ERRORS IN THE MEASUREMENT OF ULTRASONIC PHASE VELOCITY IN THE CONTEXT OF MATERIALS EVALUATION

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ABSTRACT. The influence of timing jitter and additive noise on estimates of the phase velocity of acoustic waves propagated through a test medium is considered. The relation between the bias and standard deviation of the estimates and noise parameters is derived and verified experimentally. These expressions allow optimizing measurement conditions, either by choosing the number of measurements to ensure the desired accuracy or by assessing the accuracy of a single measurement.

INTRODUCTION

The propagation properties of ultrasound through a test medium can be used to characterize the state of the medium in the laboratory or on-line in a process. The most important parameters are the absorption coefficient (evaluates loss mechanisms) and the phase velocity (evaluates elastic moduli). Acoustic spectroscopy allows these to be determined over wide frequency ranges [1]. To reduce the measurement time pulse excitation is commonly used, and subsequent spectral analysis extracts the required information via the fast Fourier transform (FFT) [2]. This technique allows, for example, tracking phase transitions by monitoring phase velocity in a curing polymer [3]. Acoustical spectroscopy measurements are subject to errors because various kinds of noise corrupt the true signal. This raises three problems: (1) what is the accuracy of the measurement under given noise conditions; (2) how to optimize the test cell design to get highest possible accuracy; (3) how to ensure the desired accuracy in a given instrument. In principle the effect of noise can be reduced through coherent averaging of raw or partly processed data. However this approach is not always suitable for a rapidly changing medium. It is possible to assess instrument accuracy by trial and error, but this is a costly and time-consuming process. It is therefore necessary to formally investigate the influence of instrument noise on acoustical spectroscopy measurements.

This paper establishes analytical relationships between statistical descriptions of equipment noise and errors in the estimation of acoustic phase velocity, with experimental verification. The approach used is based on the expansion of the operational function used to calculate phase velocity from raw ultrasonic propagation data into a Taylor series. A similar technique has been applied earlier in the context of estimates of ultrasonic absorption coefficients as functions of frequency [4].
INFLUENCE OF RANDOM NOISE ON PHASE VELOCITY ESTIMATES

In a typical measurement a received signal $S(\omega)$ is corrupted by noise $N(\omega)$; it is related to wave propagation variables by

$$S(\omega) = S_0(\omega) + N(\omega) = S_e(\omega) \exp[-\alpha(\omega)d \text{ } \times \exp[- j \omega \frac{d}{c(\omega)}] + N(\omega), \quad (1)$$

where $S_e(\omega)$ is the excitation spectrum multiplied by the electronic signal pathway transfer function, $\alpha(\omega)$ is the absorption coefficient, $c(\omega)$ is the ultrasonic phase velocity, $d$ is the propagation distance in the medium.

The acquisition of the received signal is associated with a delay, $\tau$, between the timing signal which triggers data acquisition and the actual start time of the acquisition process. $\tau$ is a random variable associated with uncertainty in trigger and control circuits. Taking this into account we can derive the following expression for phase velocity $c(\omega)$:

$$c(\omega) = \frac{d}{\tau - \frac{\Delta \phi}{\omega}} = \frac{d}{\tau + \frac{1}{\omega} \arctg \frac{\text{Im}(S_e)}{\text{Re}(S_e)} - \frac{1}{\omega} \arctg \frac{\text{Im}(S)}{\text{Re}(S)}}, \quad (2)$$

where $\Delta \phi$ is the phase spectrum of the received signal; the second term in the denominator is to compensate for the delays in the electronic signal pathway and is constant for a given apparatus, and the third term accounts for the medium dispersion and is subject to noise influence.

