MODELS FOR CRACK DETECTION IN A CYLINDRICAL HOLE CONTAINING AN ELASTIC LAYER AND A FLUID-FILLED CYLINDRICAL HOLE

J. C. Aldrin\textsuperscript{1}, J. D. Achenbach\textsuperscript{2}

\textsuperscript{1}Computational Tools, Gurnee, IL 60031, USA

\textsuperscript{2}Center for Quality Engineering and Failure Prevention, Northwestern University, Evanston, IL 60208, USA

ABSTRACT. This paper presents a study of ultrasonic NDE models for a cylindrical hole containing an elastic layer and a fluid-filled hole. For the elastic layer case, both dispersion relations and analytical solutions were solved to assess the sensitivity of leaky Rayleigh waves to annulus properties. A BEM model was formulated for the scattering of shear waves by a fluid-filled hole with a notch. Through simulated studies, a viable inspection technique was derived and validated experimentally.

INTRODUCTION

Prior work has shown the feasibility of a two element ultrasonic transducer approach for C-141 weep hole inspection \cite{1,2}. Current weep hole inspection requires that the wet wing be completely purged of fuel before inspection can begin. Due to the time and cost of emptying and drying out a wing, the capability of weep hole inspection for a wing containing fuel is of interest to the Air Force. Prior work has been performed to understand the effect of a fluid-filled cavity on the performance of the two-element weep hole inspection technique for radial fatigue cracks \cite{3-4}. These papers indicated significant challenges for directly applying the dual transducer weep hole inspection approach for top crack detection. However, models addressing the scattering response for an incident shear wave transducer signal and the case of fluid-filled cavity with radial fatigue crack are needed to provide additional insight for a potential ultrasonic inspection approach.

Another factor that may affect the performance of the ultrasonic inspection technique is a polyurathane coating on the inside edge of the C-141 weep holes. This coating is used to protect the edge of the hole from corrosion. The current procedure requires that the coating be removed prior to inspection. A significant reduction in time and effort would be gained by eliminating the need for the removal and the subsequent reapplication of this coating. The development of a model will generate the needed understanding in order to assess the viability of using the weep hole inspection procedure for this case.

This paper presents a study of ultrasonic NDE models for both a cylindrical hole containing an elastic layer and a fluid-filled cylindrical hole. For a cylindrical hole with an elastic annulus representing the layer, the dispersion relations are first derived and solved numerically for the phase velocity of the lowest modes. An analytical solution for the scattering response to an incident plane shear wave by a cylindrical hole with an elastic annulus is also derived. Parametric studies are performed to assess the sensitivity of leaky Rayleigh waves to variation in the properties of the annulus. A BEM (boundary element
method) model is formulated for the scattering response of an incident shear wave by a fluid-filled hole with a notch. Through simulated measurement results, the measured signals are explored for potential use in crack detection.

MODELS

Figure 1(a) displays an elastic annulus embedded in an infinite elastic medium. The approach to derive the dispersion relation will be similar to the approach used by Epstein for the composite cylinder [5]. The general solution for the scalar and vector potentials for two dimensional plane strain elasticity in terms of cylindrical coordinates is given by:

\[
\phi_1(r, \theta, t) = \left( A[J_n(\alpha r)] + B[Y_n(\alpha r)] \right) e^{-i \omega t} \sin n \theta, \quad (1)
\]

\[
\psi_1(r, \theta, t) = \left( C[J_n(\beta r)] + D[Y_n(\beta r)] \right) e^{-i \omega t} \cos n \theta, \quad (2)
\]

where:

\[
k = \frac{\omega}{c}, \quad \alpha = \frac{c}{c_L}, \quad \beta = \frac{c}{c_T}.
\]

To satisfy the radiation condition for the infinite elastic medium, the general solutions for the scalar and vector potentials are written as follows:

\[
\phi(r, \theta, t) = \left[ E[H_n^{(1)}(i\alpha r)] \right] e^{-i \omega t} \sin n \theta, \quad (3)
\]

\[
\psi_1(r, \theta, t) = \left[ F[H_n^{(1)}(i\beta r)] \right] e^{-i \omega t} \cos n \theta. \quad (4)
\]

