Particle Acceleration with Strong Field Interactions

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Abstract. A number of concepts of particle acceleration by laser fields have been proposed almost since the beginning of the laser evolution. The recent tremendous progress of ultraintense lasers has created new concepts of high energy particle generation and acceleration due to strong field interactions with matter. In a state of the art of ultra-intense lasers, the highest laser intensities reach a TeV range in terms of the ponderomotive energy exerted on matter. In this paper, the laser-driven particle acceleration concepts are reviewed on basic mechanism of strong field-particle beam and plasma interactions. New concepts of super high energy particle acceleration mechanism based on super-strong laser-plasma interactions are presented.

INTRODUCTION

A number of concepts of particle acceleration by laser fields have been proposed almost since the beginning of the laser evolution. Recently great advances of ultraintense ultrashort pulse lasers have brought about tremendous experimental and theoretical progress in maturity of laser-driven particle accelerator concepts. In particular relativistic electron acceleration mechanism due to ultraintense laser-plasma interaction can be expected to make a compact high energy accelerator. In this context there is a great interest growing in the world-wide community of laser, plasma and accelerator physics. Although an accelerating field limit of conventional accelerators increases as wavelength and pulse duration of driving electric fields decrease, material destruction due to surface heating limits accelerating fields to around 1 GV/m even for optical wavelengths less than 10 μm at a short pulse duration of a few psec[1]. In vacuum, intense focused optical fields can exceed a few TV/m. Such ultrahigh fields have evolved a great deal of particle acceleration concepts by ultraintense laser interaction with particle beams and plasmas.

A novel particle acceleration concept was proposed by Tajima and Dawson[2], which utilizes plasma waves excited by intense laser beam interactions with plasmas for particle acceleration, known as laser-plasma accelerators. In particular recently there has been a great experimental progress on the laser wakefield acceleration (LWFA) of electrons since the first ultrahigh gradient acceleration experiment made by Nakajima et al. [3]. Recent experiments have successfully demonstrated that the self-modulated LWFA mechanism is capable of generating ultrahigh accelerating gradient of ~ 100 GeV/m[4]. In the self-modulated LWFA, however, the maximum energy gain has been limited at most to 100 MeV with energy spread of ~ 100 % because of dephasing and wavebreak-
ing effects in plasmas where thermal plasma electrons are accelerated. The first high energy gain acceleration exceeding 200 MeV has been observed with the injection of an electron beam at an energy matched to the wakefield phase velocity in a fairly underdense plasma[5].

The ponderomotive motion of charged particles in the field of non uniform electromagnetic wave is under investigation for a long time, being initiated by the P. L. Kapitza and P. A. M. Dirac [6]. A new concept of laser ponderomotive acceleration named as "Dirac accelerator" is presented. Particles are accelerated due to direct strong field interactions with laser pulses of which a group velocity is less than the vacuum speed of light. In this regime, the energy gain of particles is determined by the group velocity and does not depend on laser intensity. The laser intensity determines a probability for the particle to be reflected from a leading edge of the pulse, and, thus, the number of particles captured in the acceleration. Here we present some examples of applications of a "Dirac accelerator" concept to superhigh energy particle accelerators with GeV to TeV energies.

The peak amplitude of the transverse electric field of a linearly polarized laser pulse is given by

$$E_L [TV/m] \approx 2.7 \times 10^{-9} l^{1/2} [W/cm^2] \approx 3.2a_0 / \lambda_0 [\mu m],$$  \hspace{1cm} (1)

where $I$ is the laser intensity, $\lambda_0$ is the laser wavelength, and $a_0$ is the laser strength parameter defined by $a_0 \equiv eA_0 / mc^2$ in terms of the peak amplitude of the laser vector potential $A_0$ and the electron rest energy $mc^2$. Using the laser peak intensity $I = cE_L^2 / 8\pi = c\kappa A_0^2 / 8\pi$, the laser strength parameter is given by

$$a_0 = (2e^2 \lambda_0^2 l / \pi\phi^2 c^5)^{1/2} \approx 0.85 \times 10^{-9} \lambda_0 [\mu m] l^{1/2} [W/cm^2].$$  \hspace{1cm} (2)

Physically $a_0$ is equal to the normalized momentum of the electron quiver motion in the laser field. In a strong laser field, an electron absorbs energy and momentum from the wave to cause mass shift from $mc$ to $mc^2\gamma_L$, where $\gamma_L = (1 + a_0^2 / 2)^{1/2}$. The effective potential of the electron inside the laser field for $a_0 << 1$ is

$$U_{eff} = mc^2 (1 + a_0^2 / 2)^{1/2} \approx mc^2 + mc^2 a_0^2 / 4$$  \hspace{1cm} (3)

In the nonrelativistic regime, the ponderomotive force or the field gradient force can be defined as

$$F = -\nabla U_{eff} \approx -mc^2 a_0^2 / 4.$$  \hspace{1cm} (4)

The laser pulse propagating in underdense plasmas expels plasma electrons exerted by the ponderomotive force to excite plasma waves. In this regime particle acceleration with strong laser fields is mainly attributed to the laser wakefield acceleration mechanism. In the highly relativistic regime for $a_0 >> 1$, if the initial momemtum of a free electron is smaller than the quiver momentum, the electron is reflected from the laser pulse. The reflection of particles from the laser pulse moving in plasmas results in their acceleration. This mechanism is the basis of the "Dirac accelerator".
LASER ACCELERATION IN VACUUM

Lawson-Woodward theorem

Particle acceleration in vacuum can eliminate the difficulties associated with gas and plasmas where the accelerating field is limited due to the gas breakdown and the plasma wave-breaking besides suffering from beam-gas and plasma collisions and laser-plasma instabilities[7]. A major shortcoming of laser-vacuum acceleration is attributed to the phase velocity of the electric field in the accelerated direction of particles greater than the vacuum light velocity \(c\) for a focused laser beam.

