Particle Acceleration and Radiation with Super-Strong Field Interactions

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Abstract. New concepts of particle acceleration and coherent x-ray generation in the super – strong laser - beam interactions are investigated, the quantum effects are treated using the Klein – Gordon theory. Particles are accelerated up to TeV energy range in the scattering by the leading edge of the laser pulse propagating at the group velocity which is less than the speed of light. The quantum regime in the x-ray Compton FEL is investigated, in which the FEL becomes the two-level quantum oscillator with a completely inverted active medium.

INTRODUCTION

The physics of interaction of high-power laser pulses with relativistic particle beams is of the great importance for the progress both in the accelerator technology and in the development of novel radiation sources. The study of these processes have been initiated by P. L. Kapitza and P. A. M. Dirac [1]. Recent advances in high-intensity laser systems [2] have renewed interest in the next generation of high-gradient laser-driven accelerators [3,4]. The laser synchrotron source concept, utilizing the Compton scattering of laser photons by relativistic electrons, opens a prospective route for the development of intense high-brightness sources of x-ray and γ radiation [5,6].

In this paper, the quantum effects in laser – electron beam interactions at relativistic intensities are investigated with the use of Klein-Gordon field theory. In the second section, we discuss a new concept of TeV-range laser ponderomotive acceleration [7], in which particles are accelerated in the point-like scattering by the leading front of the laser pulse, propagating at the group velocity which is less than the vacuum speed of light. The acceleration is treated as the scattering of de Broglie waves by the laser ponderomotive potential, in analogy with the scattering of electromagnetic wave by the overdense plasma. The peak laser intensity determines the threshold and the quantum probability of acceleration, and have no influence on the energy gain, which is determined by the group velocity and the initial energy of a particle.

The quantum operation regime in the x-ray Compton free-electron laser (FEL) is analyzed in the third section. This regime emerges when the quantum emitted exceeds the energy spread of the electron beam and FEL homogeneous linewidth, determined by the optical undulator length [8]. Such a FEL becomes a two-level quantum oscillator with a completely inverted active medium. In the nonlinear interaction
regime, inversion can be completely removed in one pass, in analogy with π-pulse formation in coherently amplifying medium. Estimates show the quantum FEL is a promising compact high-power high-brightness source of coherent x-ray radiation.

**HIGH-ENERGY LASER PONDEROMOTIVE ACCELERATION IN A PLASMA**

In the conventional theory of laser ponderomotive acceleration (LPA) in vacuum [9], the laser pulse group velocity is assumed to be equal to the vacuum speed of light c, \( \beta^* = v_{gr} / c = 1 \). The acceleration thus originates from the transverse gradient of the ponderomotive potential. Expelled particle inevitably has a transverse velocity. At sufficiently high intensities, the quiver amplitude of motion in laser wave becomes comparable to the beam waist, and electron is effectively expelled out from the focal region due to the transverse gradient of the laser field. The laser-particle interaction is thus terminated within a wavelength, and the gain in energy of the accelerated particle increases monotonously with the laser peak intensity.

In a plasma, the laser pulse group velocity is less than c, \( \beta^* < 1 \). With an increase in laser intensity, the particle velocity inevitably reaches \( v_{gr} \). The expelled particle has begun to move in front of the laser pulse, and qualitatively a new LPA regime has to occur, in which the particle is accelerated as a result of the scattering by the leading front of the pulse [10].