These solutions can subsequently be applied to evaluate the displacements, \( u_r \) and \( u_\theta \), and the stresses, \( \tau_r \) and \( \tau_\theta \), for the two regions. Following application of traction free boundary conditions at \( r = a \), and continuity of displacement and traction at \( r = b \), with some manipulation, the homogeneous system of equations is obtained. In order to solve the dispersion relation, solutions in terms of complex angular wave numbers are necessary. The resulting imaginary part of the angular wave number corresponds to the attenuation of the modes necessary for an infinite elastic domain. A program was written in Maple to search for the complex roots in \( n \) for the lowest modes of interest over the range of frequencies corresponding with the inspection problem. With each solution for the complex angular wave number, the phase velocity of the circumferential mode was calculated. For additional details on the solution of this problem, see reference [6].

An extension of this analytical derivation was performed by deriving the transient scattering response due to a plane shear wave incident on a cavity with an elastic layer. Pao and Mow previously investigated the case of a cavity in an infinite elastic medium with an elastic liner for an incident plane longitudinal wave [7]. For the present study, the case of an incident plane shear wave will be solved. To obtain the transient response, a plane wave pulse is first defined in the frequency domain. Then, the expansion coefficients are solved for a selected range of frequencies, and an inverse Fourier transform is applied to calculate the solution for the locations of interest. For each frequency, the six expansion coefficients are solved for \( N \) angular wave numbers in order to properly construct the response. The value of \( N \) is determined through verification of convergence for each frequency. Hassan and Nagy [4] used a similar approach to study the ‘leaky’ Rayleigh wave for the fluid-filled hole case. For additional details on the solution of this particular problem, see reference [6].

Figure 1(b) displays the configuration of a fluid-filled cylindrical hole with a radial notch emanating from the hole. To address this problem, the boundary element method...
FIGURE 1. Diagrams of (a) elastic annulus bonded to a cylindrical cavity in an infinite elastic solid, and (b) fluid-filled cylindrical cavity in an infinite elastic medium with a fluid-filled notch.

(BEM) was used. The frequency domain BEM formulation presented here is based on an extension of the derivation for a fluid inclusion presented by Niwa et al [8]. For this problem, the notch is finite in width and filled with a fluid. To minimize computational time, a single boundary was used to define the shape of the cylindrical hole and notch. For additional details on the solution approach and parameters for the problem, see reference [6].

RESULTS FOR ELASTIC LAYER CASE

Figure 2 displays the dispersion curves where the phase velocity of the circumferential mode normalized with respect to the shear wave velocity of the elastic layer \((c / c_T)\) is plotted with respect to the normalized frequency \((\Omega = \omega \cdot b / c_T)\). In this plot, the two lowest circumferential modes for the cylindrical cavity with an elastic insert are compared with the dispersion curves for the cylindrical cavity, and for the elastic annulus. For this study, the material properties of aluminum were used for the infinite elastic medium, and the properties of polyurethane were applied to the elastic layer. As a rule of thumb, the wave speeds and density of the polyurethane are approximately one half the values for the corresponding properties in aluminum. The geometric parameter, \(h/b\), was set to 0.05, representing the worst case layer thickness found in actual applications. Of particular interest, the normalized frequency corresponding with an ultrasonic transducer center frequency of 5.0 MHz for the C-141 weep hole inspection case is 32.