Assuming the propagation of a nearly plane wave, the electric field of the Hermite-Gaussian TEM\(^{l_m}\) mode is given by[8]

\[
E_{l,m}(x,y,z) = E_0 \frac{r_0}{w(z)} H_l \left( \frac{\sqrt{2}x}{w(z)} \right) H_m \left( \frac{\sqrt{2}y}{w(z)} \right) \exp \left[ -\frac{r^2}{w^2(z)} \right] \exp(i\psi),
\]

where the phase of the transverse field \(E_{l,m}\) is

\[
\psi = kz - \omega t + z^2/(Z_R w^2(z)) - (l+m+1)\tan^{-1}(z/Z_R) + \phi_0,
\]

\(E_0\) is the maximum field amplitude, \(w(z) = r_0[1 + (z/Z_R)^2]^{1/2}\) is the laser spot radius, \(r_0\) is the minimum spot radius at focus, and \(Z_R = \pi r_0^2/\lambda_0\) is the Rayleigh length, i.e. the distance over which the spot size expands to \(\sqrt{2}r_0\), \(\lambda = 2\pi/k\) is the wavelength, \(\omega = \omega k\) is the frequency, \(k\) is the wavenumber, \(r = \sqrt{x^2 + y^2}\), \(H_l\) is the Hermite polynomial of order \(l\), \(H_0(\xi) = 1\), \(H_1(\xi) = 2\xi\), \(H_2(\xi) = 4\xi^2 - 2\), \ldots, and \(\phi_0\) is a constant.

Since a Gaussian laser field propagating in the \(z\) direction is transversely bounded, the finite longitudinal component of the electric field is obtained from \(\nabla \cdot \mathbf{E} = 0\), as \(E_z = -(1/ik)\nabla \cdot \mathbf{E}_\perp\) that can accelerate electrons in the \(z\) direction. The phase velocity of the electric field in the propagation direction is obtained from \(d\psi/dt = 0\). Along the propagation axis \(r = 0\), the phase velocity is

\[
v_{ph} = c \left[ 1 - \frac{l+m+1}{kZ_R(1 + z^2/Z_R^2)} \right]^{-1} > c.
\]

Therefore relativistic electrons with the longitudinal velocity \(v_z \approx c\) will slip in phase with respect to the accelerating field \(E_z\) and eventually decelerate. Acceleration occurs over a slippage distance \(Z_s\), defined by \(kZ_s v_{ph} - v_z \approx \pi\), which gives \(Z_s \approx \pi Z_R\). If an highly relativistic electron interacts over an infinite region (\(z = -\infty\) to \(\infty\)), a net energy gain is zero resulting from deceleration canceling out acceleration. This is pointed out by the Lawson-Woodward theorem assuming that the region of interaction is infinite, no static electric or magnetic field is present, and nonlinear effects (e.g., ponderomotive, and radiation reaction forces) are neglected[9]. In order to make a nonzero net energy gain, one or more of these assumptions must be violated.
Laser Beat Wave Accelerator

The laser beat wave accelerator is based on the nonlinear ponderomotive force[10]. Two laser beams of different wavelengths, $\lambda_1$ and $\lambda_2$, are copropagated in the presence of an injected electron beam. Properly phased electrons travelling along the same axis as the two laser beams undergo an axial acceleration from the beat term in the $v \times B$ force. The total laser field is represented by the vector potential,

$$A(z, r, t) = A_1(z, r, t) + A_2(z, r, t),$$

where $A_1$ and $A_2$ represent laser 1 and 2, respectively, and the circularly polarized laser fields are given by

$$A_i(z, r, t) = \frac{r_{0i}}{w_i(z)} \exp \left[ -\frac{r^2}{w_i^2(z)} \right] \left[ \cos(\psi_i)\mathbf{e}_x + \sin(\psi_i)\mathbf{e}_y \right],$$

where $i = 1, 2$ denotes the laser beam, $w_i(z) = r_{0i}(1 + z^2/Z_{Ri}^2)^{1/2}$, $Z_{Ri} = \pi r_{0i}/\lambda_i$, and

$$\psi_i = k_i z - \omega_i t + r^2(z/Z_{Ri})/w_i^2(z) - \tan^{-1}(z/Z_{Ri}) + \phi_{0i}.\tag{10}$$