**Kinematics Of The High-Energy LPA**

Let us assume a high-power ultrashort laser pulse of the normalized vector potential amplitude \( a_0 \) propagating in a plasma at a definite group velocity, \( \beta^* < 1 \). In a new reference frame, which is moving with the pulse group velocity in the same (z) direction, the laser pulse ponderomotive potential becomes static, so the interaction can be considered now as just an elastic potential scattering. Let the initial energy and velocity of the particle be \( \gamma_0 \) and \( \beta_0 \), respectively. The initial collision angle is \( \alpha_0 \), \( \alpha_0 = 0 \) corresponds to a co-propagating laser pulse and accelerated particle. The energy of the particle in the moving frame is

\[
\gamma^* = (1 - \beta^* \beta_0 \cos \alpha_0) \gamma_0 \gamma
\]

(1)

where \( \gamma^* = (1 - \beta^* \gamma^* \gamma^* - 1/2 \) is the characteristic \( \gamma \)-factor of the laser pulse. According to the classical scattering theory, if this energy is less than the peak ponderomotive potential,

\[
\gamma^* \leq \left(1 + a_0^2 / 2 \right)^{1/2}
\]

(2)
the particle has to be reflected, and the final energy $\gamma$ of the particle does not change after scattering (at infinity). In the laboratory frame, we have

$$\gamma (1 - \beta^* \beta \cos \alpha) = \gamma \gamma^* \tag{3}$$

In this implicit relation, $\beta$ is the particle absolute velocity, and $\alpha$ is the final scattering angle, where $\alpha = 0$ corresponds to the co-propagation case. For the particle initially at rest, the maximum energy gain is at $\alpha = 0$, i.e., $\gamma_{\text{max}} = (1 + \beta^* \beta)^{1/2} = 2 \gamma^*$, and the particle velocity exceeds the laser pulse group velocity $\beta_{\text{max}} = 2 \beta^* / (1 + \beta^* \beta) > \beta^*$. Thus, the accelerated particle is in front of the laser pulse during all the interaction process, which is qualitatively different than that for conventional vacuum LPA [9].

The energy gain for accelerated particle does not depend on the laser pulse intensity, but on the initial energy of the particles and the pulse group velocity only. From Eqs. (1) and (3) we obtain for $\alpha = 0$

$$\gamma_\pm = [(\beta^* \pm \beta)^2 + \gamma^* \gamma^*]^{1/2} \gamma_0 \gamma^* \tag{4}$$

where the signs correspond to an initially counter- (\text{"+\textquotedblleft)}) and co-propagating (\text{"-\textquotedblright)}) laser pulse and accelerated particle. This result coincides with the one-dimensional approach [10].

The value of the laser peak intensity determines the possibility of the acceleration according to the classical scattering threshold condition (2). In a more correct quantum picture, the value of ponderomotive potential determines the probability of scattering, which, in fact, gives the number of accelerated particles.

**Scattering Of De Broglie Waves By Ponderomotive Potential**

To calculate the quantum probability of scattering by the ponderomotive potential, i.e., the probability of high-energy acceleration, we seek a solution of the Klein-Gordon equation in the moving frame in the form of de Broglie waves $R(z) \exp[-i(\varepsilon / \hbar) t]$, where $\varepsilon = m\gamma^* c^2$ is the particle energy and $\gamma^*$ is given by Eq.(1). Let us consider the most interesting case of zero scattering angle $\alpha = 0$. We assume a stationary flow of particles scattered by the potential, and seek the reflected and transmitted fractions with the following asymptotic behavior of the wave function $R_{\pm \rightarrow \pm \infty} \sim \exp(\pm i k_0 z)$, as asymptotic wavenumbers $k_0 = \sqrt{\varepsilon^2 - m^2 c^4 / \hbar^2}$ should be the same for all three (i.e., incident, transmitted and reflected) waves due to the energy conservation. The spatial part of the wave function is then governed by the well-known equation

$$\frac{d^2 R}{dz^2} + \left( k_0^2 - \frac{\varepsilon^2}{h^2 c^2} - A^2(z) \right) R(z) = 0 \tag{5}$$
which is analogous to the case of scattering of the electromagnetic wave by the plasma boundary. The second term in the brackets can be regarded as the square of “photon plasma frequency”, which is proportional to the photon density, i.e., the laser intensity.

