PRINCIPAL SURFACE WAVE VELOCITIES IN THE POINT FOCUS
ACOUSTIC MATERIALS SIGNATURE V(z) OF AN ANISOTROPIC
SOLID

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ABSTRACT. This paper draws attention to the fact that point focus beam (PFB) V(z) curves of
anisotropic solids are dominated by a small number of principal SAW rays, which are associated with
directions in which Rayleigh and pseudo-SAW slownesses are stationary. This is explained on the basis of
the complex mean reflectance function of the surface, i.e. the reflectivity \( R(\theta, \varphi) \) averaged over the
azimuthal angle \( \varphi \), and also in terms of a ray model. This fact facilitates the extraction of materials
information from PFB V(z) curves. The utility of PFB AM for anisotropic solids is illustrated using
computed velocity and V(z) data for the Fe(001) surface and for Fe(001) with an overlayer of isotropic
iron.

INTRODUCTION

Acoustic microscopy (AM) investigations [1,2,3] of anisotropic solids are, more often
than not, conducted with line focus beam (LFB) [4,5] or non-axially symmetric point focus
beam (PFB) lenses. [6,7,8] In LFB AM individual V(z) curves are obtained for each value of
the azimuthal angle, \( \varphi \), yielding the directional dependence of the Rayleigh (RW) and in
certain cases, pseudo surface acoustic wave (PSAW) and lateral wave velocities. The purpose
of this paper is to draw attention to the fact that PFB V(z) signals obtained on anisotropic
solids, despite the folding of the \( \varphi \)-dependence of the reflectivity into a mean reflectance
function \( \bar{R}(\theta) \) and thereby single V(z) curve, are dominated by a small number of principal
SAW rays, which are associated with directions in which the RW or PSAW phase slowness is
stationary. This renders the extraction of materials information from a PFB V(z) curve more
manageable. Practical advantages of using PFB rather than LFB in this context are that time is
saved in measuring one, rather than a family of V(z) curves, and the technical difficulty of
maintaining the focal line accurately parallel to the specimen surface is obviated.

To illustrate the utility of PFB AM for the characterization of anisotropic solids, we
treat the surface dynamics of the Fe(001) surface. We show that the spatial Fourier spectrum of
the computed V(z) curve contains four peaks, corresponding to four principal SAW slownesses. Two of these are associated with the RW, one for propagation in the [100] direction where the RW slowness is a maximum, and the other for propagation in an oblique
direction where the RW slowness is a minimum. The other two peaks are associated with a
maximum and a minimum in the PSAW slowness. In spite of the damping of the SAW by the fluid loading of the surface, the principal SAW slownesses are obtained to within about 0.2% of the values calculated from the dynamic Green’s function for the free surface.

As a second illustrative example, we consider the surface dynamics of an isotropic iron layer bonded to the Fe(001) substrate. We show that the trend, with increasing layer thickness \( d \), towards isotropy and a single SAW velocity, can be traced with reasonable accuracy in the behaviour of the \( V(z) \) curve. The four velocities become two, and ultimately for sufficiently large \( d \), just the single velocity for isotropic iron.

**ACOUSTIC MATERIALS SIGNATURE V(z) OF AN ANISOTROPIC SOLID**

In PFB acoustic microscopy of anisotropic solids the acoustic material’s signature \( V(z) \) is given by \[ V(z) = \int_0^\pi P(\theta) \overline{R}(\theta) \exp(-2ikz \cos \theta) d\theta, \]

where \( P(\theta) \) is the aperture function of the lens, assumed here to be axially symmetric and dependent only on the angle, \( \theta \), from the surface normal,

\[ \overline{R}(\theta) = \frac{i}{2\pi} \int_0^{2\pi} R(\theta, \varphi) d\varphi, \]

is the complex mean reflectance function of the surface, i.e. the reflectivity \( R(\theta, \varphi) \) averaged over the azimuthal angle \( \varphi \), \( k = 2\pi f / v \) is the wave number in the fluid coupling medium, \( f \) is the frequency, \( v \) is the sound speed in the fluid, and \( z \) is the distance of the focal point of the lens from the specimen surface.

