THE INTERACTION OF LAMB WAVES WITH SOLID-SOLID INTERFACES

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ABSTRACT. This paper deals with the topic of the interaction of Lamb waves, more specifically the A₀ and S₀ modes, with a solid-solid interface. This solid-solid interface is the contact between two dry, rough surfaces and could represent a kissing bond in an adhesive joint or the contacting surfaces of a bolted joint. In this paper, a very thick elastomer with high internal damping is loaded against one surface of a glass plate to create a solid-solid interface. The principal effect is shown to be increased attenuation of the guided waves propagating along the glass plate. This attenuation is caused by leakage of energy from the plate into the elastomer, where it is dissipated due to high viscoelastic damping. It is shown that the increase in attenuation is strongly dependent on the compressive load applied across the solid-solid interface. This interface is represented as a spring layer in a continuum model of the multi-layered system. Both normal and shear stiffnesses of the interface are quantified from the attenuation of A₀ and S₀ Lamb waves measured at each step of the compressive loading.

INTRODUCTION

Lamb waves, and other guided waves, have long been known to offer the potential for long range, non-destructive inspection of engineering structures. Recent years have seen these techniques becoming more widely used by industry to solve a number of important inspection problems such as the rapid inspection of pipelines [1]. The possible uses of Lamb waves are numerous and it is clear that over the coming years a range of new applications will emerge. Due to the complexity of the Lamb wave propagation phenomena, a clear understanding of the wave propagation characteristics must be obtained before any practical testing is initiated. If this understanding is not present then misinterpretation of the results can easily occur. This paper deals with the topic of the interaction of Lamb waves, more specifically the A₀ and S₀ modes, with a solid-solid interface. This solid-solid interface is the contact between two dry, rough surfaces and could represent a kissing bond in an adhesive joint or the contacting surfaces of a bolted joint. Contact between the surfaces occurs between the peaks of the surface roughness. The ultrasound will "see" the solid-solid interface as an array of scatterers, which are the air gaps left between the contacting regions. The paper has two specific aims; firstly to show that a spring model of the interface is sufficient to describe the interaction of Lamb waves with a solid-solid interface. The second aim is to demonstrate how, by using the A₀
and S₀ Lamb modes, the normal and shear stiffness of the solid-solid interface between a
glass plate and semi-infinite elastomer can be measured.

DETERMINATION OF THE INTERFACIAL SPRING CONSTANTS USING
BULK WAVES

Apparatus and Results

In this section measurements of the reflection coefficient of normal incidence bulk
longitudinal waves are used to measure the normal stiffness of the interface. The approach
is based on that first developed by Kendal and Tabor [2] and used more recently by
Drinkwater et al [3]. A 1 MHz longitudinal wave ultrasonic transducer was coupled to a
glass plate through a water bath. The signal reflected from the glass-air interface was
recorded as a reference at which the reflection coefficient is known to be equal to unity.
An elastomer plate (see table 1 for material properties) of identical mechanical properties
and surface roughness to that used in the Lamb wave experiments described later was then
loaded against the glass plate. The signal reflected from the glass-elastomer interface was
recorded as a function of load. For each load increment, the reflection coefficient was
calculated by dividing the glass-elastomer measurement by the glass-air measurement in
the frequency domain. This gave the reflection coefficient spectrum over the bandwidth of
the transducer. Figure 1 shows how the measured 1 MHz longitudinal reflection
coefficient varied for a loading and unloading cycle. Note that the applied load was chosen
to generate similar contact pressures (0.47 MPa) as used in the Lamb attenuation
experiments (0.42 MPa). Figure 1 shows clear hysteresis between the loading and
unloading lines. It can also be seen that the reduction in reflection coefficient is very small
(≈4%) at the highest contact pressures. However, measurements shown in Figure 1 are
reproducible within 0.5%, thus giving good confidence in these results. This experiment
was repeated for a 1 MHz shear transducer coupled to the glass plate with viscous couplant.
In this case, no measurable change in the shear reflection was observed and so no
meaningful measurement of the shear stiffness was obtained.