Under the condition (2), when the energy of particle is less than the ponderomotive potential, i.e., \( k_0 < eA_0 / \hbar c \sqrt{2} \), where \( A_0 \) is the peak amplitude of the laser vector potential, we find for the reflection probability in the WKB-approximation [11]

\[
R = 1 - \exp \left[ -2 \int_{z_1}^{z_2} \left( \frac{e^2 A^2(z)}{2 \hbar^2 c^2} - k_0^2 \right)^{1/2} dz \right]
\]

(6)

Here, the integration is over a classically prohibited zone, between the turning points \( z_1 \) and \( z_2 \), where the integrand becomes zero. In this case, most of particles, except an exponentially small fraction, are reflected, and consequently, accelerated. In fact, the physical meaning of the integrand is the characteristic inverse “skin depth” of de Broglie waves in the “photon plasma”.

In the opposite case, when the energy is well above the ponderomotive barrier, \( k_0 \gg eA_0 / \hbar c \), the result of perturbation theory is [11]

\[
R = \left| \frac{1}{k_0} \int_{-\infty}^{+\infty} e^{2iA^2(z)/\hbar^2 c^2} \exp(2ik_0 z) dz \right|^2 \sim \left( \frac{eA_0}{\hbar k_0 c} \right)^4 \ll 1
\]

(7)

Expression in the brackets is, in fact, the ratio of the quiver momentum to the initial electron momentum in the moving frame. The transition between these two limiting cases is shown in the Fig.1, in which the dependence of the reflection coefficient \( R \) on the laser peak amplitude is calculated for a specific intensity profile of the laser pulse \( A^2(z) = A_0^2 \cosh^{-2}(z/l^*) \), where \( l^* = \gamma \beta \tau \rho c \) is the characteristic pulse length in the moving frame.

![Figure 1](image.png)

**Figure 1.** The dependence of the scattering probability (12)-(13) on the normalized peak amplitude \( \zeta = (1 + a_0^2 / 2)^{1/2} / \gamma \) for a) \( k_0 l^* = 3 \), b) \( k_0 l^* = 10 \), and c) \( k_0 l^* = 20 \).
TeV Laser Ponderomotive Acceleration Of Electrons In A Plasma

The LPA scheme under discussion makes attainable high energy gains using current laser technologies which makes this scheme a prospect for the next generation of particle accelerators.

Let us assume an electron beam with the initial energy $150 \text{ MeV} \left( \gamma_0 = 300 \right)$ injected into the interaction region in the direction of the laser pulse propagation. Using Eq. (4), one can easily estimate the laser beam and plasma parameters which are necessary to get the GeV-range energy gains. The laser pulse group velocity determines the required plasma density, $n_e [\text{cm}^{-3}] = 1.115 \times 10^{21} \gamma^{-2} \lambda^{-2} [\mu \text{m}]$. The normalized laser field amplitude is determined by the threshold condition (2), and the corresponding laser peak intensity is $I [\text{W/cm}^2] = 1.37 \times 10^{18} a_0^2 \lambda^{-2} [\mu \text{m}]$. The dependencies of the threshold normalized amplitude and laser peak intensity on the final energy of electrons are shown in Fig. 2a ($\lambda \sim 1 \mu \text{m}$), and the required plasma density and Lorentz factor $\gamma^*$, corresponding to the pulse group velocity, are shown in

![Graphs showing the dependencies of threshold laser peak intensity and normalized amplitude, and plasma density and Lorentz factor on energy gain.](image)

**FIGURE 2.** The dependence of the threshold laser peak intensity and normalized amplitude (a), and of the plasma density and the corresponding Lorentz factor $\gamma^*$ (b) on the energy gain. Electrons are injected in the direction of the pulse propagation at 150 MeV the initial energy.