The relativistic Lorentz force equation can be written as

$$du/dt = da/dt - (cu/\gamma) \times (\nabla \times a),$$

where $u = p/m_e c$ is the normalized electron momentum $\gamma = (1 + u^2)^{1/2}$ is the Lorentz factor, and where $a = a_1 + a_2$, $a_{1,2} = |e|A_{1,2}/m_e c^2$ are the normalized vector potentials. The electron energy equation is given by

$$d\gamma/dt = (u/\gamma) \cdot \partial a/\partial t.$$

In the one dimensional limit, i.e. $\lambda_j/r_{0i} \ll 1$, the transverse canonical momentum is approximately conserved, i.e. $d/dt(u_\perp - a_\perp) = 0$. Hence using $u_\perp = a_\perp$, the energy equation is described by

$$d\gamma/dz = \frac{1}{2\gamma} \partial a^2_\perp/\partial t + (u_\perp/\gamma) \partial a_\perp/\partial t,$$

where $a^2_\perp = a_\perp \cdot a_\perp$ is given by

$$a^2_\perp = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\psi_2 - \psi_1),$$

and $a_i = (a_{0i} r_{0i}/w_i) \exp(-r^2/w_i^2)$. Assuming $r_{0i} \gg \lambda_i$, the axial field $a_z$ is given by $\nabla \cdot a = 0$,

$$a_z = \sum_{i=1,2} \frac{2a_i}{k_i w_i^2} \left[ (x - \bar{z}_i y) \sin \psi_i - (\bar{z}_i x + y) \cos \psi_i \right],$$

where $\bar{z}_i = z/Z_{Ri}$. The electron energy is obtained from the equation

$$dy/dz = \frac{\hat{a}_i \hat{a}_z}{\gamma \beta_e} \Delta k \sin(\psi_2 - \psi_1) - \sum_{i=1,2} \frac{2\hat{a}_i}{w_i^2} \left[ (x - \bar{z}_i y) \cos \psi_i + (\bar{z}_i x + y) \sin \psi_i \right],$$

$$401.$$
where $\omega_2 - \omega_1 = \Delta \omega = c \Delta k > 0$ and nonresonant (slowly varying) terms proportional to $a_0^2$ are neglected. The effective accelerating gradient is inversely proportional to the electron energy. The transverse orbits are calculated by

$$\frac{dx}{dz} \simeq \frac{1}{\gamma^2 \beta_2} \sum_{i=1,2} \beta_i \cos \psi_i, \quad \frac{dy}{dz} \simeq \frac{1}{\gamma^2 \beta_2} \sum_{i=1,2} \beta_i \sin \psi_i,$$

(17)

where $\gamma = \gamma_1 \gamma_2$, $\gamma_2 = (1 - \beta_2^2)^{-1/2}$, $\beta_2 = v_z/c$, $\gamma_1 = (1 + a_1^2)^{1/2}$, and $t = \int dz/v_z$.

Along the axis $r = 0$ and near the focus $|z| < Z_{R1}$, the phase velocity of the accelerating field is

$$v_{ph} = (1 + 1/\Delta k Z_{R1} - 1/\Delta k Z_{R2})^{-1} \simeq 1 - (1 - Z_{R1}/Z_{R2})/(\Delta k Z_{R1}),$$

(18)

which is less than $c$ for $Z_{R2} > Z_{R1}$ and can be controlled by choosing the laser spot sizes. Therefore the acceleration distance is limited not by the slippage distance but by the diffraction range to be approximately two Rayleigh lengths. Setting $x = y = 0$, $t = z/c$ and $Z_{R} = Z_{R1} = Z_{R2}$, the electron energy can be found by

$$W_{F} = W_{I} - 2a_0 a_2 a_0 Z_{R} \sin(\phi - \phi_0)(\tan^{-1} z_{F} - \tan^{-1} z_{I}),$$

(19)

where $W_{F}(W_{I})$ is the final (initial) electron energy. In an infinite interaction region $z_{I} = -\infty$ and $z_{F} = -\infty$, it is $\gamma_{F}^2 - \gamma_{I}^2 = 2 \pi a_0 a_2 a_0 Z_{R}$, assuming $\sin(\phi - \phi_0) = 1$. In terms of $P_{1}(TW) = 0.043(a_0 r_{01}/\lambda_1)^2$ for $a_0 = a_2$, the electron energy is given by

$$W_{F}[\text{MeV}] = [W_{I}[\text{MeV}] + 750(\lambda_1/\lambda_2 - 1) P_{1}[TW]]^{1/2},$$

(20)

where $W_{F}(W_{I})$ is the final (initial) electron energy and $W_{I} \gg m_e c^2$. As an example, for $W_{I} = 20 \text{ MeV}$, $\lambda_1 = 2 \lambda_2 = 1 \mu m$, and $P = 10 \text{ TW}$, the final electron energy is $W_{F} = 89 \text{ MeV}$.

In the laser beat wave accelerator, an attainable maximum energy is limited by radiative losses caused by transverse quiver oscillations when electrons interact with laser fields. The power radiated by a single electron is given by the relativistic Larmor formula as

$$P_{R} \approx (2/3) r_e m_e c^3 \gamma^2 a_0^2 k^2 (1/(2 \gamma_0^2) + 1/(k Z_{R}))^2,$$

(21)

where $r_e = e^2/m_e c^2$ is the classical electron radius. Including this term, the electron energy equation is

$$d\gamma/dz \approx a_0 a_2 \Delta k / \gamma - (2/3) r_e \gamma^2 (a_0^2 Z_{R1}^2 + a_2^2 Z_{R2}^2),$$

(22)

where $k Z_{R} / 2$ is assumed. The maximum value of $\gamma$ is given by setting $d\gamma/dz = 0$. Assuming $\Delta k = k_1$, $a_0 = a_2$, and $Z_{R1} = Z_{R2}$, the maximum electron energy is given by

$$W_{\text{max}} \approx m_e c^2 \left(\pi r_0 / \lambda_1 \right)^{(3r_0 / 2r_e)^{1/3}}.$$

(23)

In terms of the practical unit, $W_{\text{max}} \approx 1.3 (r_0^{4/3} / \lambda_1)$, where $W_{\text{max}}$ is in GeV and $r_0$ and $\lambda_1$ are in $\mu m$. As an example, for $r_0 \approx 1 \mu m$, $W_{\text{max}} \approx 1.3 \text{ GeV}$.
LASER ACCELERATION IN PLASMAS

