Acceleration of Proton Beams by Relativistically Self-focused Intense Laser Pulses

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Abstract. Two dimensional particle-in-cell simulations are performed which show the formation of an extremely large electrostatic field near the front of a relativistically self-focused laser pulse propagating in an underdense plasma. The size of the field is found to reach a maximum of ~ 6.5 TV/m for a 100TW laser pulse propagating over a distance of about 1mm in a plasma at about 3% of the critical density. We use this field generated by relativistically self-focused ultraintense laser pulses to accelerate injected ions.

INTRODUCTION

Ultraintense laser interactions with matter can generate enormous numbers of highly energetic electrons, photons, and ions. Recent laser-matter interaction experiments have revealed acceleration of ~ 10^{10} electrons with the maximum energy up to ~ 100 MeV and production of ~ 10^{12} ions up to several tens of MeV[1].

In two-dimensional PIC simulations for propagation of ultraintense laser pulses of the order of 10^{20}W/cm^2 in underdense plasmas of ~ 10^{20}cm^{-3}, we have found that large amplitude of positive electrostatic fields of the order of a few TV/m are generated over a ~ 1 mm scale in the front of a relativistically self-focused laser pulse. These enormous fields imply the capability of accelerating protons. The solitary structure of the electrostatic field with a spatial and temporal size of the order of μm can produce a femtosecond ion pulse. We propose a new acceleration mechanism for ions due to enormous accelerating fields generated by relativistically self-focused laser pulses in plasmas. This mechanism may open up a new regime of ultraintense laser-matter interactions and new fields of high energy particle physics.

SIMULATION PARAMETERS

To study the self-focusing of a high intensity short pulse laser in a plasma we use the code PCUBE (Progressive Parallel Plasma Code) which is a 2 dimensional fully relativistic particle-in-cell(PIC) code[2] running on the Compaq ES40 227 node parallel computer. The simulation box is 672μm (23000 cells) by 37.4μm (1280 cells) in the x
and y directions respectively. The boundary conditions are periodic in the y direction and outgoing in the x direction. There is a vacuum region at one end of the simulation box of length 21.9μm. The plasma density is chosen to be 5.3 × 10^{19} \text{cm}^{-3} which corresponds roughly to doubly ionized Helium gas at atmospheric pressure. There are 8 electrons and 8 ions in each simulation cell with an ion to electron mass ratio of 1836. The linearly s-polarized laser pulse \((E_z, B_y)\) starts in the vacuum region and propagates into the plasma. The parameters of the laser are of the 100 TW Ti:sapphire laser at the Japan Atomic Energy Research Institute [3]. The pulse length is 19 fs with a spot size of 10μm. The wavelength is 0.8μm. The corresponding unitless laser strength parameter \(a_0 = 7.4\) where \(a_0 = eE_0/m_0\omega_0c\), \(E_0\) is the peak electric field, \(m_0\) is the electron mass, and \(\omega_0\) is the laser frequency.

The critical power \(P_{cr}\) for the relativistic self-focusing of a Gaussian laser pulse is given by [4], \(P_{cr}[GW] = 17(\omega_0/\omega_p)^2\), where \(\omega_0\) is the laser frequency and \(\omega_p\) is the plasma frequency. With the current density \(P/P_{cr} = 179\), where \(P\) is the laser power. Thus, the laser should relativistically self-focus in the plasma. Also given the condition for length of the laser pulse \(L_t\) necessary for the optimum generation of a wake field [5]: \(L_t = \pi c/\omega_p\), we find that \(L_t = 2.3\mu m\) whereas the laser pulse length is 4.9μm. Under these conditions a large wake field behind the pulse should not occur.

**SIMULATION RESULTS**

Figure 1(a) shows the laser pulse after it has propagated 256μm \((340c/\omega_p)\). The laser pulse has relativistically self-focused, has filamented, and the central portion has narrowed. From the line profile taken down the center of the pulse (Figure 1(b)) we can see that the front of the laser pulse has steepened. This is due to the fact that the front of the laser pulse has been depleted compared with the initial Gaussian profile. The amplitude is approximately a factor of 2 higher than the initial laser pulse amplitude. Figure 2(a) shows the electron density at the same propagation distance. In the central portion of the pulse the electrons have been completely ejected. An electron cavity from the main part of the pulse has formed which is \(8c/\omega_p\) wide and \(6c/\omega_p\) long. Electrons have built up in front of this cavity. Figure 2(b) shows a profile of the electron density down the center of the evacuated region. In the front of the laser pulse the density is 25 times the initial background plasma density which is just below the critical density at 32.9 times the background density.

Due to this large buildup of electrons at the front of the pulse, there is a large positive electrostatic field in the propagation direction of the laser pulse created there. Figure 3 shows the structure of the electric field after the same propagation distance. Figure 3(b) shows the line profile of the electric field down the center of the pulse. The electric field rises rapidly to a maximum of 6.5 TeV/m and gradually drops off until it becomes negative behind the laser pulse. We can compare this maximum field to the wave-breaking limit field. Using a linear group velocity for the laser pulse the wave-breaking limit field becomes \(E_{WB} = 2.17\) TV/m [6]. We see that the field generated in the front of the laser pulse is greater than the wave-breaking limit field.