We can expand the third denominator term assuming relatively low noise amplitude ($|S_0| \gg |N|$):

$$\arctg \frac{\text{Im}(S)}{\text{Re}(S)} = \arctg \frac{\text{Im}(S_0) + \text{Im}(N)}{\text{Re}(S_0) + \text{Re}(N)} = \arctg \frac{\text{Im}(S_0)}{\text{Re}(S_0)} + \frac{\text{Im}(N) \text{Re}(N)}{\text{Re}(S_0)^2} \frac{\text{Im}(S_0)^2 - \text{Re}(S_0)^2}{|S_0|^4} = \arctg \frac{\text{Im}(S_0)}{\text{Re}(S_0)} + R, \quad (3)$$

where $R$ is a random variable incorporating the influence of noise. This gives the phase velocity as

$$c(\omega) = \frac{d}{\tau_0(\omega) - \frac{R}{\omega}}, \quad (4)$$

where

$$\tau_0(\omega) = \tau + \frac{1}{\omega} \arctg \frac{\text{Im}(S_e)}{\text{Re}(S_e)} - \frac{1}{\omega} \arctg \frac{\text{Im}(S_0)}{\text{Re}(S_0)}, \quad (5)$$
In the case of relatively small errors \((R/\omega \ll \sigma_0)\) we can expand equation (4) into a Taylor series neglecting terms of order greater than unity:

\[
c = \frac{d}{\tau_0(\omega)} + \frac{d}{\tau_0(\omega)} \frac{R}{\omega \tau_0(\omega)} = c_0 \left[ 1 + \frac{R}{\omega \tau_0(\omega)} \right],
\]

where the true phase velocity is

\[
c_0(\omega) = \frac{d}{\tau_0(\omega)}. \tag{7}
\]

Therefore the relative error in the phase velocity becomes

\[
\epsilon_c = -\frac{R}{\omega \tau_0(\omega)}, \tag{8}
\]

and is determined by the influence of the random noise through \(R\).

**GAUSSIAN ADDITIVE AND FRAME JITTER NOISE AS TYPICAL RANDOM ERRORS**

Frame jitter noise derives from the random shifts in the start time of the data acquisition process. We have found experimentally that both additive Gaussian noise and frame jitter have effects on phase velocity estimates. It is convenient to analyze their influence separately and then to combine the results of both into a single expression.

The additive Gaussian noise can be determined in the frequency domain as follows \[5\]:

\[
\begin{align*}
\text{Re}(N) &= R_1\sigma_a, \\
\text{Im}(N) &= R_2\sigma_a,
\end{align*} \tag{9}
\]

where \(R_1, R_2\) are the random variables \(N[0,1]\), and \(\sigma_a\) is the standard deviation of the noise. It yields for the random term:

\[
R = \sigma_a \frac{\text{Re}(S_0)R_1 - \text{Im}(S_0)R_2}{|S_0|^2} + \sigma_a^2 \frac{\text{Im}(S_0)\text{Re}(S_0)}{|S_0|^4} \left[ R_1^2 - R_2^2 \right] + \sigma_a^2 \frac{R_1 R_2}{|S_0|^4} \left[ \text{Im}^2(S_0) - \text{Re}^2(S_0) \right].
\]

As the error in the phase velocity is proportional to \(R\), its expectation is proportional to the expectation of \(R\):

\[
E[\epsilon_c] = E(R) = 0, \tag{10}
\]

because

\[
E[R_1] = E[R_2] = E[R_1^2 - R_2^2] = E[R_1 R_2] = 0.
\]

This implies that additive noise does not cause bias in phase velocity estimates.
Considering the second order term, the variance of the relative errors in phase velocity is

\[
E[e^2] = \frac{\sigma_p^2 \left[ \text{Re}^2(S_0) + \text{Im}^2(S_0) \right]}{\omega^2 \tau_o^2(\omega) S_0^4} = \left[ \frac{\sigma_p}{\omega \tau_o(\omega) S_0} \right]^2 = \frac{1}{\omega \tau_0(\omega) \text{SNR}}^2 = \sigma_e^2, \tag{11}
\]

where SNR is signal-to-noise ratio and directly influences phase velocity error.

Using conventional ideas of time and frequency equivalence it can be shown that the frame jitter results in the following noise spectrum:

\[
\text{Re}(N(\omega)) = -\text{Im}(S_0(\omega)) \sigma_j \omega R_i \\
\text{Im}(N(\omega)) = \text{Re}(S_0(\omega)) \sigma_j \omega R_i,
\]

where \( \sigma_j \) is the standard deviation of the frame jitter, and \( R_i \) is a random variable \( N[0,1] \).