Plasma wave excitation by laser-plasma interaction

Plasmas provide some advantages as an accelerating medium in laser-driven accelerators. Plasmas can sustain ultrahigh electric fields, and can optically guide the laser beam and the particle beam as well under appropriate conditions. For a nonrelativistic plasma wave, the acceleration gradients are limited to the order of the wave-breaking field given by

$$eE_0[\text{eV/cm}] = m_e c \omega_p \approx 0.96 n_0^{1/2}[\text{cm}^{-3}],$$

(24)

where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency and $n_0$ is the ambient electron plasma density. It means that the plasma density of $n_e = 10^{18} \text{ cm}^{-3}$ can sustain the acceleration gradient of 100 GeV/m.

The dispersion relation of the transverse electromagnetic wave in a plasma is given by

$$\omega^2 = \omega_p^2 + c^2 k^2.$$  

(25)

The phase velocity and the group velocity of the electromagnetic wave in a plasma are

$$v_{ph} = \frac{\omega}{k} = c \sqrt{1 - \omega_p^2/\omega^2},$$

$$v_g = \frac{\partial \omega}{\partial k} = c \sqrt{1 - \omega_p^2/\omega^2}.$$  

(26)

(27)

An index of refraction is defined as $\eta = ck/\omega = \sqrt{1 - \omega_p^2/\omega^2}$. If $\omega > \omega_p$, an electromagnetic wave packet (a laser pulse) can propagate in a plasma with the group velocity $v_g$. This wave, however, can not efficiently accelerate particles because of the phase velocity $v_{ph} > c$. When $\omega < \omega_p$, the electromagnetic wave becomes evanescent with an imaginary refractive index.

The dispersion relation for the electron plasma wave is $\omega = \pm \omega_p$, i.e. dispersionless for a cold plasma wave. Assuming the wave numbers $k \ll k_D = \omega_p/\nu_T$, where $k_D$ is the Debye wave number and $\nu_T = k_B T/m_e$ is the thermal velocity, the phase and group velocities of the plasma wave are:

$$v_{ph} = \omega_p/k \gg \nu_T, v_g = \partial \omega/\partial k = 0.$$  

(28)

This plasma wave will not propagate, but merely oscillates at the frequency $\omega = \omega_p$. In the plasma waves, it is possible to make the phase velocity lower than $c$.

In laser-driven plasma-based accelerator, plasma waves are driven by the ponderomotive force arising from the $-e(\mathbf{v} \times \mathbf{B})/c$. In a cold fluid limit of a plasma, the momentum equation of electron fluid is

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} = -e(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B})$$

(29)

The electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields of the laser can be written as

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A},$$

(30)

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where $\mathbf{A}$ is the vector potential of the laser. For the linearly polarized laser, the vector potential is written as

$$\mathbf{A} = A_0 \cos(kz - \omega t) \mathbf{e}_z$$

(31)

In the linear limit $|\alpha| = eA_0/mc^2 \ll 1$, letting $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1$, the leading order equation of motion $m\ddot{\mathbf{v}}_0/\dot{t} = -e\mathbf{E}$ gives the quiver velocity, $\mathbf{v}_0 = e\mathbf{A}/mc = c\mathbf{a}$. The second-order motion is given by

$$m_e \frac{\partial \mathbf{v}_1}{\partial t} = -m_e[(\mathbf{v}_0 \cdot \nabla)\mathbf{v}_0 + c\mathbf{v}_0 \times (\nabla \times \mathbf{a})] = -m_e c^2 \nabla(a^2/2).$$

(32)

$\mathbf{F}_p = -m_e c^2 \nabla(a^2/2)$ is the ponderomotive force defined by averaging the nonlinear force over $2\pi/\omega_0$, exerted on plasma electrons by a laser field with frequency $\omega_0$, and the ponderomotive potential $\phi_p = m_e c^2 (a^2/2)$ can be defined. Using the linearized Poisson’s equation and the continuity equation, i.e.

$$\nabla \cdot \mathbf{E} = -4\pi e n_e, \quad \frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v} = 0,$$

(33)

excitation of the plasma wave is described by

$$\left(\frac{\partial^2}{\partial t^2} + \alpha_p^2\right) \frac{n_1}{n_0} = \frac{c^2}{2} \nabla^2 a^2,$$

(34)

where $a^2 \ll 1$ is the normalized intensity of the driving laser beam, and $n_1/n_0 \ll 1$ is the perturbed density of the plasma wave. If the electric field generated by the plasma wave is written by $\mathbf{E} = -\nabla \phi$, where $\phi$ is the electrostatic potential, Eq. (34) leads to

$$\left(\frac{\partial^2}{\partial t^2} + \alpha_p^2\right) \phi = \alpha_p^2 m_e c^2 a^2/2e.$$

(35)

**Laser Wake-Field Accelerator**

As an intense laser pulse propagates through an underdense plasma, $\alpha_p^2/\omega_0^2 \ll 1$, the ponderomotive force associated with the laser pulse envelope expels electrons from the region of the laser pulse. This effect excites a large amplitude plasma wave (wakefield) with phase velocity approximately equal to the group velocity of laser pulse. Assuming that all of the axial and time dependencies can be expressed as a function of a single variable $\zeta = z - v_p t$ with a phase velocity $v_p$ of the plasma wave, a simple-harmonic oscillator equation for the wake potential $\phi$ is

$$\frac{\partial^2 \phi}{\partial \zeta^2} + k_p^2 \phi = k_p^2 m_e c^2 a^2(n, \zeta),$$

(36)

where $k_p = \omega_p/v_p$ is the plasma wave number. The phase velocity of the plasma wave is equal to the group velocity of the laser pulse in a plasma, given by $v_p = c(1 - \alpha_p^2/\omega_0^2)^{1/2}$. 

