REPEATABILITY OF AN ABLATION SOURCE USING A TIME-FREQUENCY REPRESENTATION

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ABSTRACT. Ultrasonic Lamb waves attenuate when propagating through viscoelastic plates. Experimental attenuation values are usually obtained with transducers by comparing measured time-domain signals at two different locations. However, this method can measure only a few points on the attenuation-frequency dispersion curves, and usually only for some specific mode. It has been shown that multi-mode dispersion curves (plus their associated energy distribution) can be obtained by operating on the transient, time-domain signals, generated and detected with laser ultrasonics. This technique uses proper signal processing technique, namely, a time-frequency representation (TFR). The current research uses a similar methodology for Lamb wave attenuation measurements which provides a series of attenuation values for many modes. As a first step, the laser source repeatability (both amplitude and frequency spectrum) must be verified. This research studies the variation of a laser ablation source by considering the surface wave propagating on an isotropic half-space. A series of experimental time-domain surface wave signals from different propagation distances are operated on with a TFR and compared in, strictly, propagation-distance-independent energy slowness-frequency domain. The amplitude constant is defined for comparison purposes. Also, the proposed signal processing technique is instituted in an error-free manner.

INTRODUCTION

In plate-like structures, ultrasonic Lamb wave attenuation is one of the important characteristics of viscoelastic materials. This property is normally presented as a function of frequency and individual modes, as shown in Figure 1. Note that these curves are the imaginary parts of the wavenumbers which satisfy the complex dispersion relation at a given frequency. The dispersion relation for viscoelastic materials is the same as that of elastic materials, except that all wave speeds (longitudinal and shear) and wavenumber are complex numbers. To experimentally obtain those curves, conventional single-source generation, multi-receiver detection using transducers can be employed. Time-domain signal decay between two locations can be used to calculate attenuation. However, this method usually gives only a few points in a narrow frequency bandwidth and for only a single mode, due to the limitations of transducers. This research proposes using laser ultrasonic techniques to experimentally measure attenuation in viscoelastic plates; laser ultrasonics enables the broadband generation and detection of multi-mode Lamb waves. Moreover, the laser system is a non-contact, point-like system, which minimizes the
source/receiver variation due to other factors such as transducer coupling or pressure on transducers, and reduces diffraction and near field effects.

Previous research shows that the use of laser ultrasonics with a time-frequency representation (TFR) signal processing technique is very effective in obtaining the dispersion curves of an isotropic elastic plate [1,2]. These results show how group velocity changes with frequency (in two dimensions), and are qualitatively encouraging, providing an excellent match with numerical simulations. However, the TFR gives three-dimensional results; the third dimension, besides time and frequency, in principle, represents energy content that corresponds to each time-frequency cell. Since this energy dimension has not been quantified, this research studies it, and tries to use it to extract energy information from experimental time signals. This energy information will then lead to attenuation of ultrasonic Lamb wave modes.

To experimentally measure attenuation in viscoelastic plates, the entire process can be divided into three stages. First, source repeatability is studied and verified. Surface wave signals can be used at this stage. These surface waves are non-dispersive and single-mode; they are appropriate to be used as benchmark test signals. Second, the proposed signal processing algorithm is tested with non-stationary, dispersive, and multi-moded signals. Lamb waves in an elastic plate are fitted at this stage, because the attenuation of signal amplitudes is known to follow the $1/V$ spreading rule. Lastly, the same algorithm can be applied to time-domain Lamb wave signals from a viscoelastic plate. Attenuation in this plate can be obtained after accounting for geometric spreading. This paper presents the results of the first stage - source repeatability.

**TIME-FREQUENCY REPRESENTATION**

The time-frequency representation (TFR) shows energy localization in both the time and frequency domains. The TFR gives information as to the time at which each frequency component arrives. Among the many candidate TFRs, a windowed Fourier

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**FIGURE 1:** Attenuation of Lamb waves.
transform or short-time Fourier transform (STFT) has proved to be appropriate for transient, multi-mode Lamb wave signals, therefore an STFT will be used for transient surface wave signals.

The concept of the STFT is that the time-domain signal is segmented into overlapping portions. Each portion is multiplied by a window function with the same length and then Fourier transformed. Each portion approximates the frequency distribution at the center time in that duration. Mathematically, in discrete format, the STFT value can be written as [3]:

\[
S_f[m,l] = \sum_{n=0}^{N-1} f[n] g[n-m] \exp\left(-\frac{i2\pi n l}{N}\right)
\]  

where \( f[n] \) is the sampled time-domain signal, and \( g[n] \) is a symmetric window of unity norm and length \( N \).

In addition to the STFT, the reassignment method is used to refine the TFR’s resolution. The reassignment algorithm compresses the amplitude contours to their centers of gravity, thus reducing the spread of amplitudes in the time-frequency map [4]. Note that this reassignment algorithm does not improve the accuracy of the TFR. The reassignment procedure has the advantages that it improves the readability of the TFR, and that it refines the fixed time-frequency grid into a fixed slowness-frequency grid. The latter will be very useful when comparing Lamb wave signals with different propagation distances, since a fixed time-frequency grid will result in a variable slowness-frequency grid.