![Graph showing the variation of reflection coefficient with contact pressure.](image)

**FIGURE 1.** Measured longitudinal glass-elastomer reflection coefficient variation as a function of contact
pressure (1 MHz).
MODELLING OF THE $A_0$ AND $S_0$ INTERACTIONS WITH A COMPRESSIVELY LOADED INTERFACE

Description of Model

The interactions of Lamb modes with a compressively loaded interface were modelled using the layer stiffness matrix method [4], which is numerically more stable than the standard transfer matrix method and faster than the global matrix method [5]. As shown schematically in Figure 2 the system modelled was a 3.9 mm thick glass plate, surrounded by semi-infinite air on one side and semi-infinite elastomer on the other. There was a solid-solid interface between the glass plate and the elastomer and this interface was modelled by a 10 μm thick, isotropic layer, the properties of which were the mass density and two elastic moduli, $C_{11}$ and $C_{66}$, using the usual notation for the indices [6]. The very small thickness of this interface layer meant that the value chosen for the mass density of the interface had a negligible effect on the propagation of modes guided in the system and so the effect of the interface layer is governed by the elastic moduli, $C_{11}$ and $C_{66}$. This interface layer is therefore describing a distributed spring interface with the normal and transverse stiffnesses, given by $K_N = C_{11}/h$ and $K_T = C_{66}/h$, respectively.

Results and Discussion

Predicted dispersion curves are shown in Figure 3(a) for a 3.9 mm thick glass plate surrounded by air. The plotted frequency-thickness range corresponds to 0-1 MHz. Figure 3(b) shows that the attenuation of these modes is very small, due to the low level of leakage into the surrounding air. The through-thickness displacement fields for the glass plate in air show that both modes are composed of a combination of displacement normal to, and in the plane of, the surface of the plate. The $A_0$ displacement field is mostly normal to the surface whereas that of $S_0$ is mostly in the plane of the plate.

When the dispersion curves were predicted for the same glass plate with semi-infinite air one side and semi-infinite elastomer on the other it is found that the phase velocities of the $A_0$ and $S_0$ modes remained unchanged in the range 0.1-1 MHz. However, the attenuation of both $A_0$ and $S_0$ modes was increased by over 300%. A spring interface layer was then introduced into the model to represent the solid-solid contact as shown in Figure 2. Depending on the stiffness of the interface layer, the modes will vary between those for air-glass-air, when zero stiffness is used, and those for air-glass-elastomer, when
an infinite stiffness is used. Comparison of the $A_0$ and $S_0$ through-thickness displacement fields for an air loaded glass plate and a glass plate loaded with semi-infinite half-spaces of elastomer and air shows that they are almost identical in the low frequency-thickness region. This is therefore, consistent with the observed similar phase velocity dispersion curves.

In order to analyse this system further predictions of the normal and shear stiffness of the glass-elastomer contact at a contact pressure of 0.1 MPa were made. Firstly, the normal stiffness at 0.1 MPa was calculated from the high frequency, normal incidence experiment (see figure 1) using the spring model equation (1) below,

$$K_N = \alpha e_1 z_2 \frac{1 - |R_{12}|^2}{|R_{12}|^2 (z_1 + z_2)^2 - (z_1 - z_2)^2}$$  (1)

where $R_{12}$ is the reflection coefficient at the imperfect glass-elastomer interface, $z$ is the acoustic impedance (product of density and wave speed), the subscripts referring to the media either side of the interface and $\omega$ is the angular frequency. Secondly, this normal stiffness was used in the dispersion curve prediction software and the shear stiffness varied until the predicted $S_0$ attenuation at 120 kHz was equal to that which was measured experimentally (see next section for full details of the Lamb wave experiments). This gave $K_N = 2.8 \times 10^{12}$ N/m$^2$ and $K_T = 0.65 \times 10^{12}$ N/m$^2$. Using these stiffness values as a starting point, figure 4 shows the sensitivity of the $A_0$ and $S_0$ attenuation to changes in $K_N$ and $K_T$.

For these plots sensitivity has been defined as;

$$\text{Sensitivity} = \frac{\alpha_{0} - \alpha_{20\%}}{\alpha_{0}}$$  (2)
where $\alpha_0$ is the attenuation of the mode at the initial values of either $K_N$ or $K_T$ and $\alpha_{20\%}$ is the attenuation of the same mode if one of the stiffnesses is increased by 20% above this initial value. Note that the initial values of $K_N$ and $K_T$ chosen for this sensitivity study do not significantly effect the general result of this sensitivity study. From Figure 4(a) it can be seen that for $S_0$, $K_T$ has a significant effect on attenuation across the plotted frequency range and that the influence of $K_N$ increases with frequency. Below 0.5 MHz.mm, the $S_0$ attenuation is virtually independent of $K_N$. Conversely, from figure 4(b) it can be seen that for $A_0$, changes in $K_N$ have a significant effect on the attenuation except at very low frequencies, whereas changes in $K_T$ have a much smaller effect across the frequency range.

