SIMULATION OF EDDY-CURRENT CORROSION DETECTION USING A SENSOR ARRAY

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ABSTRACT. A computer simulation has been developed to evaluate eddy-current probes containing magnetic field sensor arrays for the detection and evaluation of hidden corrosion. The simulation is used to assess probes that incorporate magneto-resistive or Hall devices in a closely-spaced, linear array. These probes will allow rapid data acquisition over a track width determined by the length of the array. The benefit of the simulation is that adjustments to the virtual probe parameters are easily made allowing improvements in sensitivity, imaging capability and resolution. A number of probe designs have been studied in this way including the “racetrack” probe.

INTRODUCTION

In eddy-current inspection, the detection of corrosion on a hidden surface is usually carried by measuring the effects impedance changes of an inductive probe. Two basic limitations of this approach are that inductive probes are insensitive to deep lying flaws and the inspection speed is slow. The problem of sensitivity may be alleviated by using magnetic field sensors, such as giant magneto-resistive sensors or Hall devices to measure the magnetic field. Compared with induction coils, solid state sensors are more effective for magnetic field detection at the low frequencies needed for subsurface flaw detection. The speed of inspection may be increased by using an array of field sensors.

A computer simulation is been developed to evaluate eddy-current probe designs that contain magnetic field sensor arrays. As well as simulating probes with circular excitation coils, coils with a racetrack geometry are also modelled, Figure 1. These have two straight sections parallel to the line of sensors and two bends that connect the two straight sections. A method for computing the electromagnetic field of a racetrack coil has been devised in which the electric field due to the “bends” and the “straights” are determined separately [1]. The interaction of the probe with the flaw is calculated numerically using a code based on a volume integral formulation that determines an approximate representation of the field in the flaw region [2]. The magnetic field at the site of each sensor is then evaluated from the flaw field using an expression derived using a reciprocity principle.

RACETRACK COIL THEORY

The analysis of the unperturbed electric field inside the conductor due to a time-harmonic current in the racetrack coil is given in this section. In the analysis of both straights and bends, it is implicit that a closed path for current is formed by using current sheets. These sheets are located across the ends of the straights to form a rectangular coil and their effect is cancelled by current sheets added to the bends to form the straight segments of two D-coils.

The analysis gives the electric field in a homogeneous half-space conductor in the region z < 0, and in an infinite conducting slab in the region −d < z < 0. The results for a multilayered
Consider a half-space conductor with a permeability equal to that of free space occupying the region $z < 0$, excited by an external current source $J(r)$, whose components are transverse to the $z$-direction. The electric field in adjoining half spaces is a solenoidal solution of

$$
\nabla^2 E(r) = j\omega\mu_0 J(r), \quad z > 0 \quad \text{and} \quad \left(\nabla^2 - j\omega\mu_0\sigma\right) E(r) = 0, \quad z < 0,
$$

(1)

where $\sigma$ is the conductivity of the conductor. The electric field, being transverse to the $z$-direction and having zero divergence, can be expressed in terms of a transverse electric (TE) scalar potential as

$$
E(r) = -j\omega\mu_0 \nabla \times [\hat{z}\psi(r)].
$$

(2)

For the same reasons, the current source can be expressed in a similar way, in terms of a scalar function, $M(r)$ say, as

$$
J(r) = \frac{1}{\mu_0} \nabla \times [\hat{z}M(r)].
$$

(3)

The function $M(r)$ is an equivalent $z$-directed magnetic dipole density giving rise to the same field as the source current, $J(r)$. This use of an equivalent magnetic source is a generalization of the magnetic shell model for a filamentary current loop in which the filamentary current is replaced by a magnetic shell bounded by the loop.