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Consider laser wakefields driven by a circularly polarized laser pulse with a normalized intensity profile given by $|a(r, \zeta)| = a_0 \exp\left(-\frac{r^2}{r_0^2} - \frac{\zeta^2}{2\sigma_z^2}\right)$, where $\sigma_z$ is the temporal 1/e half-width of the pulse and $r_0$ is the spot size. The axial and radial wakefields are given by

$$eE_z(r, \zeta) = \sqrt{\frac{\pi}{4}} m_e c^2 k_p^2 \sigma_z a_0^2 \exp\left(-\frac{2r^2}{r_0^2} - \frac{k_p^2 \sigma_z^2}{4}\right)[C(\zeta) \cos k_p \zeta + S(\zeta) \sin k_p \zeta],$$

$$eE_r(r, \zeta) = -\sqrt{\frac{\pi}{4}} m_e c^2 k_p \sigma_z a_0^2 \exp\left(-\frac{2r^2}{r_0^2} - \frac{k_p^2 \sigma_z^2}{4}\right)[C(\zeta) \sin k_p \zeta - S(\zeta) \cos k_p \zeta],$$

where $C(\zeta) = 1 - \text{Re}\{\text{erf}(\zeta/\sigma_z - ik_p\sigma_z/2)\}$, and $S(\zeta) = -\text{Im}\{\text{erf}(\zeta/\sigma_z - ik_p\sigma_z/2)\}$. For $\zeta \ll \sigma_z$, i.e. the region away behind the laser pulse, $C(\zeta) \to 2$ and $S(\zeta) \to 0$. Hence the radial wakefield is zero along the axis and its phase is shifted by $\pi/2$ from that of the axial wakefield. An electron displaced from the axis will experience simultaneous axial accelerating and radial focusing forces in a phase region of $\Delta \zeta \approx \pi/4$.

The maximum accelerating gradient is achieved at the plasma wavelength $k_p = \pi \sigma_z$ as $(eE_z)_{\text{max}} = 2\sqrt{\pi} e^{-1} m_e c^2 a_0^2/\sigma_z$. For linear polarization, $a_0^2$ is replaced with $a_0^2/2$ in all equations describing wakefields. In practical units, the maximum axial wakefield occurs at the plasma wavelength, $\lambda_p [\mu\text{m}] \approx 0.57 \tau$ in a plasma with the resonant electron density, $n_0 [\text{cm}^{-3}] = 1/(\pi r_e^2 \sigma_z^2) \approx 3.5 \times 10^{21} / \tau^2$ in terms of a FWHM pulse duration $\tau$ [fs], where a FWHM pulse width is given by $c\tau = 2\sqrt{\ln 2} \sigma_z$. When a Gaussian driving laser pulse with the peak power $P$ [TW] is focused on the spot size $r_Q [\mu\text{m}]$, the maximum axial wakefield yields $(eE_z)_{\text{max}}[\text{GeV/m}] \approx 8.6 \times 10^4 P \lambda_0^2/(\gamma_L \tau [\text{fs}]),$ where $\gamma_L = (1 + a_0^2/2)^{1/2}$ takes account of nonlinear relativistic effects, and $a_0 = 6.8 \lambda_0 P^{1/2}/r_0$ for the linear polarization.

### Acceleration Energy Gain

Assuming a Gaussian beam propagation of the laser pulse with the peak power $P$ in an underdense plasma ($\omega \gg \omega_p$), the effective acceleration length can be limited to a diffraction length $L_{df} = \pi Z_R$. For a properly phased electron, the maximum energy gain is given by

$$\Delta W_{df}[\text{GeV}] \approx 0.85 P[\text{TW}] \lambda_0 [\mu\text{m}]/(\gamma_L \tau [\text{fs}]).$$

Note that the maximum energy gain is independent of the focusing property of the laser beam due to diffraction effects in the limit of $a_0^2 \ll 1$. For example, the maximum energy gain of the LWFA driven by the laser pulse with $\lambda_0 = 0.8 \mu\text{m}$, $P = 2$ TW, $\tau = 100$ fs, and $r_0 = 10 \mu\text{m}$ is limited to $\Delta W_{df} = 12$ MeV by the diffraction length $L_{df} = 1.2$ mm.

