ANALYTICAL AND COMPUTATIONAL STUDY OF ONE-DIMENSIONAL IMPACT OF GRADED ELASTIC SOLIDS

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Abstract. Some of the recent efforts to improve the ballistic performance of lightweight armors utilize functionally graded materials to provide a continuous transition in properties between dissimilar materials. It has been conjectured that the elimination of abrupt acoustic impedance changes may result in beneficial stress wave attenuation. We examine this issue for some idealized one-dimensional problems in which all materials are linear elastic. Exact solutions are compared with DYNA3D simulations of the same problems.

INTRODUCTION

It is well known that the ballistic performance of metallic armor can be improved by the addition of a ceramic front plate. A typical design for appliques to combat vehicles consists of ceramic tiles bonded to metal backings by a low strength polymeric adhesive. However, there is evidence that a strong coupling between the ceramic and metal plates results in increased ballistic performance in addition to providing the capability of supporting structural loads (see Gooch et al. (1)). One approach to the elimination of low strength interfaces is the introduction of a functionally graded material (FGM) that transitions between the hard ceramic front plate and the tough metal backing plate. It has been conjectured that a continuous gradation in acoustic impedance may also reduce the amplitude of reflected tensile waves that can ultimately result in failure of the more brittle ceramic. In view of the complexity of the waves generated by ballistic impact of real armor systems, the influence of any particular grading on stress wave propagation can be difficult to sort out experimentally. Numerical simulations may provide valuable insight in such cases. However, practical limitations on the number of elements in 2D and particularly 3D simulations may prohibit the use of meshes that are fine enough to accurately resolve some property gradings of interest. In such cases the simulations may not capture the peak stresses that lead to material failure.

In this paper we study the effects of continuous variations in acoustic impedance on the propagation of longitudinal waves for some idealized one-dimensional (uniaxial strain) impact problems which are simple enough that exact solutions can be obtained by Laplace transform techniques. This necessitates the assumption that all materials are linear elastic. These exact solutions are also used as a check on numerical simulations of the same problems with the Lagrangian finite element code LLNL-DYNA3D.

APPLIED STEP IN STRESS

The first problem is described by the left illustration in Fig. 1. At time \( t=0 \) a uniform compressive stress \( \sigma_0 \) is applied and maintained at the left face \( (x=0) \) of an initially unstressed linear elastic plate. The right face \( (x=1) \) is fixed. The acoustic impedance \( Z \) varies with position \( x \) according to
the rule

$$Z(x) = Z_0(ax + 1)^2, \quad 0 \leq x \leq 1, \quad (1)$$

where $Z_0 = Z(0)$ and $a$ is a parameter that characterizes the grading. The wave speed $c$ is assumed to be constant, so that the longitudinal modulus $L = cZ$ and the density $\rho = Z/c$ are proportional to $Z$. Throughout this paper the principal longitudinal stress $\sigma$ is taken to be positive in compression. Because of the applied stress boundary condition, the solution for the normalized stress $E = CT/CT_0$ depends only on the parameter $a$ in eq. (1), i.e., it is independent of $Z_0$.

For $a = 1$, the impedance, longitudinal modulus, and density increase by a factor of four from the left to the right end (see left illustration in Fig. 1). The exact solution for the time history of the normalized stress $\Sigma$ at the midpoint ($x = 1/2$) of the specimen is given by the thin solid curve in the top two plots in Fig. 2. The time $t$ has been nondimensionalized so that the wave traverses the length of the specimen in one unit of time. The first reflected wave from the rear surface arrives at the center at $t = 1.75$; the first reflected wave from the front surface at $t = 2.5$.

DYNA3D solutions with 30 and 300 hexahedral elements through the thickness of the specimen are also shown in the top and middle plots, respectively, in Fig. 2. The relative change in impedance per element varies from 3%-7% for 30 elements, and 0.3%-0.7% for 300 elements. The 30 element DYNA3D solution captures the initial compressive and tensile stress peaks but grossly underestimates the tensile spikes. The 300 element solution captures the initial compressive and tensile stress peaks, but substantially underestimates the compressive and tensile stress spikes at later times. These results were relatively insensitive to the value of the artificial viscosity.