Solutions of (1) that vanish as $r \to \infty$ and ensure continuity of the tangential electric and magnetic field at $z = 0$ can be expressed as in (2), with the TE potential given by

$$
\psi(r) = \frac{1}{\mu_0} \int_{\Omega_0} G(r, r') M(r') d\mathbf{r},
$$

(4)

where the Green's function, $G(r, r')$, is continuous at the air-conductor interface, has a continuous normal gradient and vanishes at infinity. Fourier transformation with respect to $x$ and $y$, gives

$$
\tilde{\psi}(u, v, z) = \frac{1}{\mu_0} \int_{h-c}^{h+c} \tilde{g}(\kappa, z, z') \tilde{m}(u, v, z') dz',
$$

(5)

where the integration is between the lower and upper limits of the source coil. The functions $\tilde{g}$ and $\tilde{m}$ are the Fourier transforms of $G$ and $M$ respectively. With $u$ and $v$ being the Fourier-space coordinates corresponding to $x$ and $y$ respectively and, taking the positive root, $\kappa = \sqrt{u^2 + v^2}$. 

\textbf{FIGURE 1.} Racetrack probe showing the coil geometry and magnetic field sensor array.
With a singular source in the half-space \( z > 0 \), \( \tilde{g} \) for the half-space \( z < 0 \) is given by
\[
\tilde{g}_{\text{H-S}}(\kappa, z, z') = \frac{1}{\gamma + \kappa} e^{\gamma z - \kappa z'},
\]
where \( \gamma = \sqrt{\kappa^2 + j\omega \mu_0 \sigma} \), taking the root with a positive real part. Similarly, the Green's function for a slab \(-d < z < 0\) with an external singular source, is computed by taking into account the continuity conditions at the surfaces to give
\[
\tilde{g}_{\text{slab}}(\kappa, z, z') = \frac{1}{\gamma + \kappa} e^{\gamma z - \kappa z'} \frac{1 + \Gamma e^{-2\gamma(d+z)}}{1 - \Gamma e^{-2\gamma d}},
\]
where \( \Gamma = \frac{\gamma - \kappa}{\gamma + \kappa} \), the reflection term and \( d \) is the height of the slab.

**Electric Field Due to the Straights**

Let the \( y \)-component of the current density in the straight parts of the coil be uniform and given by the product of the turns density \( n \) and the coil current \( I \). Then it can be deduced from (3), with the coil parameters as shown in Figure 1, that
\[
M(x, y, z) = \begin{cases} 
\mu_0 n I f(x, y), & h - c \leq z \leq h + c \\
0, & \text{otherwise},
\end{cases}
\]
where
\[
f(x, y) = \begin{cases} 
a - b_i, & 0 \leq |x| \leq b, \\
|y| \leq d, \\
a - |x|, & b \leq |x| \leq a, \\
|y| \leq d, \\
0, & \text{otherwise}.
\end{cases}
\]
Because \( f(x, y) \) is even in \( x \) and \( y \), the Fourier transform can be expressed in the form of a double cosine integral and evaluated to give
\[
\tilde{f}(u, v) = -\frac{4}{u^2 v} \left[ \cos(ua) - \cos(ub) \right] \sin(vd).
\]

By taking Fourier transform of (2) and using (5) and (8), it is found that
\[
\tilde{e}(u, v, z) = -2\omega \mu_0 n I \frac{\tilde{v}}{\kappa} \tilde{f}(u, v) \tilde{g}(u, v, z, h) \sinh(\kappa c), \quad z < 0.
\]
The electric field can now be computed using a fast-Fourier-transform algorithm.