An electron can be accelerated along the z-axis by an electrostatic plasma wave of the form $E_z = E_{z_{\text{max}}} \sin \omega_0 (z/v_{ph} - t)$. As the electron is accelerated, its velocity $v_z$ will increase and approach the speed of light, $v_z \to c$. If the phase velocity of the plasma wave is constant with $v_{ph} < c$, the electrons will eventually outrun the accelerating phase and move into the decelerating phase. This dephasing effect in the plasma wave limits the energy gain of the electron. The dephasing length is defined as the length the electron

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must travel before its phase slips by one-half of a period with respect to the plasma wave. For a highly relativistic electron with \( v \approx c \), the dephasing length is given by

\[
L_d = \frac{\lambda_p}{2}\left(1 - \frac{v_{ph}}{c}\right) \approx \lambda_p \left(\frac{\omega_0^2}{\omega_p^2}\right) = \lambda_p \gamma_p^2, \tag{40}
\]

where \( \gamma_p = \left(1 - \frac{v_{ph}^2}{c^2}\right)^{-1/2} \) is the relativistic factor associated with the phase velocity of the plasma wave. The energy gain of a highly relativistic electron is obtained from integrating \( eE_z \) over the acceleration length \( L_d \). The maximum energy gain is given by

\[
\Delta W_d = \frac{2}{\pi} eE_{z\text{max}} L_d \approx \frac{2}{\pi} eE_{z\text{max}} \lambda_p t_p^2. \tag{41}
\]

For the optimum plasma condition, \( \lambda_p = \pi \sigma_z \), the dephasing length is given by

\[
L_d[\text{cm}] = 0.18 \times 10^{-4} \tau^3[\text{fs}] \gamma_p^2/\lambda_0^2[\mu\text{m}], \tag{42}
\]

and the maximum energy gain is

\[
\Delta W_d[\text{GeV}] = 0.01 P[\text{TW}] \tau^2[\text{fs}] / r_0^2[\mu\text{m}]. \tag{43}
\]

For example, the maximum energy gain of the LWFA driven by the laser pulse with \( \lambda_0 = 0.8\mu\text{m} \), \( P = 2 \text{ TW} \), \( \tau = 100 \text{ fs} \), and \( r_0 = 10 \mu\text{m} \) is limited to \( \Delta W_d = 2 \text{ GeV} \) by the dephasing length \( L_d = 32 \text{ cm} \).

In the laser-plasma accelerator, as the laser driver excites a plasma wave, it loses energy. The pump depletion length \( L_{pd} \), in which the laser pulse loses a half of its total energy to excite plasma waves, is estimated by equating the laser pulse energy to the energy left behind in the wakefield, \( E_z^2 L_{pd} = (1/2)E_L L \), where \( E_z \) is the laser field. For a Gaussian laser pulse, the pump depletion length is given by

\[
L_{pd} = \frac{8}{\sqrt{\pi}} \frac{\gamma_p^2}{\sigma_z^2} \exp \left(\frac{k_p^2 \sigma_z^2}{2}\right), \tag{44}
\]

For the optimum condition, \( \lambda_p = \pi \sigma_z \), the pump depletion length is \( L_{pd} \approx 2.65 \lambda_p \gamma_p^2 \sigma_z^{-2} \). Note that if the laser strength parameter is \( a_0 < 1.6 \), the pump depletion length is larger than the dephasing length, \( L_d < L_{pd} \). The energy gain limited by the pump depletion is given by

\[
\Delta W_{pd} = eE_z L_{pd} = 2m_e c^2 \gamma_p^2 \exp(k_p^2 \sigma_z^2/4). \tag{45}
\]

For the optimum plasma condition, \( \lambda_p = \pi \sigma_z \), the pump depletion length is given by

\[
L_{pd}[\text{m}] = 1.06 \times 10^{-8} \tau^3[\text{fs}] \gamma_p^2/\lambda_0^2[\mu\text{m}]/(\lambda_0^4[\mu\text{m}] P[\text{TW}]), \tag{46}
\]

and the maximum energy gain is

\[
\Delta W_{pd}[\text{GeV}] = 0.91 \times 10^{-3} \tau^2[\text{fs}] \gamma_p^2/\lambda_0^2[\mu\text{m}]. \tag{47}
\]

For example, the maximum energy gain of the LWFA driven by the laser pulse with \( \lambda_0 = 0.8\mu\text{m} \), \( P = 2 \text{ TW} \), \( \tau = 100 \text{ fs} \), and \( r_0 = 10 \mu\text{m} \) is limited to \( \Delta W_{pd} = 18.5 \text{ GeV} \) by the pump depletion length \( L_{pd} = 1.9 \text{ m} \).
TABLE 1. The design parameters of the capillary-guided laser wakefield accelerators.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.5</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy gain $\Delta W$ [GeV]</td>
<td>0.5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Pulse duration $T$ [fs]</td>
<td>20</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Peak power $P$ [TW]</td>
<td>100</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Spot radius $r_0$ [$\mu$m]</td>
<td>30</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Laser strength $a_0$</td>
<td>1.8</td>
<td>1.7</td>
<td>2.4</td>
</tr>
<tr>
<td>Plasma density $n_e$ [cm$^{-3}$]</td>
<td>$8.8 \times 10^{18}$</td>
<td>$1.4 \times 10^{18}$</td>
<td>$3.5 \times 10^{17}$</td>
</tr>
<tr>
<td>Accelerating gradient $eE_{\text{max}}$ [GeV/cm]</td>
<td>1.9</td>
<td>0.7</td>
<td>0.55</td>
</tr>
<tr>
<td>Diffraction length $L_{\text{dif}}$ [cm]</td>
<td>1.1</td>
<td>0.5</td>
<td>0.12</td>
</tr>
<tr>
<td>Dephasing length $L_d$ [cm]</td>
<td>0.4</td>
<td>5.5</td>
<td>56</td>
</tr>
<tr>
<td>Capillary length $L_c$ [cm]</td>
<td>No</td>
<td>1.5</td>
<td>10</td>
</tr>
<tr>
<td>$N_{\text{max}}$ [$10^9$]</td>
<td>7</td>
<td>1.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