This sensitivity study points to an exciting possibility, which is that combined low frequency $S_0$ and $A_0$ attenuation measurements can be used to measure $K_T$ and $K_N$. Of particular importance is the fact that $K_T$ can potentially be measured with good accuracy whereas such a measurement proved to be very difficult using normal incidence bulk waves.

MEASUREMENTS OF THE INTERACTIONS OF $A_0$ AND $S_0$ WITH A COMPRESSIVELY LOADED INTERFACE

The experimental set-up shown in Figure 5 was used to generate and detect the Lamb waves ($A_0$ and $S_0$) in a 3.9 mm glass plate. The plate measured 600 mm by 300 mm. Based on the sensitivity study results shown in figure 4 measurement positions were chosen centred on 120 kHz (or 0.47 MHz.mm) for $S_0$ and 300 kHz (1.17 MHz.mm) for $A_0$. At 0.47 MHz.mm the aim was to calculate $K_T$ from the $S_0$ measurement, where there is little sensitivity to $K_N$. At 1.17 MHz.mm $A_0$ is sensitive to $K_N$ and so the aim was to use this measurement to calculate $K_N$, based on the previously measured $K_T$. For the measurement of $A_0$ at 300 kHz an air-coupled transmitter and an air-coupled receiver were used. The transducers were capacitive and had an active diameter of 45 mm. Their frequency bandwidth was centred at 200 kHz with −15 dB points at 50 and 400 kHz. A 10 cycle Hanning windowed tone burst was input to the transmitting transducer centred on 300 kHz. Both transducers were inclined at 8°, with respect to the normal of the plate,
which was calculated from the dispersion curves shown in figure 3(a) using the coincidence principle. The air coupling means that both the transmitter and receiver are sensitive to normal displacement of the plate and insensitive to in-plane displacement. This means that they are ideally suited to the measurement of $A_0$, which has a large normal displacement component. As $S_0$ is made up of mostly in-plane displacement a different experimental set-up was used for this measurement. As can also be seen in figure 5 a piezoelectric contact transducer (active element size was 100 mm long by 40 mm wide) was placed against the edge of the plate. This generates a large in-plane displacement, strongly exciting $S_0$. An air coupled receiver, inclined at 3.5°, with respect to the normal of the plate, was then used to detect $S_0$ by measurement of its small normal displacement component. Figure 3(a) also shows the experimentally measured phase velocities for $A_0$ and $S_0$ for the glass plate surrounded by air from which it can be seen that there is excellent agreement with the predicted dispersion curves.

Experiments were then performed in order to measure the interaction of $A_0$ and $S_0$ Lamb waves with a compressively loaded interface. The experimental set-up shown in figure 5 was used to perform measurements in which a 26.4 mm thick elastomer plate measuring 160 mm by 60 mm was compressively loaded against the previously described glass plate. The elastomer plate was loaded through a thick (60 mm) aluminium block. This acted as an effectively rigid block and so the load was applied evenly across the top surface of the elastomer plate. The glass plate was supported by a 40 mm thick block of rigid foam. This foam was very porous and had a very low acoustic impedance and so simulated air backing of the glass plate.

The elastomer plate was thick (26.4 mm) and highly attenuative (see table 1), so that any ultrasonic waves, transmitted into the elastomer were heavily attenuated. This meant that the amplitudes of the reflections (both longitudinal and shear) from the back face of the elastomer were small (<12% of the transmitted amplitude for a longitudinal wave at 300 kHz) and so the elastomer was modelled as semi-infinite in later analysis.

At each load step the transmitted signal was recorded and transformed to the frequency domain using a Fast Fourier Transform. The transmission coefficient was then calculated by dividing the spectra of the transmitted pulse by a reference measurement.
TABLE 1. Acoustic properties of glass, elastomer and air used in the modelling. Properties of air are those at 20°C. The velocities and attenuations of the glass and elastomer were measured at 300 kHz.