The substitution of (12) into the expression for \( R \) yields:

\[
R = \sigma_j \omega R_i. \tag{13}
\]

Consequently frame jitter does not bring about any bias in phase velocity estimates provided that it is distributed symmetrically about zero time shift. The standard deviation of the relative error in the phase velocity influenced by frame jitter is

\[
\sigma_e = \frac{\sigma_j}{\tau_o(\omega)}. \tag{14}
\]

This is equivalent to a simple timing error with imposed statistical variation. The overall standard deviation for phase velocity errors is obtained by combining the result for frame jitter and additive noise, thus

\[
\sigma_e = \frac{1}{\tau_0(\omega)} \sqrt{\sigma_j^2 + \left( \frac{1}{\omega \text{SNR}} \right)^2}. \tag{15}
\]

**OPTIMAL CHOICE OF MEASUREMENT DISTANCE**

The acoustic path length in the test medium (the gauge length) affects the received SNR through the influence of attenuation in the test medium. The wave propagation time, on which phase velocity estimates are based, is proportional to the gauge length. Gauge lengths which are either below or above an optimal value bring about uncertainties in attenuation measurement [4]. It is therefore appropriate to derive an expression that relates phase velocity errors to gauge length. This will enable gauge length to be set to a value which minimizes errors in estimates of phase velocity. Using equation (1), equation (15) can be re-expressed as follows:
Here the first term reflects the influence of the frame jitter noise, and the second term reflects the influence of the Gaussian additive noise. In the general case it is necessary to find the minimum of this expression which gives an optimal value \( d \). It can be shown that the additive noise term reaches a minimum when

\[ \alpha d = 1. \]  

This is the condition for minimizing errors in the absorption coefficient measurement [4]. It is perhaps obvious that an increase in \( d \) will improve the estimate of phase velocity through the jitter noise term in equation (16). However this effect is limited by the influence of \( d \) on the second term in equation (16). Fig. 1 shows the standard deviation of phase velocity plotted versus \( d \) for a typical measurement in a low absorption medium such as water (solid line, \( f = 10 \text{ MHz}, c_0 = 1500 \text{ m/s}, \sigma_j = 1 \text{ ns}, \sigma_a = 2 \text{ Nepers/m}, \text{SNR} = 50 \)). In this case the minimum error is achieved at \( d = 0.75 \text{ m} \). The dotted line in fig. 1 shows that an increase in SNR to 100 reduces the error and the minimum is achieved at \( d = 0.95 \text{ m} \). With an increase in absorption coefficient to 20 Nepers/m the optimum value of \( d \) reduces to 0.07 m (dashed line in fig.1) and the minimum error increases. Clearly the optimal gauge length depends on both terms, although equation (17) can be used as a rough initial estimate.

The above considerations apply to estimates of absolute values of phase velocity. In many circumstances it is convenient to estimate phase velocity dispersion, which is the difference between phase velocity at any frequency in the spectrum and the phase velocity at some notional reference frequency. In this case the influence of jitter is the same for both the test frequency and the reference in a given frame and only the second term in equation (16) applies; the optimal measurement condition then reduces to equation (17).

![Graph showing the relationship between relative standard deviation and gauge length](image-url)

**FIGURE 1.** Simulated errors in estimates of phase velocity at 10 MHz with jitter noise standard deviation \( \sigma_j=1 \text{ ns} \), low frequency phase velocity \( c=1500 \text{ m/s} \) and (solid line) SNR=50, \( \alpha=2 \text{ Nepers/m} \); (dotted line) SNR=100, \( \alpha=2 \text{ Nepers/m} \); and (dashed line) SNR=50, \( \alpha=20 \text{ Nepers/m} \).
EXPERIMENTAL RESULTS

Experimental results were obtained from a pulse echo system with water as the test medium and a gauge length of 1.5 mm (propagation distance 3 mm). A 20 MHz center frequency transducer was used (Panametrics V317) connected to a pulser receiver (UPR, NDT Solutions Ltd) [6]. A LeCroy 9450 digital storage oscilloscope was used as a digitizer and 1000 frames were acquired at a sampling frequency of 400 MHz. The average spectrum is shown in fig.2 together with the equipment noise level calculated using a simple model [4]. The average phase spectrum, which is equal to $\arg(S_e)$, is presented in the fig.3 in both wrapped and unwrapped forms.