In order to achieve the acceleration energy gains higher than 1 GeV in a single stage of cm-scale, it is necessary to extend the acceleration length limited by diffraction effects of laser beams. We propose the capillary-guided laser wakefield accelerators in which both the driving laser pulses and particle beams can be guided through the capillary discharge plasmas of cm-scale[12]. The design parameters to test electron acceleration of GeV energies are shown in Table 1. The design of the laser wakefield accelerators is based on availability of the 10 Hz table-top ultrashort, ultrahigh peak power Ti:Sapphire laser with 20 fs and 100 TW developed by JAERI-KANSAI[13]. The parameters are optimized according to the criteria described in ref. [14] so as to maximize the energy gain in the single-stage acceleration under the conditions that the wakefield amplitudes are less than the relativistic wave-breaking field and that the longitudinal wakefield should be larger than the transverse field. The energy gain of 5 GeV will be achieved with a 10 cm long capillary plasma waveguide driven by a 20 TW, 100 fs laser pulse in the plasma density of $n_e = 3.5 \times 10^{17}$ cm$^{-3}$ without wave-breaking. The maximum number of electrons capable of accelerating with 100% energy spread is estimated to be $N_{\text{max}} \sim 3.55 \times 10^9 P \lambda_0^2/(\tau \gamma_0^2)[15]$.

PONDEROMOTIVE ACCELERATION IN PLASMAS

Ponderomotive potential scattering

Let us consider the ultraintense ultrashort laser pulse propagating in a plasma with the electron density $n_e$ and the plasma frequency $\omega_p$. In the reference frame that is moving with the group velocity $v_g = c(1 - \omega_p^2/\omega^2)^{1/2} < c$ of the laser pulse along the $z$ axis. In the reference frame, since the laser ponderomotive potential becomes static, the interaction can be considered as the elastic potential scattering. Let the initial energy and velocity of the particle $\gamma_0$ and $\beta_0$, respectively, and the initial angle of collision is $\theta_0$. The Lorentz transformation gives the energy of the particle in the reference frame:

$$\gamma' = (1 - \beta_0 \cos \theta_0) \gamma_0 \gamma_0,$$  \hspace{1cm} (47)
where \( \gamma_g = (1 - \beta_g^2)^{-1/2} \), and \( \beta_g = v_g/c \). According to the classical scattering theory, if this energy is less than the field peak ponderomotive potential, i.e.

\[
\gamma^* \leq \gamma_e = (1 + \alpha_0^2/2)^{1/2},
\]

the particle must be reflected. As an effective Hamiltonian is time-independent in the reference frame, the final energy \( \gamma \) of the particle does not change after scattering, and after the Lorentz transformation, we have in the laboratory frame

\[
\gamma_g \gamma(1 - \beta_g \beta \cos \theta) = \gamma^*.
\]

In this relation, \( \beta \) is the particle velocity, and \( \theta \) is the final scattering angle, where \( \theta = 0 \) corresponds to the co-propagation case. From these equations, the following conservation law is given by

\[
\gamma(1 - \beta_g \beta_z) = \gamma_0(1 - \beta_g \beta_{0z}) = \text{const.}
\]

This result of simple kinematics approach coincides with that of electrodynamics consideration[16, 17]. The maximum final energy of particle is given for scattering at \( \theta = 0 \);

\[
\gamma = [(\beta_g - \beta_0)^2 + \gamma_0^2]^{1/2} \gamma_0.
\]

In a quantum mechanical analysis, the value of ponderomotive potential determines the probability of scattering. To estimate its value, one can start with Klein-Gordon equation, as the spin effects are evidently not important for the scattering. It is convenient to consider the problem in the reference frame, where the potential is time-independent. We are interested in the backscattering, i.e. scattering at \( \theta = 0 \), and, consequently, one can use the 1D approximation. In the Lorentz gauge and paraxial ray approximation, the vector potential \( A(z) \) is the transverse one, and the Klein-Gordon equation is reduced to the following equation

\[
\frac{\partial^2 \psi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \frac{m^2 c^2}{\hbar^2} (1 + \frac{e^2 A^2(z)}{m^2 c^4}) \psi = 0.
\]

Here and below we omit (*) in the coordinates of the reference frame. As the potential is time-independent, the energy of the particle is conserved. One can obtain the solution in the form

\[
\psi(z,t) = R(z) \exp(-i\epsilon t / \hbar),
\]

where \( \epsilon \) is the energy of a particle with the mass \( m \) in the reference frame, \( \epsilon = mc^2 \gamma^* \). Assuming that the stationary flow of particles collides with the potential, we obtain reflected and transmitted parts of this flow. The correspondent asymptotic behavior of solution must be the following

\[
R(z \to \infty) = \exp(-ik_0 z) + a \exp(ik_0 z), \quad R(z \to -\infty) = b \exp(-ik_0 z),
\]

where \( a \) and \( b \) are the reflection and transmission amplitudes, normalized to the amplitude of the incident flow. As the Klein-Gordon current is given by

\[
j = -i \frac{\hbar}{2mc^2} (\psi \nabla \psi^* - \psi^* \nabla \psi),
\]
and asymptotic wavenumbers are the same for all three waves, the absolute values $|a|^2$ and $|b|^2$ are the reflected and transmitted fractions of the incident flow, respectively. The asymptotic wavenumber $k_0 = \sqrt{\epsilon^2 - \frac{m^2c^4}{\hbar c}}$ corresponds to the field-free momentum of an electron of a given energy. The spatial part of the wave function is governed by

$$\frac{d^2 R}{dz^2} + \left(k_0^2 - \frac{e^2}{\hbar^2 c^2} A^2(z)\right) R(z) = 0$$ (56)