The source of the large density created at the front of the laser pulse can be determined
FIGURE 1. (a) $E_z$ field of the self-focusing laser pulse, (b) profile down the center of the pulse after the laser has propagated $340c/\omega_\lambda$.

FIGURE 2. (a) Electron density of the background plasma after the laser has propagated $340c/\omega_\lambda$. (b) Profile of the electron density down the center of the simulation box.

From the equation of continuity: $\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_x)}{\partial x} = 0$ where $n_e$ is the electron density, $x$ is the propagation direction of the laser pulse, and $v_x$ is the velocity of electrons in the propagation direction. In the frame comoving with the laser pulse we get[7]:

$$n_e = \frac{n_{e0}}{1 - \beta_x}$$  \hspace{1cm} (1)

where $n_{e0}$ is the initial electron density and $\beta_x$ is $v_x/c$. From this equation we see that where the background electron velocity is high the density is high. We can determine the
peak of this electron density relative to the laser field by using the equations of motion of an electron in a plane wave. For a plane wave of the form \( E(\phi) = a_0 \sin(\phi) \) where \( \phi = \omega t - x \) is the phase, \( a_0 \) is the normalized amplitude, and \( \omega \) is the frequency of the laser pulse, the velocity of the electron \( \beta_x \) is given by [8]:
\[
\beta_x = 1 - \frac{2}{2 + a_0^2 (\cos(\phi) - 1)^2}.
\]

Using Equation 1 we get:
\[
\frac{n_e}{n_0} = \frac{2 + a_0^2 (\cos(\phi) - 1)^2}{2}.
\]

The maximum in the electron density occurs at \( \phi = \pi \). Compared to the simulation results in Figures 1(b) and 2(b), this corresponds to the same position in the laser wave for the maximum density. Using the value of \( a_0 = 5.57 \) corresponding to the field at the same time as Figure 1(b) from Equation 2 we get \( n_e/n_0 \approx 63 \) for the maximum density. This is about 2.5 times larger than the maximum value of the density from the simulation.

The maximum density can also be estimated from the electrostatic potential between the front and back of the laser pulse occurring from the buildup of electrons at the front of the pulse and the resulting evacuation at the back. The maximum attainable potential, \( \Delta \Phi \), can be estimated by:
\[
e|\Delta \Phi| \approx 2\pi n_e d^2 = mc^2 a_0^2 / 2
\]
where \( n_e \) is the electron density, \( d \) is the width of the electron bunch, and the last term represents the maximum energy that electrons can attain in a finite time duration plane wave. Rewriting this equation we get:
\[
n_e = \frac{a_0^2}{4\pi n_0 d^2 r_e^2}
\]
where \( r_e \) is the classical electron radius. Using \( a_0 = 5.57 \) and assuming \( d = \lambda_0 \) we get \( n_e/n_0 \approx 26 \). This is in close agreement with the simulation results.

Figure 4 shows the maximum normalized amplitude of the laser pulse \( E_z \) and the corresponding electrostatic field \( E_x \) generated by the pulse as a function of propagation distance of the laser pulse. The laser pulse relativistically self-focuses to a peak nor-
malized amplitude of $a_0 = 15.7$ after propagating $160c/\omega_p$. This is more than twice the initial amplitude. After the initial peak in self-focusing amplitude, the pulse begins to deplete. This depletion is due to the absorption of the laser pulse by the plasma. By the end of the simulation $\approx 89\%$ of the laser energy has been deposited in the plasma. The laser is propagating in a near vacuum at the speed of light while the electron cavity created by the laser pulse propagates at near the plasma group velocity. Theoretically in one dimension the depletion distance is $l_{\text{pulse}} \approx 257 \mu m$. This value is smaller than that seen from the simulation results. One factor may be the effect of the relativistic factor $\gamma$ of the background electrons on the plasma frequency $\omega_p$ or two dimensional effects.

As shown in Figure 4 the electrostatic field peaks after the laser field peaks. The maximum electrostatic field is 6.5 TeV/m ($a_0 = 1.75$). Throughout most of the laser propagation the electrostatic field amplitude is about 10\% of the laser field amplitude. Also, in conjunction with the laser pulse depletion the electrostatic field decreases.

**ION ACCELERATION SCHEME**

We propose to use the large field created at the front of the pulse to accelerate injected protons to higher energies. In order to determine the minimum energy of injection of the protons we can use the theory developed for determining the minimum energy of injection for electrons in a wakefield [10]:

$$\gamma_{\text{max/min}} = \gamma_p (1 \pm \beta_p \sqrt{2 \gamma_p \Delta \phi})$$  \hspace{1cm} (3)

assuming $\gamma_p \Delta \phi \ll 1$ where $\gamma_{\text{max/min}}$ refers to the maximum and minimum energy of electrons which can be trapped in a potential well of a normalized potential difference
FIGURE 5. (a) Initial and (b) final $\gamma-x$ phase space of the central portion of the injected proton beam at $630\Delta \leq y \leq 650\Delta$ where the center of the simulation is at $y = 640\Delta$.