The relative standard deviations obtained experimentally and using equation (16) are shown in fig.4 together with the standard deviation due to the frame jitter contribution (first term in equation (16)). The standard deviation of the frame jitter noise was estimated to be 0.72 ns. Fig.5 presents the experimentally obtained bias in the phase velocity estimates which was very small and commensurate with our theoretical estimate of zero.

The results for the velocity dispersion measurement are presented in fig.6. In this case the phase velocity at 21 MHz was used as the reference value, the standard deviation being zero at this frequency. The relative bias for the dispersion measurement was found to be similar to the data in fig.5 for the absolute phase velocity measurement.

FIGURE 2. Amplitude spectrum of the received ultrasonic signal $S(\omega)$ (solid line) after 1000 coherent averages, and the modeled additive noise (dashed line).

FIGURE 3. Averaged phase spectrum of the received signal $S(\omega)$: wrapped (dashed line) and unwrapped (solid line).
FIGURE 4. Errors in estimates of phase velocity expressed as standard deviations: solid line – calculated values, crosses – experimental values, dashed line – calculated frame jitter noise term.

FIGURE 5. Relative bias in experimental phase velocity.


The results indicate that the theoretical and experimental standard deviations are in close agreement. The effects of frame jitter dominate the phase velocity errors in the center of the transducer passband, around 20 MHz. Outside of the passband errors rise steeply due
to diminishing signal levels which result from the combined effects of the transducer response and attenuation in the test medium. At the extremes of the passband the theory overestimates observed errors although the trends are compatible between theory and experiment. This overestimate is probably due to the approximations implicit in equation (3) combined with our use of a much simplified noise model for the electronic system. The experimental values of the bias are very small in the passband, which was predicted theoretically, and increase as SNR reduces. Comparison of figures 4 and 5 indicate clearly that the dispersion measurement, by avoiding the effect of jitter, results in significantly reduced variance compared to measurements of absolute velocity.

CONCLUDING REMARKS

Errors in ultrasonic phase velocity estimation arise from a combination of frame jitter noise and additive noise in the electronic equipment used for the measurements. We have presented a simple statistical theory which enables these two effects to be quantified and it has been verified through experiment. The effect of frame jitter diminishes as gauge length is increased as would be expected on the basis of proportionally increased propagation times. The effect of additive noise is more complex since it depends on the product of gauge length and the attenuation coefficient, expressed in Nepers. The optimal attenuation which minimizes the influence of electronic noise on phase velocity measurement is around 1 Neper, higher or lower values bring about increases in estimation error. In earlier work the same condition was found to apply to the measurement of ultrasonic attenuation [4]. For a given measurement condition (gauge length, attenuation value) the optimum value of gauge length will depend on both frame jitter, attenuation, and equipment SNR. This can be calculated using the analysis proposed in this paper. In an experiment in water we found that under the favorable signal to noise conditions around the transducer center frequency errors in phase velocity estimates were dominated by frame jitter noise. Away from the transducer midband additive noise was seen to dominate. In regions where both types of noise are significant equation (16) can be used to calculate required SNR for a given gauge length and a required maximum limit to the variance of the phase velocity measurement. This, combined with knowledge of the raw equipment SNR will enable the required number of coherent averages to be established a priori. Bias errors in phase velocity estimates were found to be small and independent of both frame jitter and additive noise. Overall, our experiments confirm that the relative standard deviation of errors in phase velocity estimates as low as 5*10^-4 can be achieved using commercially available equipment. It is therefore possible to employ ultrasonic phase velocity measurements as an adjunct to attenuation measurements in ultrasonic spectroscopy applied to materials evaluation.

REFERENCES

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