\( R(\theta, \varphi) \) can vary considerably with \( \varphi \), as shown by Figs. 1 and 2, which depict the magnitude \( R \) and phase \( \Phi \) of the reflectivity \( R(\theta, \varphi) \) for the water loaded Fe(001) surface for (010) (\( \varphi = 0^\circ \)) and (1 10) (\( \varphi = 45^\circ \)) orientation of the sagittal plane. Each pronounced feature in the reflectivity, at angle \( \theta \), is in one-to-one correspondence with a SAW slowness, \( s = s_{\text{water}} \),

\[ V(z) = \int_0^\pi P(\theta) \overline{R}(\theta) \exp(-2ikz \cos \theta) d\theta, \]

where \( s_{\text{water}} \) is the sound speed in water, taken as 1.509 mm/\( \mu \)s here.

For \( \varphi = 0^\circ \) the features labeled a, b, c and D correspond respectively to the longitudinal lateral wave slowness \( s = \sqrt{\rho / C_{11}} \), the transverse lateral wave slowness \( s = \sqrt{\rho / C_{44}} \), the threshold bulk wave slowness (for which \( \vec{s} \) is inclined to the surface but with the corresponding ray velocity \( \vec{v} \) parallel to the surface), and the RW slowness. Beyond c there is no bulk wave into which the incident wave can be transmitted, and so \( R = 1 \). The Rayleigh wave slowness is characterized by a rapid decrease in the phase \( \Phi(\theta) \) by approximately \( 2\pi \), and there are sharp kinks in \( \Phi(\theta) \) at a, b and c.

For \( \varphi = 45^\circ \) the features labeled d, e and A correspond respectively to the longitudinal lateral wave slowness \( s = \sqrt{2\rho / (C_{11} + C_{12} + 2C_{44})} \), the threshold slowness for sagittally polarized displacements, and the PSAW slowness. The PSAW in this symmetry plane is a pure two component supersonic SAW. The bulk wave continuum of SH polarized waves extends some way beyond the PSAW slowness, but is uncoupled from the incident pressure wave in the fluid.
FIGURE 1. $R(\theta, \varphi = 0^o)$ for the Fe(001) surface.

FIGURE 2. $R(\theta, \varphi = 45^o)$ for the Fe(001) surface.

FIGURE 3. Im $G_{33}(\xi_n)$ for the Fe(001) surface.

A global view of the surface dynamics of the Fe(001) surface is provided by Fig. 3, which shows, in gray scale, Im $G_{33}(\xi_n)$, the imaginary part of the Fourier domain surface dynamic Green’s function for force and displacement normal to the surface. The darkness of the image is a measure of the weighted density of bulk and surface modes. The lateral waves show up as lighter lines against a dark background, the threshold slownesses form the boundary between dark (supersonic) and bright (subsonic) regions, and the RW and PSAW are the intense narrow dark lines. A small amount of damping has been incorporated into the
calculations to give the RW a finite width and render it visible in the diagram. The seven symmetry plane slownesses a, b, c, d, e, D and A are evidently all limiting values of SAW slownesses with variation of $\varphi$, as one would expect on symmetry grounds. In addition there are two non-symmetry limiting slownesses, C in a direction 21° from the <100> axis, where the RW slowness is a minimum, and B in a direction 27° from the <100> axis, where the PSAW slowness is a maximum. Table 1 lists all the limiting slownesses $s_{ij}$ for the Fe(001) surface, and related information which is discussed below.

Figure 3 has been calculated for the free Fe(001) surface. With fluid loading, the RW and PSAW become appreciably damped through being able to radiate their energy into phase matched bulk waves in the fluid, and a similar calculation to Fig. 3 shows the sharp RW and PSAW curves broadened into bands. Correspondingly, there is a finite rather than infinitesimal angular range over which $\Phi(\theta)$ undergoes its large decrease. The broadening of the RW and PSAW resonances are accompanied by only very slight shifts to their slownesses, and so these slownesses can still be measured accurately. In calculating the mean reflectance function $\bar{R}(\theta)$, it is nevertheless illuminating to consider the limiting case of weak loading of the solid surface by a fluid of low acoustic impedance $Z_f$ compared with that of the solid, $Z_s$. Such would be the case, for instance, with air-coupled transduction.