$\Delta \phi$. The potential difference is:

$$\Delta \phi = \phi_{\text{max}} - \phi_{\text{min}} = a_0 m_e$$

where $a_0$ is the normalized amplitude of the electrostatic field generated by the laser pulse. Using the maximum electrostatic field $a_0 = 1.75$ we get $\Delta \phi = 9.53 \times 10^{-4}$ and with $\gamma_0 = 5.73$ resulting in $\gamma_0 \Delta \phi = 5.46 \times 10^{-3}$ satisfying the condition for the validity of Equation 3. Using these values we get: $\gamma_{\text{max/min}} = 6.18/5.3$ or correspondingly, $T_{\text{max/min}} = 4.72/3.92\text{GeV}$ This implies that if we inject protons at the minimum energy and can accelerate them to the maximum energy we can get an increase of up to 0.8 GeV. Due to the fact that the actual group velocity of the self-focused laser pulse differs from the one predicted by linear considerations we measured the velocity of the pulse from the simulations. The values are $\beta_{\text{sim}} = 0.952$ and $\gamma_{\text{sim}} = 3.27$. Using these values we get $T_{\text{sim}} = 2.06\text{GeV}$ giving a lower injection velocity. In order to insure that some protons are injected in front of the electrostatic field a proton beam with an initial energy spread was input into the simulation. The beam was uniformly distributed in the $y$ direction and placed in the same initial position as the laser pulse with a length of $29.2\mu\text{m} (1000\Delta)$. The proton beam had a very low density so that the protons are only pushed by the electrostatic field generated by the laser pulse. Figure 5 shows the (a) initial and (b) final $\gamma-x$ phase space of the central portion of the injected proton beam at $630\Delta \leq y \leq 650\Delta$ where the center of the simulation is at $y = 640\Delta$. In Figure 5(a) is shown the initial proton beam with a spread in $\gamma$ between 2.47 and 6.85. After the proton beam has propagated with the laser over $642\mu\text{m} (880c/\omega_p)$ the proton beam has spread due to the initial spread in energies (Figure 5(b)). There is a gap between the front portion of the beam and the back part of the beam. This may be due to the electrostatic field generated by the laser pulse. It can be seen in the plot that there are now protons which have
energies greater than $\gamma = 7$. In Figure 6 is shown the energy distribution of the injected proton beam. In Figure 6(a) one can see that the final distribution (solid line) is shifted from the initial distribution (dotted line). In a close-up of the peak in Figure 6(a)(inset) the shift corresponds to $\Delta \gamma \approx 0.1$ or an equivalent acceleration of 90 MeV. In a log plot of the distribution in Figure 6(b) one can see that there is an increase in the high energy tail of the proton beam. One can see that this increase corresponds to $\Delta \gamma \approx 0.5$ or an equivalent acceleration of 450 MeV. The 90 MeV and 450 MeV increases in energy correspond to average acceleration gradients of 0.14 and 0.7 TeV/m, respectively. These gradients are reasonable in comparison to the maximum electrostatic field gradient of 6.5 TeV/m.

The optimal energy and position of the injected protons still need to be determined. The phase space area occupied by the beam increased in the transverse momentum by about a factor of 2 from the initial spread. For this paper the main point was to show the possibility of acceleration of protons by the large electrostatic field generated at the front of the laser pulse.

There are several factors which affect or limit the acceleration of ions. One is the depletion of the laser pulse. In Figure 4 we showed the maximum electric field as a function of propagation distance. The maximum field was found to decrease in accordance with the depletion of the laser pulse. Thus with the depletion of the laser pulse the acceleration efficiency also drops. Since the depletion is slower at lower densities according to theory [9], it is better to accelerate protons using low density plasmas. At lower densities, however, the initial injection energy of the protons needs to be higher. Another factor affecting the acceleration is the "snaking" instability which causes the laser pulse to deviate from its original propagation direction [11]. All these factors need to be considered for the optimal acceleration of protons.
CONCLUSION

In this paper we have proposed using the large electrostatic field created in the front of a relativistically self-focused laser pulse to accelerate ions. We have shown from 2 dimensional PIC simulations that an accelerating field of the order of 6.5 TeV/m can be excited by a 100 TW laser pulse propagating in a plasma at a density of 5.3 \times 10^{19} \text{ cm}^{-3}. The large field is found to be generated by the buildup of electrons at the front of the relativistically self-focused laser pulse. The buildup of electrons can be accounted for by using the continuity equation. Protons injected in the front of such a self-focused pulse can be accelerated in coincidence with the electric field which moves at a velocity nearly equal to the laser group velocity in plasmas. The maximum kinetic energy gain for injected protons with an average energy of 2 GeV is found to be 450 MeV for a laser intensity of the order of 10^{20} \text{ cm}^{-3} with a pulse duration of \sim 20 \text{ fs}. The acceleration will be limited by the depletion of the laser pulse. The next stage of this investigation will be the acceleration of stationary protons using an inhomogeneous plasma with a density gradient. These type of laser-matter interactions will open up a new regime in high energy beam science.

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REFERENCES