At lower frequencies, the first circumferential mode equates well with the dispersion curve for the cylindrical hole case. Clearly, when the wavelength of the circumferential mode is large, the relative thickness of the layer is small, and thus the layer has little impact on the response. Therefore, very thin layers composed of soft material produce little change in the ‘leaky’ Rayleigh wave for the empty cylindrical hole case. As the frequency is increased, the first circumferential mode decreases in phase velocity and shifts toward the lowest mode of the elastic annulus case. Likewise for the second circumferential mode, a decrease in the phase velocity is observed, where the second lowest mode for the elastic annulus case is approached. Thus, as the wavelength becomes small with respect to the layer thickness, the mode becomes concentrated within the elastic layer. At the center frequency of the inspection transducer, the existence of the layer produces a significant difference in the phase velocity with respect to the empty cylindrical hole case. Clearly, a measurable change in the ‘leaky’ Rayleigh wave is expected for the selected layer parameters. Due to the thickness of the layer being relatively thin, only a few significant modes are observed within the range of frequencies and phase velocities solved. However, increases in the thickness of the layer would further impact the nature of the circumferential modes.
To better explore the sensitivity of the ‘leaky’ Rayleigh wave to variation in the thickness of the elastic annulus, the transient response for the analytical model was studied. Figure 3 displays the transient in-plane deflection amplitude at \( r = 3b, \theta = 180^\circ \), for a cylindrical hole with an elastic layer where \( h/b \) was varied from 0.00 to 0.20. This choice of location corresponds to the approximate location of the pulse-echo transducer. First, with an increase in the width of the elastic layer, the time of flight of the ‘leaky’ Rayleigh wave increases. This can be correlated to a decrease in the phase velocity of the ‘leaky’ Rayleigh wave as observed in the dispersion relation shown in Figure 2. In addition, as the layer width is increased, loss into the layer and subsequent re-radiation from the layer produces the observed spreading of the ‘leaky’ Rayleigh wave signal. In terms of the dispersion relation, the higher circumferential modes contribute more to the response as the layer thickness is increased. Lastly, due to the spreading of the signal, the peak amplitude of the ‘leaky’ Rayleigh wave signal is generally reduced. In conclusion, given the low levels of noise present within the measurement gate, the observed variation in the signal will have only a limited impact on the peak-to-peak measurement performed for top crack detection. However, one can likewise conclude that this added variation would greatly hinder any methodology to characterize the crack size.

**FIGURE 2.** Comparison of dispersion curves for circumferential waves propagating around a cylindrical cavity with an elastic layer (solid gray) with curves for a cylindrical cavity (black), and an elastic annulus.

**FIGURE 3.** Total field in-plane response (peak to peak) for an incident shear plane wave at location \( r = 3b, \theta = 180^\circ \) for various layer geometric parameter settings \( (h/b) \).
FLUID-FILLED HOLE RESULTS WITH EXPERIMENTAL COMPARISON

Figures 4(a) and 4(b) display contour plots of the total field response to an incident in-plane shear pulse on a fluid-filled cylindrical cavity for no-notch and with-notch cases respectively, for eight snapshots in time. The magnitude of the in-plane displacement for each field location is represented by a gray scale, where the darkest and lightest regions represent locations of greatest and lowest amplitude, respectively. The properties of water were used for the fluid. See reference [6] for the transducer and material property values.

In Figure 4(a), initial reflection of longitudinal and transverse waves is first observed. Also, a longitudinal wave is transmitted into the fluid-filled cavity that propagates as a halo wave [3]. The incident transverse wave also generates a Rayleigh wave that propagates along the surface of the weep hole. The Rayleigh wave can clearly be observed leaking energy into the surrounding solid due to the curvature of the weep hole and thus generating a significant circumferential shear wave propagating from the hole. However, a significant loss of the ‘leaky’ Rayleigh wave in the form of a longitudinal wave into the fluid at the Rayleigh angle is observed. Additionally, as time marches forward, a halo wave continues to propagate through the fluid, becomes incident upon the wall of the cavity, and transmits a portion of the wave back into the elastic media. Thus the reradiated waves from the fluid dominate the signal which returns to the catch transducer, as was previously shown [3-4].

FIGURE 4. Contour plots of the total displacement field response to an incident in-plane shear pulse on a 1/4” fluid filled weep hole for (a) no crack and (b) with crack cases for eight time steps, t = 1,2,3,4,5,6,7,8.
For the notch case in Figure 4(b), observations can be made from the scattering response to examine potential signals for crack detection. The incident transverse wave again generates a Rayleigh wave that propagates along the surface of the weep hole. With a top notch present, a portion of the ‘leaky’ Rayleigh wave is reflected. The reflected Rayleigh wave leaks energy into the riser and thus generates a small but detectable circumferential shear wave that propagates from the hole. Time step $t = 4$ displays this detectable signal. As for the empty cylindrical hole case, the pitch transducer can be used to detect this signal. Additional observations can be made for the notch case. Again, a significant loss of the propagating ‘leaky’ Rayleigh wave into the fluid as a halo wave is observed. However, the magnitude of the halo wave transmitted into the fluid for the notch case is not as great as for the no-notch case. Since a portion of the ‘leaky’ Rayleigh wave is reflected before the signal is fully leaked into the fluid (and solid), the transmitted halo wave in the fluid is reduced. This results in the first reradiated halo wave being significantly smaller for the notch case. Time step $t = 8$ clearly displays this reduction in the magnitude of the first reradiated halo wave. As for the empty cylindrical hole case, a second transducer located above the pitch transducer can be used to detect this signal. These observations indicate two potential methods for detecting the existence of a top notch.