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Density (kg/m³)</th>
<th>Longitudinal Velocity (m/s)</th>
<th>Longitudinal Attenuation @300 kHz (dB/cm)</th>
<th>Shear Velocity (m/s)</th>
<th>Shear Attenuation @300 kHz (dB/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>3.9</td>
<td>2460</td>
<td>5880</td>
<td>0</td>
<td>3490</td>
<td>0</td>
</tr>
<tr>
<td>Elastomer</td>
<td>8.8</td>
<td>1250</td>
<td>1960</td>
<td>3.5</td>
<td>566</td>
<td>15</td>
</tr>
<tr>
<td>Air</td>
<td>-</td>
<td>1.225</td>
<td>343</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

taken when no elastomer was present and the attenuation was known to be zero (hence transmission coefficient equalled unity). All signals were captured at the same distance between transducers (300 mm) and so the geometrical attenuation (beam spreading) was equal in the measured and reference signals. In this way measured transmission coefficient reductions are due, either to the leakage of energy into the surrounding media, or reflections back from the edges of the elastomer contact. Note it was found that the leakage into the foam was negligible, as were the reflections from the edges of the contact. This measured transmission coefficient was then converted into attenuation and is shown in Figure 6(a) for a cycle of loading and unloading. From this figure it can be seen that as the load (and hence contact pressure) was increased, so the attenuation increased. It can also be seen from Figure 6(a) that there was a distinct difference between the loading and unloading parts of the cycles. This effect has been observed in earlier work on metal-metal contact [3] and was explained as a contact adhesion effect. On loading the contacting asperities (peaks of the surface roughness) adhere to one another. On unloading these adhered asperities tend to maintain the contact area. This causes the degree of contact, and hence attenuation, to be greater on unloading when compared to loading.

The following procedure was then adopted to determine the normal and shear stiffnesses of the glass-elastomer interface as a function of loading from the experimental data. For each load increment (using $K_N = 2.8 \times 10^{12}$ N/m$^3$ and $K_T = 0.65 \times 10^{12}$ N/m$^3$ as a

![Graphs](image-url)
starting point) $K_T$ was adjusted until agreement was obtained with the $S_0$ attenuation measurement at 120 kHz. Using this new value of $K_T$, $K_N$ was then adjusted until agreement was obtained with the $A_0$ attenuation measurement at 300 kHz. The result of this $K_N$ and $K_T$ optimisation procedure is shown in figure 6(a). Figure 6(b) shows how the $K_N$ and $K_T$ values extracted from this procedure varied as a function of loading. Also shown on figure 6(b) are the $K_N$ values obtained from the 1 MHz normal incidence measurements. The good agreement between these stiffness measurements for the loading cycle, demonstrates that such Lamb wave measurements can be used to measure interfacial stiffness. The unloading data are not directly comparable as the normal incidence experiment was loaded to a higher load and so the hysteresis loop is shifted to the right. Note that the differences at low loads are thought to be due to errors in the normal incidence measurement which is very sensitive to reflection coefficient errors as the reflection coefficient varies over a very small range (between 1.0 and 0.965).

CONCLUSIONS

Measurements of the velocity and attenuation of $A_0$ and $S_0$ in a glass plate loaded through a solid-solid contact with an effectively semi-infinite elastomer have been made. These results show that, for both modes (below the $A_1$ cut-off frequency), the phase velocity was virtually unaffected by the elastomer and the applied load whereas the attenuation was strongly dependent on the applied load. It was proposed that the coupling across the solid-solid interface can be modelled by a spring interface having both normal and shear stiffness. As the load across the solid-solid interface is increased, so the interface stiffnesses increases. The dispersion characteristics of this system (air-glass-"spring interface"-elastomer) have been modelled. The normal stiffness was found to affect the attenuation of $A_0$ to a much greater extent than that of $S_0$. Conversely, the shear stiffness was found to affect the attenuation of $S_0$ to a much greater extent than that of $A_0$. This fact means that the measurement of $A_0$ and $S_0$ can be used to independently quantify both the normal and shear stiffnesses of an interface. In a separate experiment, high frequency (1 MHz) normal incidence longitudinal wave measurements were used to measure the glass-elastomer interfacial normal stiffness as a function of load. Good agreement was found between stiffness obtained from these normal incidence measurements and stiffness measured via $A_0$ and $S_0$ attenuation measurements.

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REFERENCES