} \left( \frac{dR}{dt} \right)^2 + \frac{4\eta}{\rho R^2} \frac{dR}{dt} = \frac{T}{\rho R}$$

(1)

Here R is the radius, t is the time, \(\eta\) is the coefficient of viscosity, \(\rho\) is the density, and T is the applied tension. The third and fourth terms represent the usual conditions for small voids and their solution leads to the constant value of \(V/R\) equal to \(T/4\eta\), where V is the void growth rate \(dR/dt\). The first two are acceleration terms; they lead to a reduced growth velocity for larger voids. From this equation we solved for the growth rates for aluminum, copper, and tantalum for a viscosity coefficient of 20 Pa-s using a fourth order Runge-Kutta method. The resulting curves are shown in Fig. 1. The inertial effect (reduction in velocity for larger voids) is enhanced for larger densities.

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![Figure 1. Void growth velocity for Al, Cu, and Ta, assuming \(\eta = 20\) Pa-s.](image-url)
CONDITIONS FOR OUR SIMULATIONS

To study both the thermal and inertial effects we performed dynamic explicit one-dimensional finite-element simulations of a spherical void in the center of a spherical block of material. The inner void surface was given an initial velocity and a constant tensile stress was applied to the outer boundary. The material was treated as a fluid with a linear viscous behavior.

To verify the simulation method, we made short simulations with several initial void radii and no thermal strength reduction effect. The internal energy in these short cases never approached the melting energy. The computed growth rates were close to the curve for aluminum given in Fig. 1; therefore, we assumed our method was sound.

We performed the next finite-element simulations with a thermal effect which modified the viscosity as a function of the current internal energy as shown in Fig. 2. Our intention was to approximate the model of Steinberg et al (4), and also data of Razorenov et al (5) and Hanim and Klepaczko (6). We assume that this form is reasonable for very high rate loading as in our case near the rim of the void. Then as the energy approaches melting there is a sudden drop in the viscosity from 20 Pa-s (from our earlier work with ductile fracture in aluminum (7)) to 0.02 Pa-s, or about 16 times the measured value (1.2 mPa-s (8)) for liquid aluminum near the melt temperature.

We used the larger 0.02 Pa-s for the liquid case to minimize oscillations in our simulations.

The simulations showed that void growth velocity varies essentially as $1/R^2$ to maintain a nearly constant density throughout the block. The circumferential deviator stresses are proportional to the circumferential strain rate $V/R$, where $V$ is the void growth rate.

$$\sigma'_c = 2\eta V/R$$

The radial deviator stress is $-2\sigma'_c$ to balance the two circumferential deviator stresses, and hence is given by

$$\sigma'_r = -4\eta V/R$$

This compressive deviator stress is usually (without the thermal reduction effect) balanced by the large tensile pressure which exists throughout the block. But when the viscosity reduces because of the thermal effect, the pressure must also approach zero at the void surface so that the zero-stress boundary condition is maintained. The variation of the deviator stress, pressure, and total stress are shown in Fig. 3 at some time in a simulation (starting radius of 5 micrometers) after a steady condition has been achieved for an applied tensile stress of 0.8 GPa. The computation was initialized with the internal energy at melting, about 586 J/g. Thereafter the rapid reduction in the viscosity causes the continuing increase in energy at the rim to proceed at a very slow pace. Hence, although the energy reaches incipient melt at the void.

![Figure 2](image1.png)

**FIGURE 2.** Viscosity variation according to the thermal strength reduction effect

![Figure 3](image2.png)

**FIGURE 3.** Stress and pressure variation with radius.
surface, it does not significantly exceed this value. Therefore, one might not see indications of melting on the void surface.

**RESULTS**

We made a series of short simulations with starting void radii of 1, 3, 10, 30, 100, 300, and 1000 micrometers and an imposed radial stress of 0.8 GPa at the outer boundary. The internal energies were initialized at melting at the void rim, decreasing with the inverse 6th power of radius. Velocities were initialized with an estimated velocity at the rim and then varied inversely with the square of radius. This initialization was undertaken so that a steady-state condition could be achieved rapidly and consistent with the imposed external stress. Similarly, the deviator stress and pressure were initialized to represent our estimate of the steady-state conditions. The resulting velocity-to-radius ratio and the velocity of void growth are shown in Figs. 4 and 5. The velocities from the simulations appear to oscillate somewhat initially and then reach a steady or smoothly varying condition until a reflected wave returns from the outer boundary. The results of the many small simulations do not fit well together: this probably indicates that a steady state was not yet reached. These simulation results are compared with the solution of Eq. (1) in these figures.

The thermal effect appears to increase the velocity by about 35% for small voids, but much less for larger voids.

**AN APPROXIMATE SOLUTION**

We sought a simple approximation to the void growth equation which can be used economically to follow the growth of large arrays of voids. Through our analyses of Eq. (1) we found that the first term, the second derivative, has a small effect on the solution over the whole range of radii. Therefore, we omitted this term and the quadratic formula gives the growth rate:

\[
V = - \frac{4Q_v \eta}{3pR} + \frac{1}{3p} \sqrt{\left( \frac{4Q_v \eta}{R} \right)^2 + 6pQ_t T}
\]

This solution is also shown in Figs. 4 and 5. This equation includes the "adjustment factors" \(Q_v\) and \(Q_t\) which are each equal to one for Poritsky's equation, but here are added to account for the thermal effects. The curve is made using \(Q_v = 0.81\) and \(Q_t = 1.1\). Clearly this solution is an approximation that can be applied appropriately in many situations.
CONCLUSIONS

Temperature has an effect on the rate of void growth. When thermal effects are not accounted for, there is a misinterpretation of the magnitude of the viscosity. The effects of inertia become important for larger voids, and especially for higher density materials. A simple approximation for the altered void growth process was presented to facilitate the separate treatment of a range of void sizes.

REFERENCES