The regime proposed makes possible acceleration of electrons up to the TeV energy range using modern laser technology. For the injection energy $275 \text{ MeV} \left( \gamma_0 = 550 \right)$ and plasma density $n_e \sim 1.115 \times 10^{12} \text{ cm}^{-3} \left( \gamma^* \sim 3.2 \times 10^4 \right)$ the energy gain is $\sim 1 \text{ TeV} \left( \gamma \sim 2 \times 10^6 \right)$. The threshold normalized laser field amplitude required is then $a_0 \sim 42.3$ which corresponds to a laser peak intensity of $\sim 2.47 \times 10^{21} \text{ W/cm}^2$. The plasma density (and consequently the plasma frequency) is sufficiently low, and no plasma instability and relativistic self-focusing can affect the pulse propagation at a reasonable Rayleigh lengths.
X-RAY COMPTON FEL IN THE QUANTUM REGIME

In the x-ray Compton FEL, the high-intensity laser pulse is scattered by counter propagating relativistic electron beam [12], and the high-frequency component is generated as a result of resonant interaction of electrons with the slow ponderomotive wave of frequency $\omega = \omega_i - \omega_s$ and wavenumber $k = k_i + k_s$, $\omega_{x,i}$ and $k_{x,i}$ are frequencies and wave numbers of pumping (i) and signal (s) waves respectively. The dependencies of the emission and absorption probabilities for high-frequency photons in FEL on the beam energy, i.e., laser absorption and emission lines, are thus of resonant nature and situated at the energies $\epsilon_{a,e} = \epsilon_0 \pm \hbar \omega / 2$, where the energy $\epsilon_0 = mc^2 (1 - \omega / kc)^{-2}$ corresponds to the synchronism between the electron beam and ponderomotive wave.

The quantum properties of an FEL are determined by the ratio of the separation $\hbar \omega$ between the absorption and emission lines and their effective width,

$$\eta = \frac{\hbar \omega}{\Delta \epsilon},$$

which in X-ray Compton FEL is mainly determined by the beam energy spread $\Delta \epsilon = mc^2 \Delta \gamma$. In the classical FEL operation regime $\eta \ll 1$, the emission and absorption profiles practically coincide which lead to the classical gain coefficient formula [12,13].

The quantum operation regime emerges under the condition $\eta \gg 1$, when the emission and absorption lines are completely separated, so the state of electron changes dramatically within the scale of $\hbar \omega$. The FEL dynamics reduces to the quantum transitions between these two energy levels, $\epsilon_a$ and $\epsilon_e$. Adequate description of lasing in this regime thus requires the quantum theory and can not be done within the classical electrodynamics [8].

To describe correctly two-level x-ray FEL dynamics in the quantum regime, we will seek the electron wave function as a sequence of de Broglie waves with the momentum $p_n = p_0 + nk$ and energy $\epsilon(p_n) = \left(p_n^2 c^2 + m^2 c^4 \right)^{1/2}$ correspondent to the quasienergy states. The mass of an electron in the field of relativistically intenselaser pulse is $m = m_0 (1 + a^2 / 2)^{1/2}$. $a_1$ is the normalised vector potential. The Klein Gordon equation leads to the following set of equation for amplitudes of these de Broglie waves [13]

$$id_n = g_n a_{n-1} \exp(-i \Delta n t) + g_{n+1} a_{n+1} \exp(i \Delta n+1 t)$$

$$g_n = (\epsilon^2 / \hbar) A_i A_s (\epsilon_{n-1} \epsilon_{n})^{-1/2}$$

Here, $A_{i,s}$ are the vector potential amplitudes of the pumping (i) and x-ray signal (s) coherent waves. Eqs. (9) describes an anharmonic oscillator, which is characterized by the detuning from the resonance, increasing with the number of level.
Under the condition $\eta \gg 1$, all the exponents in (9) with $n \neq 0$ oscillates sufficiently fast during the interaction time, so the excitation of correspondent states is negligible. As a result, when electrons are injected at the optimum energy $\varepsilon = \varepsilon_s$, the Compton FEL becomes a two-level ($n = 0, -1$) quantum oscillator with a completely inverted active medium [8].