GEOMETRIC SPREADING OF SURFACE WAVES

Laser excitation on the surface of semi-infinite medium (or half-space) naturally generates surface waves. For a given excitation function of source, the transient wave propagation problem, which is a boundary-value problem, can be solved conventionally by integral transform techniques [5]. Since every function can be decomposed into the superposition of time-harmonic functions by the use of a Fourier series, it is reasonable to consider the time-harmonic solution (or solve the problem in frequency domain). As a result, the problem requires only a single transformation in space. In the present case, when the point excitation is the normally applied stress, time-harmonic solutions (stresses and displacements) for isotropic elastic half-space can be written in the form:

\[
a(r, z, t) = A(r, z) e^{i(br - \omega t)}
\]  

where \( a \) is any field quantity, \( A \) is a complex amplitude, and cylindrical coordinates are employed – \( r \) is distance from the source, \( z \) is the distance into the medium, and \( t \) is time. Following Miller and Pursey [6], the amplitude on the surface, \( A \) can be separated as:

\[
A(r, 0) = \frac{A_0}{\sqrt{r}}
\]  

where \( A_0 \) is a constant amplitude. From (3), clearly, the magnitudes of any quantities decrease as \( 1/\sqrt{r} \).

EXPERIMENTAL PROCEDURE

This study uses an aluminum half-space specimen. A Q-switched, Nd:YAG laser, with the pulse energy that generates ultrasonic waves in ablation regime, is used to generate
ultrasonic waves. The ablation source behaves like a normally applied stress source, which naturally generates a surface wave. On the detection side, a heterodyne interferometer is used to detect the out-of-plane motions at the specimen surface. The measured time-domain signals, for propagation distances ranging from 30 to 79.5 mm every 1.5 mm, are digitally recorded by an oscilloscope, and transferred to a personal computer.

The time-domain signals are operated on with the STFT. The resulting TFR is then reassigned. In order to make the TFR independent of propagation distance, the time axis is normalized to energy slowness ($S_L$) axis by dividing by propagation distance. The TFR now presents the dispersion curve in energy slowness-frequency domain, which in this case, is expected to consist of a single, non-dispersive mode propagating with the surface wave slowness. From the reassigned STFT, the amplitude constants, defined as the real part of $A_0$ in (2), at some typical frequencies are calculated, and then plotted against the propagation distance, $r$. Note that in this study, a rectangular window of 385-point length and overlapping of 192-point length (from the sampling frequency of 100 MHz) is used in the STFT algorithm.

To make sure that the results are free from any possible errors introduced by the processing, the cases when the window does not cover the whole part of the surface wave signal are excluded. The variation in the remaining results is the variation of laser ablation source only.

RESULTS AND DISCUSSION

A typical time-domain surface wave signal and its spectrum calculated by the normal Fourier transform is shown in Figure 2. This plot shows the high signal-to-noise ratio and broad frequency bandwidth of the optical system. Figure 3 shows typical a TFR in the energy slowness-frequency domain, and its cross-section. This plot shows that energy localization arrives with the slowness of 0.00033-0.00034 s/m which corresponds to the surface wave slowness in aluminum. To calculate the amplitude constant at some specific frequency, the cross-sectional area at that frequency is computed and multiplied by the square root of the propagation distance. The amplitude constants are then plotted versus propagation distances for some specific frequencies as shown in figure 4 and 5. Most of the errors in the amplitude constants (about 94% of the results) are in the order of 7%. A large

![FIGURE 2: Typical time-domain and frequency spectrum of surface wave signals.](image)
variation (larger than 10%) is observed at high frequencies and a large propagation distance. The reasons are discussed as follows.

Looking closely at each time-domain signal, the major differences in shape from one signal to the others is in the trail following the main portion (the smaller peaks following two positive peaks). That part of the signal is higher in frequency than the main part. As a result, the spectrum of the surface wave portion is naturally more sensitive, i.e. more varied, from measurement to measurement, at higher frequencies than at lower frequencies, when compared to the center frequency. Since the rectangular window (which results in the largest sidelobes) is used, this variation at high frequencies is then more pronounced. For large propagation distances (and their associated lower signal-to-noise ratio), the main characteristics of the waves are more easily distorted by other factors such as imperfections in material (both inside and on the surface) and noise in the system. These distortions will introduce some variation to the results especially, again, for the high frequency components.

CONCLUSION

This research uses the STFT technique to verify the $1/r$ geometric spreading of the amplitudes of surface waves. The variation of the amplitude constants is the variation of laser ablation source that is due to the complicated ablation mechanism. The results show that the variation is about 7%, provided that the signal-to-noise ratio is sufficiently large as presented in this research. This condition emphasizes the careful design of the experiments. Besides that, in order to use the TFR to correctly extract the energy (in terms of amplitude) distribution, the parameters of the algorithm are critical and need to be properly selected. In the results shown here, both the repeatability of the source and the processing procedure support the idea of using this methodology to experimentally measure attenuation of Lamb waves in viscoelastic plates.

*FIGURE 3:* Energy slowness-frequency dispersion curve and its cross-section of the surface wave.
FIGURE 4: Experimental amplitude constants vs propagation distance at 0.6, 1.0, 1.4, 1.8 MHz.

FIGURE 5: Experimental amplitude constants vs propagation distance at 2.2, 2.6, 3.0, 3.4 MHz.
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