The solution is obtained by considering a specific intensity profile of laser pulse, such as the modified Pöschl-Teller potential[18], $A^2(z) = A_0^2 \cosh^{-2}\left(\frac{z}{\tau_p}\right)$, where $\tau_p = \gamma_0 \beta_p c \tau_p$ is the characteristic pulse length in the reference frame for the pulse duration $\tau_p$ in the laboratory frame. Then the reflection and transmitted probabilities are calculated by

$$|a|^2 = \frac{1}{1 + \Delta^2}, \quad |b|^2 = \frac{\Delta^2}{1 + \Delta^2}, \quad \Delta = \frac{\tanh \delta_+ + \tanh \delta_-}{1 - \tanh \delta_+ \tanh \delta_-},$$ (57)

where the characteristic phases $\delta_\pm$ are determined by the relation between the particle momentum and the ponderomotive potential as

$$\delta_\pm = \frac{\pi}{2} \left( k_0 l^* \pm \left( \frac{e^2 A_0^2 l^*}{\hbar^2 c^2} - \frac{1}{4} \right)^{1/2} \right).$$ (58)

For an incident particle energy higher than the laser ponderomotive potential barrier, these large positive phases result in a large $\Delta$ so that the reflection probability vanishes, whereas the transmission probability is close to 1. When the ponderomotive potential exceeds the particle energy, $\delta_+$ becomes negative to lead to a small $\Delta$ as the laser intensity increases. Then the reflection probability rapidly reaches unity as shown in Fig. 1. In this regime the ponderomotive scattering off the laser pulse works as a particle acceleration mechanism which we call as "Dirac Accelerator".
The ponderomotive potential scattering results in acceleration of electrons via a point-like interaction with the strong laser fields. This acceleration length will be at most a half of the laser pulse length when the reflection condition Eq. (48) is satisfied. Let us consider acceleration of an electron from the injection energy $\gamma_0$ to the accelerated final energy $\gamma$ via the ponderomotive interaction with the laser field in a plasma with the density $n_e$. The required group velocity and the corresponding Lorentz factor of the laser pulse are given by

$$\beta_g = \frac{\gamma_0 \beta_0 + \gamma \beta}{\gamma_0 + \gamma}, \quad \gamma_g = \frac{\gamma_0 + \gamma}{\sqrt{2[1 + \gamma \gamma_0 (1 - \beta \beta_0)]}^{1/2}}.$$ (59)

The plasma density can be determined as $n_e [\text{cm}^{-3}] \approx 1.115 \times 10^{21} \gamma_0^{-2} \lambda_0^{-2}$. For this condition of "Dirac accelerator", the minimum laser field is given by

$$a_0 = \sqrt{2[\gamma_0^2 \gamma_0^2 (1 - \beta_0 \beta_0)^2 - 1]^{1/2}}.$$ (60)

The corresponding laser intensity is $I [\text{W/cm}^2] = 1.37 \times 10^{18} \lambda_0^{-2} \mu \text{m} a_0^2$. The laser pulse with the ponderomotive energy $\gamma_L$ can accelerate the electron with the initial energy $\gamma_0$ up to the maximum final energy $\gamma_{\text{max}}$ given by

$$\gamma_{\text{max}} = 2 \gamma_0 \gamma_0 (1 + \beta_0 \beta_L) - \gamma_0,$$ (61)

where $\beta_L = (\gamma_L^2 - 1)^{1/2}/\gamma_L$. Fig. 2 shows the maximum final energy for the electron acceleration from the injection energy $E_{\text{inj}} = 150$ MeV as a function of the laser intensity.

The "Dirac accelerator" makes it possible to accelerate electrons up to the super high energy, such as 1 PeV ($\gamma \approx 2 \times 10^9$), provided with the injection energy 250 MeV.
\( n_e \approx 1.1 \times 10^9 \text{ cm}^{-3} \) \( (\gamma_0 \approx 500) \), the plasma density \( n_e \approx 1.1 \times 10^9 \text{ cm}^{-3} \) \( (\gamma_0 \approx 10^6) \), and the laser intensity \( 2.7 \times 10^{24} \text{ W/cm}^2 \) \( (\gamma_0 \approx 1000, a_0 \approx 1414) \) for the wavelength \( \lambda_L = 1 \mu\text{m} \). This laser intensity can be produced by focusing \( \sim 400 \text{ PW} \) on the spot radius \( r_0 = 3 \mu\text{m} \).

**CONCLUSIONS**

The particle acceleration with strong laser fields is reviewed on interactions in vacuum and plasmas. In vacuum, the laser beat wave scheme can produce efficient particle acceleration in the energy range less than 1 GeV. In plasmas, wakefields excited by ultrashort intense laser pulses can produce ultrahigh gradient particle acceleration of the order of 100 GeV/m, which is 3 magnitudes higher than the conventional accelerators. The high energy gain exceeding 1 GeV will be achieved by using the optical guiding technique, such as a capillary discharge plasma. In the ultra-relativistic regime of the laser intensity higher than \( 10^{20} \text{ W/cm}^2 \), the laser ponderomotive scattering results in particle acceleration in plasmas, which is called as "Dirac accelerator" based on the strong field-particle interaction different from classical acceleration mechanism such as RF accelerators and the laser wakefield accelerators. The "Dirac accelerator" may accelerate electrons up to PeV range energies with the laser intensity of \( > 10^{24} \text{ W/cm}^2 \).

**REFERENCES**