Material equations (9) at $n = 0, -1$ joint with the wave equation for the x-ray signal amplitude form the Maxwell - Bloch system of equations, solution to which is usually seeking in the form of coherent oscillations $a_0 = \cos(\chi(z)/2)$, $a_{-1} = \cos(\chi(z)/2)$, with the “pulse area” of the signal

$$\chi(z) = \frac{\mu}{e c h} \int_0^z E_0(z) dz, \quad \mu = e^2 E_0 / m \omega_s^{1/2} \omega_0^{3/2}$$

(11)

here $\mu$ is an effective dipole moment of the two-level system of electrons. The pulse area is guided then by the wave equation which becomes in our case the pendulum equation [8]

$$\frac{\partial^2 \chi}{\partial z^2} = \alpha^2 \sin \chi, \quad \alpha = \left( \frac{2 n e^4 E_0^2}{\hbar \omega_s^2 \omega_0 m^2 e^2} \right)^{1/2}$$

(12)

Note, the linear gain coefficient $\alpha$ contains the Plank constant in explicit form. It means that the regime under consideration has essentially quantum nature and qualitatively differs from all the conventional FEL operation regimes.

**Generation Of Coherent X-ray $\pi$-Pulses In The Compton FEL**

The coherent nonlinear oscillations of electron and signal wave amplitudes lead to the effect of $\pi$-pulse generation, which is well-known in the conventional nonlinear optics. When the total square of the pulse reaches the value $\chi(L) = \pi$, the system becomes in the lower energy state, and the complete removal of inversion emerges.

Using the solution of Eq. (12), one can determine the nonlinear evolution of the signal intensity over the interaction region

$$I_s(z) = d n^{-2} \left( \frac{\alpha c}{\kappa} \right) I_s(0)$$

(13)

where the parameter for the Jacobi function is $\kappa = 1/(1 + I_s(0)/I_b)$, and $I_b = n_b \hbar \omega_s c$. The solution (13) is a periodical function of $z$, the maximum value of which is $I_{s_{\text{max}}} = I_s(0) + I_b$. That is, $I_b$ is the maximum x-ray intensity which can
be emitted by the bunch of electrons, when each “two-level” electron emits one x-ray quanta. The optimum interaction length, corresponding to the maximum of intensity, is determined by the half-period of the Jakobi $dn$-function

$$L_{opt} = K(x)\sqrt{x\alpha^{-1}}$$

(14)

where $K(x)$ is the complete elliptic integral.

The coherent generation of x-ray photons in the quantum FEL can be realized at conditions close to the recent experiment on nonlinear laser-beam Thomson scattering at BNL & KEK [14]. In this experiment, the ~6 keV spontaneous photons were produced in the scattering of 600 MW 200 ps CO$_2$ laser pulses ($\lambda = 10.6\mu m$) by the InC 1ps bunches of 60 MeV electrons. Taking for estimate the electron density in the bunch $\sim 10^{17} \text{ cm}^{-3}$, one can easily find that the coherent interaction effects dominates $\alpha L_{sp} > 1$ (here $L_{sp}^{-1} = (4\pi / 3)(e^2 / hc)\beta^2(1 + a_i^2 / 2)^{-1}$ is the characteristic length of spontaneous scattering [8]) in the domains of normalized laser amplitudes $a_i < 0.5$ and $a_t > 2.4$. At $a_t = 0.5$ ($I_t = 3 \times 10^{15} \text{W/cm}^2$) the linear gain coefficient is $\alpha = 7.6 \text{ cm}^{-1}$, so that the characteristic interaction length can be choosen close to the optimum length $L_{opt}$. In the x-ray $\pi$-pulse each of $\sim 6 \times 10^9$ bunch electrons will emit x-ray quantum coherently, which corresponds to the signal intensity $I_{s, max}$ $\sim 2.9 \times 10^{12} \text{W/cm}^2$. Due to coherence, the divergency of x-ray pulse is determined by the diffraction limit $< 20 \mu \text{rad}$ that lead to the high brightness of the quantum x-ray FEL. Really, the estimate for the photon flux in the spherical angle determined by divergency is $\sim 6 \times 10^{27} \text{ mm}^{-2} \text{ mrad}^{-2} \text{ sec}^{-1}$.

REFERENCES