ULTRASONIC ATTENUATION AS INFLUENCED BY ELONGATED GRAINS

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ABSTRACT. Reliable nondestructive evaluation of structural/machine components fabricated from polycrystalline materials require the knowledge of attenuation and dispersion of an ultrasonic wave propagating through such microscopically inhomogeneous medium. Expected propagation characteristics of ultrasonic waves in randomly oriented equiaxed grains are relatively well understood. But when the grains are elongated and/or preferentially oriented, the wave propagation constants exhibit anisotropic behavior. The authors have published a number of research articles on propagation of ultrasonic waves in materials with macroscopic texture and elongated grains. The present paper sheds more light on the effect of grain shape on the attenuation and dispersion of ultrasonic waves in polycrystals. Specifically, theoretical results are presented showing the effects of different grain aspect ratios. It is observed that for the same effective grain volume, grain elongation has smaller effect on attenuation.

INTRODUCTION

A polycrystalline material is composed of numerous discrete grains, each having a regular, crystalline atomic structure. The elastic properties of the grains are anisotropic and their crystallographic axes are oriented differently. When an acoustic wave propagates through such a polycrystalline aggregate, it is attenuated by scattering at the grain boundaries, with the value of this attenuation and the related shift in the propagation velocity depending on the size, shape, orientation distributions, and crystalline anisotropy of the grains. If the grains are equiaxed and randomly oriented, these propagation properties are independent of direction, but such is not the case when the grains are elongated and/or have preferred crystallographic orientation. Therefore, reliable ultrasonic testing of engineering alloy components require the knowledge of the anisotropies in the attenuation and velocities of ultrasonic waves due to preferred grain orientations and elongated shapes.

The propagation of elastic waves in randomly oriented, equiaxed polycrystals has received considerable attention, with most recent contributions for the cubic
TABLE 1. Material Properties

<table>
<thead>
<tr>
<th>Materials</th>
<th>$c_{11}$ $N_m^{-1}$</th>
<th>$c_{12}$ $N_m^{-1}$</th>
<th>$c_{44}$ $N_m^{-1}$</th>
<th>$\rho_g$ $m^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless Steel$^{13}$</td>
<td>$2.16 \times 10^{11}$</td>
<td>$1.45 \times 10^{11}$</td>
<td>$1.29 \times 10^{11}$</td>
<td>$7.86 \times 10^3$</td>
</tr>
<tr>
<td>Aluminium$^{13}$</td>
<td>$1.034 \times 10^{11}$</td>
<td>$0.571 \times 10^{11}$</td>
<td>$0.286 \times 10^{11}$</td>
<td>$2.76 \times 10^3$</td>
</tr>
</tbody>
</table>

materials being made by Hirsekorn [1,2], Stanke and Kino [3,4], Beltzer and Brauner [5], and Turner [6]. Stanke and Kino present their "unified theory" based on the second order Keller approximation [7] and the use of a geometric autocorrelation function to describe the grain size distribution. Stanke and Kino argue that their approach is to be preferred because i) the unified theory more fully treats multiple scattering, ii) the unified theory avoids the high frequency oscillations which are coherent artifacts of the single-sized, spherical grains assumed by Hirsekorn, and iii) the unified theory correctly captures the high frequency "geometric regime" in which the Born approximation breaks down. The theoretical treatment of ultrasonic wave propagation in preferentially oriented grains is more limited. Hirsekorn has extended her theory to the case of preferred crystallographic orientation while retaining the assumption of spherical grain shape [8], and has performed numerical calculations for the case of stainless steel with fully aligned [001] axes [9]. Turner, on the other hand, derives the Dyson equation using anisotropic Green's functions to predict the mean ultrasonic field in macroscopically anisotropic medium [6]. He then proceeds to obtain the solution of the Dyson equation for the case of equiaxed grains with aligned [001] axes.

Previously we have employed the formalism of Stanke and Kino [3,4] in [001] aligned stainless steel polycrystal to compute the mean attenuation and phase velocity of plane ultrasonic waves [10,11]. In this paper we revisit our earlier calculations for the case of elongated grains and focus our attention on the effect of grain shape on the mean propagation characteristics. Specifically, we consider two cases: 1) the [001] crystallographic axes are aligned with the z-axis of the laboratory coordinate system while remaining two axes are randomly oriented and 2) all the crystallographic axes are randomly oriented. In both cases, the crystallites have cubic symmetry and the grains are considered to be ellipsoidal with either their major or minor axis parallel to the z-direction of the laboratory coordinate system. Numerical results for the attenuation and phase velocity of longitudinal wave in these two polycrystals are presented here. The material properties of the two media are listed in Table 1.