**Electric Field Due to the Bends**

The same source-equivalence principle, equation (3), is used for the analysis of the bends or D-coils as that used for the straight parts. However, it is convenient to express the Green’s function in this case using the cylindrical polar coordinates as
\[
G(\mathbf{r}, \mathbf{r}') = \frac{1}{2\pi} \sum_{m=0}^{\infty} \epsilon_m \cos[m(\phi - \phi')] \int_0^\infty J_m(\kappa \rho) J_m(\kappa' \rho') \tilde{g}(\kappa, z, z') \kappa d\kappa,
\]
which can be derived using an approach given by Morse and Feshbach [3]. In (12), \( \epsilon_m \) is the Neumann factor: \( \epsilon_0 = 1 \) and \( \epsilon_m = 2 \) \((m = 1, 2, 3, \ldots)\). The function \( \tilde{g} \) needed to give the field in the conductor is given by either (6) or (7) depending on whether the conductor is a half-space or slab. In order to evaluate (4), the explicit form of \( M(\mathbf{r}) \) for cylindrical polar coordinate system is found by writing
\[
M(\rho, \phi, z) = \begin{cases} 
\mu_0 n I f_D(\rho), & h - c \leq z \leq h + c, \\
0, & 0 \leq \phi \leq \pi, \\
\text{otherwise}.
\end{cases}
\]
and deducing from (3) that

\[ f_D(\rho) = \begin{cases} 
  a - b, & 0 \leq \rho \leq b, \\
  a - \rho, & b \leq \rho \leq a, \\
  0, & \text{otherwise.} 
\end{cases} \tag{14} \]

From equations (2), (12), (13) and (4) the electric field components can be written as,

\[ E_\rho(r) = -\frac{4j\omega\mu_0 I}{\pi\rho} \sum_{\lambda=0}^{\infty} \cos((2\lambda + 1)\phi) \int_0^\infty J_{2\lambda+1}(\kappa\rho) F_{2\lambda+1}(a, b, \kappa) \tilde{g}(\kappa, z, z_0, h) \sinh(\kappa c) d\kappa \tag{15} \]

and

\[ E_\phi(r) = j\omega\mu_0 I \left( \frac{1}{\pi} \sum_{\lambda=0}^{\infty} \frac{2\lambda + 1}{2\lambda + 1} \sinh((2\lambda + 1)\phi) \right) \int_0^\infty \left[ \kappa J_{2\lambda}(\kappa) - \frac{2\lambda + 1}{\rho} J_{2\lambda+1}(\kappa) \right] F_{2\lambda+1}(a, b, \kappa) \tilde{g}(\kappa, z, h) \sinh(\kappa c) d\kappa 
- \int_0^\infty \kappa J_1(\kappa) F_0(a, b, \kappa) \tilde{g}(\kappa, z, h) \sinh(\kappa c) d\kappa \right), \tag{16} \]

where

\[ F_\nu(a, b, \kappa) = \int_0^a f_D(\rho) J_\nu(\kappa\rho) d\rho 
= \frac{1}{\kappa^3} \left[ a J_\nu^{(1)}(\kappa a) - b J_\nu^{(1)}(\kappa b) \right] - \frac{1}{\kappa^3} \left[ J_\nu^{(2)}(\kappa a) - J_\nu^{(2)}(\kappa b) \right] \tag{17} \]

and

\[ J^n_\nu(z) = \frac{1}{\kappa} \int_0^z x^n J_\nu(x) dx. \tag{18} \]

The function in (18) is evaluated for \( \nu > 3 \) with the aid of a recursion relationship

\[ (\nu - n)J_{\nu+1}^{(1)}(z) = -2\nu z^n J_\nu(z) + (\nu + n)J_{\nu-1}^{(1)}(z), \tag{19} \]

derived using equation 11.3.6 of reference [4]. At large arguments it is necessary for numerical evaluation to use Hankel's asymptotic expansion (equations 9.2.5, 9.2.9 and 9.2.10, reference [4]) along with Fresnel integrals (equations 7.3.9, 7.3.10, 7.3.27 and 7.3.28, reference [4]).