Figures 5 and 6 display the comparison of pulse-echo and pitch catch signals respectively for four cases: (1) no notch, no fluid, (2) no notch, with fluid, (3) with notch, no fluid, no top notch, Pulse-echo, No fluid, 0.07" top notch, Pulse-echo, Fluid-filled, No top notch, Pulse-echo, Fluid-filled, 0.07" top notch, Pulse-echo.

**FIGURE 5.** Simulated and experimental pulse-echo signals for a transducer signal incident on a rib clip hole with and without an insert and/or a notch.
No fluid, No top notch, Pitch-catch transmitted leaky Rayleigh wave

Fluid-filled, No top notch, Pitch-catch

No fluid, 0.07" top notch, Pitch-catch

Fluid filled, 0.07" top notch, Pitch-catch

FIGURE 6. Simulated and experimental pitch-catch signals for a transducer signal incident on a rib clip hole with and without an insert and/or a notch.

no fluid, and (4) with notch, with fluid. The magnitude of the BEM simulations was adjusted to match the average specular reflection for the experimental data. Overall, there is good agreement between the simulated and experimental data. For the pulse-echo response in Figure 5, the notch case is shown to generate a significant reflected 'leaky' Rayleigh wave signal for both the empty and the fluid-filled hole cases. Although the magnitude of the reflected 'leaky' Rayleigh wave signal for the fluid-filled hole case (4) is about 30% of the magnitude for the empty hole case (3), this signal is still significantly greater than the noise level for the no-notch case (2). It can be observed that accurate placement of gating can improve the signal to noise ratio significantly. For the pitch-catch response in Figure 6, the transmitted circumferential Rayleigh wave is reduced considerably by the addition of fluid to the insert. In addition, two significant reradiated halo waves are also shown for the fluid filled cases. As observed in the contour plots, there is a significant reduction in the magnitude of the first reradiated halo wave due to the existence of a top notch. Simulated results produce a reduction of about 50% for larger notch cases.

Figure 7(a) displays the peak to peak response for the BEM simulation of the reflected 'leaky' Rayleigh wave signal in the pulse-echo mode and the first reradiated halo wave signal in the pitch-catch mode for various notch lengths. The symbols indicate the results of the BEM calculations, while the lines indicate the general trend of the peak to peak
response versus notch length. A proposed measure consists of the ratio of the amplitudes
of the reflected ‘leaky’ Rayleigh wave by the pulse-echo transducer over the first reradiated
halo wave by the catch transducer. Figure 7(b) displays the results for this ratio for various
numerically simulated notch length cases, clearly indicating that notches of 0.010 in. and
larger can be properly classified using this approach.

Seven no-notch and seven notch weep hole samples were investigated experimentally.
The sawcut notch lengths varied between 0.05 and 0.08 in. Gates were used to measure the
peak-to-peak signals from the reflected ‘leaky’ Rayleigh wave and the first reradiated halo
wave. Proper classification was made of all no-notch and notch samples. Ratio values
ranged from 0.04 to 0.08 for the no-notch cases and from 0.29 to 0.43 for the notch cases.
Any discrepancies with the simulation may be due to variation in experimental top-notch
locations ($\theta_n = 45^\circ$ for the BEM simulation), saturation of 8 bit A/D, transducer model
approximations, and coherent noise. In conclusion, the results of this paper support the
claim that an NDE technique for crack detection of fluid-filled cylindrical holes is feasible.

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