Figure 4 shows, for loading by a hypothetical fluid with velocity that of water but with its density and acoustic impedance smaller by a factor of 50, $\bar{R}(\theta)$ for Fe(001) in the angular range of the RW and PSAW. Both the phase and magnitude of $\bar{R}(\theta)$ show characteristic singularities at the four limiting RW and PSAW slownesses, in the form of a discontinuity, with finite slope to one side and a power law type variation on the other side of the discontinuity. The sharp dips C and D in $\bar{R}$ arise in the integration over an angular range in which $|\varphi(\theta, \varphi)| = 1$. There is, however, pronounced variation of $\Phi(\theta, \varphi)$ in this range, and as a result $\bar{R}$ ends up being less than 1 through phase cancellation. As the integration path, with variation of $\theta$, approaches tangency with the RW curve at the stationary point C or D, singular behavior is to be expected in $\bar{R}$ and $\Phi$. Similar comments apply to the PSAW stationary slowness points A and B.

Figure 5 shows $\bar{R}(\theta)$ for the water-loaded Fe(001) surface. Even though $\bar{R}(\theta)$ differs markedly from the weak loading case, there are still clearly identifiable sharp peaks and troughs in $\Phi(\theta)$ occurring within about 0.2% of the angles corresponding to the limiting RW and PSAW slownesses.

### Table 1. Principal RW and PSAW for the Fe(001) surface.

<table>
<thead>
<tr>
<th>Slowness</th>
<th>$\varphi$(deg)</th>
<th>$\Theta$(deg)</th>
<th>$s_{ij}$ ((\mu s/mm))</th>
<th>$v_{ij}$ (mm/(\mu s))</th>
<th>$\Delta z$ (mm)</th>
<th>$v_{SAW}$ (mm/(\mu s))</th>
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</thead>
<tbody>
<tr>
<td>A RW min</td>
<td>45</td>
<td>28.28</td>
<td>0.3140</td>
<td>3.185</td>
<td>0.02819</td>
<td>3.190</td>
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<tr>
<td>B PSAW max</td>
<td>29</td>
<td>28.90</td>
<td>0.3210</td>
<td>3.115</td>
<td>0.02668</td>
<td>3.109</td>
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<tr>
<td>C RW min</td>
<td>21</td>
<td>30.80</td>
<td>0.3384</td>
<td>2.955</td>
<td>0.02382</td>
<td>2.950</td>
</tr>
<tr>
<td>D RW max</td>
<td>0</td>
<td>31.50</td>
<td>0.3461</td>
<td>2.889</td>
<td>0.02287</td>
<td>2.895</td>
</tr>
</tbody>
</table>
SAW VELOCITIES OBTAINED FROM PFB $V(z)$

Figure 6 shows the calculated PFB $V(z)$ of the water loaded Fe(001) surface, for an operating frequency of 225 MHz. Evidently there are several periods in the complicated variation of $V$ with $z$, and this becomes clear in the fast Fourier transform (FFT) $F(k)$ of $V(z)$, shown in Fig. 7 (differing windowing functions have little effect on this result). In the wave number interval $4k \in [130,190] \text{mm}^{-1}$ there are four peaks in $F(k)$, which correspond to the spatial periods, $\Delta z = 1/k$, listed in Table 1. The corresponding SAW velocities, calculated by the standard ray model relation [1,2,3],

$$v_{SAW} = \frac{v_{water}}{\sqrt{1 - \left(1 - \frac{v_{water}}{2f\Delta z}\right)^2}} ,$$

agree with the limiting RW and PSAW velocities $v_p$ to within about 0.2% (see Table 1).
RAY MODEL INTERPRETATION