**RESULTS**

Two examples of the array probe responses to subsurface material loss in a slab are presented here. In the first of these, the material loss is a square recess 25.4 mm × 25.4 mm ×
TABLE 1. Parameters for magnetic sensor measurements.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Outer radius</th>
<th>Inner radius</th>
<th>Axial length</th>
<th>Straight length</th>
<th>Nominal lift-off</th>
<th>Number of turns</th>
<th>Height of sensors</th>
<th>Distance between sensors</th>
<th>Number of sensors</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.89 mm</td>
<td>1.55 mm</td>
<td>5.99 mm</td>
<td>27.0 mm</td>
<td>2.5825 mm</td>
<td>517 ± 1 mm</td>
<td>0.869 mm</td>
<td>2.0 mm</td>
<td>33</td>
<td>2000 Hz</td>
</tr>
<tr>
<td>Plate</td>
<td>Conductivity</td>
<td>Thickness</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.82 \times 10^7$ S/m</td>
<td>4.85 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

FIGURE 3. Variation in the magnitude of the magnetic field for the center sensor with probe position for square subsurface recess, figure 2(a).

3 mm deep, Figure 2(a). In the second, the material loss is a 0.8 mm x 25.4 mm x 3 mm deep slot at the bottom of a slab, Figure 2(b). Other related dimensions, parameters of the probe and material are listed in Table 1. Probe current of 1 Amp is assumed for the analysis.

Figures 3 and 4 show the magnitude and phase respectively of the vertical magnetic field at the center sensor due to the square recess. Figures 5 and 6 show similar results for the long subsurface slot in a plate. Two coils are used to predict the results. The first coil is a racetrack coil with straight length of 27 mm and second coil is a circular coil, equivalent to a racetrack coil with no straight parts. Both the coils has the same inner and outer radii, vertical height and number of turns as listed in Table 1. A scan is performed in the x-direction, Figure 2.

By using a set of sensors at 2 mm intervals, a surface plot of the data can be generated using simulated measurements from a one-dimension scan of the flaw. Figures 7 and 8 show surface plots using predicted measurements from a 33 sensor array for the long slot with a racetrack coil of straight length 27 mm. For Figure 7, the scan is performed in the x-direction, perpendicular to the slot length, whereas for Figure 8, the scan is in y-direction.

An examination of the magnitude of the sensor signals from the racetrack coil and a circular coil for the square recess, Figure 3, shows a distinct region in the racetrack coil response between two inflection points. These points mark the boundary of the flaw region. A similar indication of the flaw dimensions is given in the surface plot of the magnitude of the magnetic field due to a long slot with scan in y-direction, Figure 8. Figure 7, in contrast, has four peaks, two along x-axis and other two along y-axis. Peaks along x-axis corresponds to the interaction of
FIGURE 4. Variation in the phase of the magnetic field for the center sensor with respect to probe position for square subsurface recess, figure 2(a).

FIGURE 5. Variation of the magnitude of the magnetic field at a central sensor with respect to the x probe coordinate for a subsurface long slot [Figure 2(b)] oriented in y-direction.
FIGURE 6. Variation of the phase of the magnetic field at a central sensor with respect to the \( x \) probe coordinate for a subsurface long slot [Figure 2(b)] oriented in \( y \)-direction.

FIGURE 7. Variation of the magnitude of the magnetic field as detected by an array of 33 sensors in a racetrack probe aligned parallel with the \( y \)-axis. The field is due to a long slot [Figure 2(b)] oriented in \( y \)-direction and is displayed as a function of \( x \)-coordinate of the probe position.
FIGURE 8. Variation of the magnitude of the magnetic field as detected by an array of 33 sensors in a racetrack probe aligned parallel with the x-axis. The field is due to a long slot [Figure 2(b)] oriented in y-direction and is displayed as a function of y-coordinate of the probe position.

the flaw boundaries with that of the racetrack straight parts and that along y-axis corresponds to the matching of the recess dimension with that of the coil. From these two surface plots it is possible to estimate the lateral dimensions of the recess.

CONCLUSION

The racetrack coil analysis provides a method for evaluating the unperturbed electric field in a multi-layered structure via a two-parts calculation. The first part gives the field due to the straights and the second gives the field due to separate D-coils. The unperturbed field calculated in this way is used in a further calculation to determine the magnetic field due to a flaw.

The dimension chosen for probe simulated here are suitable for scanning in one dimension along a row of fasteners with the array orientated perpendicular to the scan direction.

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REFERENCES