A NEW APPROACH TO THERMAL TOMOGRAPHY

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ABSTRACT. In this paper, a new approach to producing tomographic images of material properties from heat propagation data is described. The approach uses a finite difference forward propagation model in conjunction with a modified version of the algebraic reconstruction technique. Results from simulated experimental data are presented to illustrate the potential utility of this approach.

INTRODUCTION

Thermal imaging offers many attractive features as a nondestructive testing technique. It is rapid, sensitive to a wide range of potential defects, cost-effective in many applications and provides full field coverage that is difficult to obtain with other approaches. Tomographic reconstruction is also an extremely useful approach to producing full field images of local material property variations in nonhomogeneous media. Typically, tomography is used for reconstructions from wave propagation (either electromagnetic or acoustic) data. In this research we seek to apply tomographic imaging techniques explicitly to a diffusion problem: heat conduction through a solid media.

BACKGROUND

Thermal tomography has been studied by several investigators recently [1 - 6]. Two main approaches have been utilized to this date: 1) Temporal Based Imaging and 2) Thermal Wave Analogy. Temporal imaging is used to reconstruct planar slices of thermal image information based principally upon surface arrival times. By windowing the time record of the surface temperatures, one then obtains signatures (more or less) from different depths within the specimen. In practice, simple time gating procedures have been inefficient in resolving depth information with sufficient accuracy for imaging purposes and alternative signal processing methods have been utilized. Several investigators have empirically observed that depth estimates can be improved by using derivatives of the temperature versus time curves on a pixel-by-pixel basis. Several researchers report that the peak slope of the temperature contrast curve is a more reliable indicator of depth than straight time measurement [1 – 3]. Winfree et.al. use this parameter as a characteristic time for obtaining depth estimates that are then used to reconstruct image slices through the thickness of a sample [4]. However, this approach provides no quantitative information about the composition of a material, nor is it truly tomographic as it is based on single source measurements and is subject to shadowing and other imaging limitations.
An alternate approach to the problem is based on the use of a pseudo-wave propagation analogy. If one introduces a harmonically varying thermal source, the governing diffusional differential equation for heat conduction becomes the Helmholtz pseudo-wave equation. This equation admits a wave type solution that is extremely dispersive and highly attenuative so there is a major question as to its physical significance. Nonetheless, this approach does provide the computational framework for alternative methods of extracting depth information from thermal data. Mandelis and co-workers at the University of Toronto have used this formalism for two different types of image reconstruction [5-8]. In their work, a point source (focused laser) is used as the source. The localized nature of the source is needed to better establish the source-sensor geometry needed for reconstruction. One approach of the Toronto group is a ray optics solution with ART used in the reconstruction. This is a truly tomographic approach to thermal imaging using a well-established acoustic tomography reconstruction algorithm (ART). A second approach taken by this group uses a wave diffraction based reconstruction algorithm to achieve a similar result. While these approaches are interesting and do point the way towards a solution, they suffer from some major drawbacks, most notably image quality. The wave analogy basis for the analysis is imperfect at best as there is no well-defined wavefront or established ray trajectories to deal with as in acoustics. This is a diffusion phenomenon and must be analyzed as such in order to obtain an accurate solution.

This is the objective of this research effort. The method is designed to take advantage of the rapid, full field capabilities of thermal imaging. Of critical importance in this development is the introduction of an alternative to ray optics (or other wave analogies) in the reconstruction algorithm so one can appropriately weight the contributions from each pixel in the reconstruction domain in the context of thermal diffusion.

**RECONSTRUCTION ALGORITHM DEVELOPMENT**

Since the fundamental physics of the problem are well understood and there are a variety of established tomographic reconstruction algorithms available, attention here will be focused on a modification to existing wave based reconstruction algorithms, which properly reflect the diffusional character of the problem. Our choice as a starting point in this modification process is the algebraic reconstruction technique ART since it is rapid, iterative and relatively tolerant of experimental limitations/noise.

The main problem in adapting wave based imaging algorithms for thermal tomography is properly accounting for diffusional heat transfer. Straight-line paths and ray optics assumptions can produce distorted images. In order to properly modify ART for thermal imaging, one must identify a diffusion analog to acoustic transit and construct an alternative weighting factor to cell path length used in ART for acoustic wave propagation. With regard to transit time analogs, the question has not been fully resolved as there is no single accepted measure in the literature. However, there is a reasonable body of evidence to demonstrate that the time associated with the peak slope in the thermal contrast (PCT) versus time curve is a measure of a thermal signal arrival analogous to a wavefront arrival in acoustics and is the starting point for this development effort.

Next one must deal with the development of a ray path alternative that is compatible with diffusion phenomena. Here, we use an approach that is similar to that developed for acoustic tomography in anisotropic media where material inhomogeneity...
and beam skew phenomena can produce significant deviations from straight line ray paths [9-12].