Figure 8 suggests a simple ray model interpretation of the limiting slownesses (a similar line of reasoning has been advanced by Kushibiki et al. [8]). The oscillations in $V(z)$, as is well known, can be considered to result from the interference of two (or more) rays. One is the ray along the axis of the lens, which is incident normally on the specimen surface, specularly reflected and then retraces its path back up the lens. The other is the ray which is incident on the surface at the Rayleigh angle,

$$\theta_R = \arcsin\left(\frac{v_{\text{fluid}}}{v_{\text{solid}}}\right),$$

exciting a phase matched leaky Rayleigh wave in the surface, which subsequently radiates back into the fluid. It is only the ray which is radiated from the diametrically opposite point to the point of incidence that ends up traveling directly up the lens and contributing to the signal. It is precisely this argument which, for an isotropic solid or LFB AM of an anisotropic solid, leads to Eq. (4). In the case of an anisotropic solid, there is a distribution of Rayleigh incidence angles $\theta_R(\phi)$, which correspond to rays incident on the surface along the non-circular curved path depicted schematically in Fig. 8. The surface wave which is excited at point 1, which is a point of stationary slowness (not necessarily lying in a symmetry direction) has surface ray and slowness vectors which are parallel to each other (since the ray velocity is normal to the slowness curve), and so the ray passes through the origin 0 to arrive at the inversion point 1' where its radiation into the fluid ends up traveling directly up the lens, and contributing to $V(z)$. The ray which is excited at the non stationary slowness point 2, on the other hand, veers off to the side (an effect known as beam walk off [7]) and does not pass through the origin to reach its symmetrically opposite point. Instead it arrives at the excitation contour at point 2', from where its radiation does not pass directly up the lens. As a result, this ray does not contribute to $V(z)$. Thus, only stationary slowness points such as 1 play a significant role in determining $V(z)$.

![Figure 8. Ray model for the principal rays.](image-url)
ISOTROPIC LAYER ON AN ANISOTROPIC SUBSTRATE

We consider here the effects on the surface dynamics of a polycrystalline Fe layer (with Voigt averaged elastic constants) bonded to the Fe(001) surface. The evolution of the RW and PSAW velocities with increasing layer thickness $d$ is shown in Fig. 9. For $d$ less than about $1 \mu m$, there are two limiting RW velocities, which converge towards each other with increasing $d$, and two limiting PSAW velocities which likewise converge towards each other. Beyond about $1 \mu m$ there is only one RW and one PSAW limiting velocity, and these two velocities converge towards each other with increasing $d$. By $d = 20 \mu m$ there is effectively only one directionally independent SAW velocity, which is that of isotropic iron.

Figure 10 shows the variation with $d$ of the limiting RW and PSAW velocities, as determined from $\text{Im} G_{33}(s, \omega)$ (the dotted curves) and from the FFT of $V(z)$ (the solid curves). There is overall consistency between the two sets of curves, albeit not exact agreement. For $d < 0.1 \mu m$ four clear peaks can be resolved in the FFT of $V(z)$, and the surface wave velocities they yield are in reasonable agreement with the values determined from $\text{Im} G_{33}(s, \omega)$. Beyond $d = 0.2 \mu m$ the two RW peaks in the FFT overlap to the extent that they can no longer be resolved, yielding a single peak intermediate between the limiting RW velocities determined from $\text{Im} G_{33}(s, \omega)$. Similar comments apply to the PSAW velocities. Beyond $d = 1 \mu m$ the single RW and PSAW velocities, as obtained by the two means, show similar behavior, converging towards each other, and merging at about $d = 20 \mu m$.

![Figure 9](image-url). Directional dependence of the RW and PSAW velocities for the Fe(001) surface with an isotropic layer of iron of various thicknesses $d$ bonded to it.
DISCUSSION

We have shown that point focus beam acoustic microscopy, in spite of the azimuthal averaging of the surface dynamics that it entails, yields the limiting values of the directionally dependent RW and PSAW slownesses for an anisotropic solid. It is also able to trace the variation of these velocities when a layer of different elastic characteristics is bonded to the substrate. These findings provide a case for the more widespread use of PFB AM in the characterization of anisotropic solids and thin supported layers. Other problems to which this approach can be applied are, for instance, the determination of crystallographic orientation of grains, and the measurement of near surface elastic properties.

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REFERENCES