For comparison, consider the analogous problem for a homogeneous material ($a = 0$). Exact and DYNA3D stress histories at the center are shown at the bottom in Fig. 2. DYNA3D captures the stress peaks for this case, with continued widening of the shock front as the number of wave reflections increases. Observe that for a homogeneous material the stress is periodic in time and never tensile. Thus for this problem the impedance grading introduces tensile stresses that are not present in a homogeneous material.

**IMPACT OF GRADED TARGETS**

Most of the literature on analytical solutions of “impact” problems for graded materials assumes an applied step in stress (as in the previous section); cf. Lee et al. (2). However, for a true impact, a continuous impedance grading in the target introduces continuously distributed reflected waves, so that the history of the stress $\sigma$ and the particle velocity $v$ at the impact face are not known in advance. If, as assumed here, the impactor is semi-infinite, linear elastic and welds to the target, then for impact at time $t = 0$ of a target initially at rest, the appropriate boundary condition at the impact face $x = 0$ is

$$Z_\text{imp} v(0,t) + \sigma(0,t) = H(t) Z_\text{imp} v_\text{imp}, \quad (2)$$

where $Z_\text{imp}$ and $v_\text{imp}$ are the acoustic impedance and velocity of the impactor.
FIGURE 2. Exact vs. DYNA3D stress histories at the center for an applied step in stress with rigid boundary. Top and middle figures for a graded material with $a = 1$ in eq. (1). Bottom figure for a homogeneous material.

The targets considered here are illustrated in Fig. 1 and motivated by those in Gooch et al. (1). The left (impact) face has properties of titanium diboride (TiB$_2$) and the right (stress-free) face that of titanium (Ti). The impedance of TiB$_2$ is 1.78 times that of Ti. These materials have nearly the same density, so the density is taken to be constant throughout the target. The impactor is tungsten, with an impedance of 2.06 times that of TiB$_2$. Target 1 is a TiB$_2$ front plate welded to a Ti back plate. Targets 2–4 have a continuous variation in acoustic impedance. For Target 4 the impedance is $Z(x) = Z_{TiB_2}(bx/2 + 1)^{2/3}$ for $0 \leq x \leq 2$, where $b = (Z_{Ti}/Z_{TiB_2})^{3/2} - 1 = -0.58$. Analogous relations hold for the graded sections in Targets 2 and 3. These impedance gradings are chosen to simplify the analytical solution (the derivations will be given elsewhere). Although the impedance varies nearly linearly in the graded sections of Targets 2–4, an exact linear variation yields a more complicated solution involving integrals of Bessel functions; cf. Lee et al. (2). The exact and the DYNA3D solutions for the normalized stress $\Sigma \equiv \sigma/(Z_{imp}v_{imp})$ in Targets 1 and 2 are plotted in Figs. 3 and 4. The stress histories are at a point 1/4 of the way in from the left (impact) face, which in Target 2 corresponds to the TiB$_2$/graded interface.

The DYNA3D simulations used 60 uniformly spaced hexahedral elements in the target. For the graded section in Target 2, the relative impedance change per element varies from 1.5%–
2.6%. DYNA3D does a a good job of capturing the stress peaks for these problems, except for the overshoots at the initial and first unloading shock in the graded Target 2. This was due in part to the use of the “tied” (type 2) interface condition, which forces the impactor to stick to the target. This condition was imposed for comparison with the analytical solutions, which do not allow for separation of the impactor and target. The overshoot at the initial shock dropped by 60% when a sliding interface with separation (type 3) was used, though this had little effect on the overshoot at the first unloading shock. The (exact) stress histories for Targets 3 and 4 are compared in Fig. 5.

CONCLUSIONS

It is reasonably easy to construct simple dynamical problems for smoothly graded materials that tax the abilities of Lagrangian hydrocodes. A small relative impedance change per element does not necessarily guarantee an accurate solution. However, DYNA3D did much better for the more realistic impact problems.

As can be seen from Figs. 3–5, for these problems the introduction of a continuous grading in impedance, as opposed to the discrete interface in Target 1, had relatively little effect on the peak compressive and tensile stresses in the target.

REFERENCES