THEORY

Formal Approach

As has been reported by us in an earlier paper [10], the application of the unified theory of Stanke and Kino [3] to a polycrystalline aggregate yields the generalized Christoffel's equation

$$\left[\Gamma_{ik} - \rho \omega^2 \delta_{ik}\right] \ddot{u}_k = 0,$$

(1)
where

\[ \Gamma_{ik} = \left\{ C_{ijkl}^0 + \epsilon \langle \Delta_{ijkl} \rangle + \epsilon^2 \left[ \langle \Delta_{ijkl\eta} \rangle - \langle \Delta_{ij\eta} \rangle \langle \Delta_{\eta kl} \rangle \right] \right\} \times \int G_{\alpha\eta}(\bar{s}) \left[ W(\bar{s}) e^{ikr\hat{k}} \right] d\nu \hat{k}_j \hat{k}_l, \]

Equation (2) describes the expected propagation constant of plane waves of the form \( u_i = a_u e^{-i\omega t - ik\hat{k}} \), where \( \omega \) is the angular frequency and \( \hat{k} \) is the direction of propagation. \( k \) is related to phase velocity \( v_p \) and attenuation coefficient \( \alpha \) through the relationship \( k = \omega / (\alpha \hat{k}) \).

Equation (1) admits solutions for \( u \) only if the determinant of the matrix in brackets on the left-hand side vanishes. In the absence of scattering, these occur for three distinct real values of \( u^2 / k^2 \) one for each of the two quasi-shear waves and one for the quasi-longitudinal wave. The wave polarizations are given by the corresponding eigenvectors.

**Particular Case for Calculations**

We have extended our previous calculations [10] for polycrystals of cubic symmetry to accommodate grain elongation along the direction of preferred orientation. Generalizing on Stanke and Kino [3], the geometric autocorrelation function \( W(\bar{s}) \) is assumed to have the form [14]

\[ W(\bar{s}) = e^{-2d/\sqrt{1+(h/d^2-1)\cos^2\theta}} \]

where \( d \) is the mean grain diameter in the plane perpendicular to the preferred direction, \( h \) is the grain height along the preferred direction, and \( \theta \) is the angle measured with respect to the preferred direction. Stanke and Kino pointed out the suitability of this choice when \( h = d \) for real materials and have used it in a previous publication [6,15]. It is to be noted that small values of \( d/h \) correspond to elongated (cigar shaped) grains, while large values correspond to flattened (pancake shaped) grains. As mentioned before, the particular texture considered in this work has the [001] crystallographic axes of all grains parallel to the z-axis of the laboratory coordinate system while the [100] and [010] axes are randomly oriented about this direction. This simplifies the averaging procedure. Thus, if \( \phi \) is the rotation of the [100] axis from the x-axis in the laboratory system,

\[ \langle f \rangle = \frac{1}{2\pi} \int_0^{2\pi} f(\phi) \, d\phi. \]

Following the general procedure to obtain the complex propagation constants and polarizations as described before, we were able to develop an integral equation for the expected propagation constant for elastic waves propagating along arbitrary
directions in the \( y-z \) plane. In order to do this, it was found convenient to rotate the laboratory coordinate system \((x, y, z)\) by an angle \( \theta \) about the \( x \)-axis resulting in a primed \((x', y', z')\) coordinate system and choose the \( z' \)-axis (direction 3) as the propagation direction. The forms of the two-point averages appearing in equation (2) are then
\[
D_{a3klmn} = \varepsilon^2 \left[ \langle \Delta_{a3kl} \Delta_{mn3} \rangle - \langle \Delta_{a3kl} \rangle \langle \Delta_{mn3} \rangle \right],
\] (6)

Waves with arbitrary propagation direction are, in general, not purely longitudinal or shear in a medium with macroscopic texture. However, when the single crystal anisotropy \( A \) is not large compared to \( C^o_{ijkl} \), the deviations of the polarizations from those of pure modes are not expected to be large. Therefore, we have neglected the deviation of the polarizations from the pure mode values in the polycrystalline aggregate under consideration. With this assumption, we only need the one- and two-point averages \( \epsilon \langle \Delta_{3333} \rangle \) and \( D_{klmn} = D_{3333mn33} \). Finally carrying out the algebraic operations in equation (1) as far as possible, the expected propagation constants for \( L \)-waves propagating in an arbitrary direction in the \( y-z \) plane is given by the integral equation
\[
[1 + F(\theta)] \chi_L = \chi_{ol}^2 + \frac{\chi_L^2}{\chi_{ol}^2 (C^o_{3333})^2} \Psi_L (\chi_L, \chi_{ol}, \chi_{os}).
\] (7)

where \( F(\theta) = \left[ A (\cos^4 \theta + 3 \sin^4 \theta/4) - 3A/5 \right] /C^o_{3333} \) for the \([001]\) aligned stainless, \( F(\theta) = 0 \) for randomly oriented grains in aluminum, \( A \) is the single crystal anisotropy factor for cubic crystals, \( \chi = kd, \chi_{ol} = k_{ol}d, \chi_{os} = k_{os}d, \) and
\[
\Psi_L (\chi_L, \chi_{ol}, \chi_{os}) = \int_0^{2\pi} \int_0^\pi \int_0^\infty D_{klmn} B_{ln}(\chi_L) (r_k \bar{r}_m G_1 + \delta_{km} G_2) s^2 ds dv d\phi,
\] (8)

In equations (8), \( \delta_{km} \) is the Kronecker delta, \( s \) is a dummy variable of integration, \( \bar{r} = (\sin \psi \cos \phi, \sin \psi \sin \phi, \cos \psi), \)
\[
G_1 = \frac{1}{4\pi} \left[ \left( \frac{3}{s^3} + i \frac{3 \chi_{os}}{s^2} - \frac{\chi_{ol}^2}{s} \right) e^{-i \chi_{os} s} - \left( \frac{3}{s^3} + i \frac{3 \chi_{ol}}{s^2} - \frac{\chi_{os}^2}{s} \right) e^{-i \chi_{ol} s} \right],
\] (9)
\[
G_2 = \frac{1}{4\pi} \left[ \left( \frac{1}{s^3} + i \frac{\chi_{ol}}{s^2} \right) e^{-i \chi_{ol} r} - \left( \frac{1}{s^3} + i \frac{\chi_{os}}{s^2} - \frac{\chi_{ol}^2}{s} \right) e^{-i \chi_{os} s} \right],
\] (10)

and
\[
B_{ln}(\chi) = \frac{\partial^2}{\partial x_l \partial x_n} \left[ e^{-2r \sqrt{1 + ((d/k)^2 - 1) \cos^2 \theta / d}} x' \right].
\] (11)

RESULTS

To obtain the attenuation per wavelength \( \alpha/k_{ol} \) and the normalized shift in phase velocity \((V - V_0)/V_0\) for plane waves in the considered textured medium, the integral equation for the expected propagation constants, Eq. (8), was solved numerically. Here and in what follows, \( V_0 \) and \( k_0 \) refer to phase velocity and wave number, respectively, based on Voigt averaged elastic constants in the absence of preferred grain orientation. The subscripts ‘l’ and ‘s’ associated with the Voigt averaged quantities
FIGURE 1. Normalized attenuation coefficient for $L$-waves in stainless steel: Frequency dependence for propagation in $y$-direction.

refer to $L$- and $S$-waves respectively and $V_{\text{mean}}$ refers to mean grain volume. In the process, the single crystal elastic constants used for stainless steel [13] and aluminum [13] are listed in Table 1.

Longitudinal Wave

Figure shows the dependence of the normalized attenuation coefficient $\alpha/k_0$ in [001] aligned stainless steel on the normalized frequency $k_0V_{\text{mean}}^{1/3}$ for $L$-waves propagating in the $y$-direction for different grain aspect ratios ($d/h$). At very low frequencies (Rayleigh frequency regime), for the same mean grain volume, attenuation per wavelength is hardly affected by grain elongation. Careful observation however reveals that elongation in the direction perpendicular to the wave propagation direction causes slightly greater attenuation. It is also seen that in the stochastic region where $k_0V_{\text{mean}}^{1/3} = O(1)$, deviation from the sphericity of the grains decreases the attenuation. Looking at Fig. 2, we observe that grain elongation delays the transition to the “geometric” frequency regime where attenuation varies inversely as $V_{\text{mean}}^{1/3}$ which is a measure of the mean grain volume. It is clear that in the “stochastic-geometric” transition regime, slender grains cause more attenuation than the more flattened grains. This is consistent with the intuitive notion that when ultrasonic waves behave as rays, slender grains having more projected area, remove more energy from the beam through reflection at the grain boundaries.

In Fig. , we have plotted the normalized shift in phase velocity $(V_t - V_0)/V_0$ [001] aligned stainless steel versus the normalized frequency $k_0V_{\text{mean}}^{1/3}$ for $L$-waves propagating in $y$-direction with $d/h$ as a parameter. In this case we observe that, at low frequencies, the acoustic wave becomes increasingly dispersive with grain elongation. In the entire frequency regime considered here, it is observed that grains flattened in the direction of mean propagating wave are less dispersive. The complicated behavior of the phase velocities for different grain shapes when $k_0V_{\text{mean}}^{1/3} \geq 10$ is believed to be associated with differing “Rayleigh-stochastic-geometric” transition regimes.

Figure shows the variation of the normalized attenuation coefficient in aluminum with randomly oriented grains with the normalized frequency propagating in the y-direction for different grain aspect ratios $(d/h)$. Again we observe that attenuation per wavelength is hardly affected by grain elongation in the Rayleigh regime when the same effective grain volume is considered. Similar to the case of the textured stainless steel, it is seen that when $k_{0}V_{\text{mean}}^{1/3} = O(1)$, deviation from the sphericity of the grains decreases the attenuation slightly. Furthermore, the grain elongation causes a delay in transition to the “geometric” frequency regime. It is also observed that in the “stochastic-geometric” transition regime, slender grains cause more attenuation than the more flattened grains.

**CONCLUDING REMARKS**

We have applied the unified theory of Stanke and Kino [3] to determine the propagation constants in a textured polycrystalline material with elongated grains where the crystallites have cubic symmetry. We have presented computed ultrasonic wave propagation characteristics in two different medium: 1) [001] aligned stainless steel and 2) aluminum with randomly oriented grains. Our numerical results show that the attenuation is largely controlled by grain volume. The predominant effect of grain shape is to alter the onsets of the “Rayleigh-stochastic” and the “stochastic-geometric” transition regimes and the extent of each of these frequency regimes. The effect of grain shape on phase velocity is also quite small.

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REFERENCES