Here, we first develop a finite difference algorithm for the forward propagation problem. For this study a thin square plate with variable thermal conductivity and specific heat is assumed. The plate is assumed to be thin enough to ignore through the thickness variations in temperature, reducing the analysis to a two-dimension form. For this analysis convection losses are also ignored, yielding a basic governing equation for heat conduction is given by:

$$\frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} = \rho c_p \frac{\partial T}{\partial t}$$

where $T$ is temperature, $k$ is thermal conductivity, $\rho$ is density, $c_p$ is heat capacity and $t$ is time.

We discretize the domain into a $10 \times 10$ grid representing a $3 \text{ cm} \times 3 \text{ cm}$ sample. Each element of the grid is assumed to have a given thermal conductivity and heat capacity. An explicit finite difference solution is developed that explicitly ensures the heat flow and temperature is continuous across grid lines. This solution for a multilayered configuration is in excellent agreement with a one-dimensional solution of the same multilayered configuration.

For the simulation of thermal tomography, an instantaneous heat flux is inserted at one of the edge nodes of the model. Temperature is calculated as a function of time for every node in the domain. The numerical code simulates 10 seconds of experimental data with a time step given by 0.125 times the square of the size of the grid divided by the maximum diffusivity in the specimen. From the forward propagation data, we can then determine the PCT at any point along the perimeter of the sample to represent experimental measurements. For this code, it was found that propagation data in the immediate vicinity of the source were inaccurate with the time step selected. Using a smaller time step for near field calculations is, of course possible, but results in an inordinately long computational time. Accordingly, to avoid this problem, we deleted the three nearest neighbor pixels on each side of the source in the reconstructions. This meant that a total of 36 possible source positions and 29 possible receiver locations (or 1044 source-receiver pairs) were used in this study. This formulation of the problem uses an oversampled data set of 1044 measurements to reconstruct 100 unknowns. As is expected, increasing the oversampling decreases the overall error in reconstruction but lengthens the computational time. The experimental configuration (1044 pairs) used here was found to be a good trade off between the time required and the reconstruction accuracy.

This formulation also allowed us to numerically calculate the sensitivity of each measurement to a particular cell's material parameters at each stage of the iteration process by slightly perturbing the parameter of the cell in question (while holding the other values constant), determining the resulting change in the measurement and calculating the resulting derivative. This parameter is then used in place of the cell path length as a measure of the relative influence of a given cell properties on heat propagation in the structure. This computational framework is directly adaptable from acoustic wave propagation to diffusion simply by changing the governing equation in the finite difference algorithm to diffusion, changing the fundamental material parameter characterizing each cell from velocity to thermal diffusivity (or conductivity or heat
capacity) and using PCT instead of using an acoustic transit time. In this case the correction factors for ART become

\[ \Delta \alpha_i = \frac{ \left( PCT^{\text{predicted}}_{\text{measured}} - PCT^{\text{predicted}}_{\text{measured}} \right) \frac{\partial PCT^{\text{predicted}}_{\text{measured}}}{\partial \alpha_i} }{ \sum_j \left| \frac{\partial PCT^{\text{predicted}}_{\text{measured}}}{\partial \alpha_i} \right| } \]

(2)

where the sensitivity factor is calculated numerically by perturbing the local thermal diffusivity i.e.

\[ \frac{\partial \text{PCT}^\alpha}{\partial \alpha_i} \frac{\text{PCT}^\alpha}{\text{PCT}^\alpha} \left( \alpha + \Delta \alpha_i \right) - \frac{\text{PCT}^\alpha}{\text{PCT}^\alpha} \left( \alpha \right) \]

(3)

This thermal tomography formulation is explicitly diffusion based yet still builds upon the vast amount of imaging work done for wave based problems.

RESULTS AND DISCUSSION

In this research effort a computational framework for diffusion based thermal tomography reconstruction of local thermal properties (either thermal conductivity or heat capacity) from thermal measurements was developed. First a target domain for the reconstruction process is chosen. Then, the forward, heat conduction finite difference code was used to generate a set of simulated data to represent experimental results. Finally, the ART algorithm is used to iteratively reconstruct the target domain. Initially, a sample with uniform properties (here aluminum) is used to start the process. Here, pixel properties are systematically adjusted to bring the time required for heat propagation for the model domain into agreement with those of the simulated experiment.

Results are presented below for a simulated “cracklike” defect in an aluminum plate (Figure 1 a-d). The target domain is illustrated in Figure 1a, here using thermal conductivity as the reconstruction variable. The bulk material is assumed to be aluminum and the defect has a significantly reduced thermal conductivity (~15% of the bulk value). Figure 1b) shows the reconstruction after 10 iterations using “perfect” experimental data i.e., data with no experimental errors. As can be seen, excellent agreement between the target domain properties and the reconstructed values is observed. Next, random experimental noise is introduced into simulated experimental data. Here, we perturb each source-receiver transit time a random amount between 0 % and +/- 1 % to simulate measurement error that might be seen in the laboratory. Results are shown in Figure 1c. Clearly, a larger amount of error is seen in the reconstruction for the same number of ART iterations, but the basic qualitative and quantitative features of the target domain are well reproduced. This convergence process is shown in Figure 1d where error (sum of the squares of the differences between the actual and predicted conductivity values for each of the 100 pixels) for the ART reconstruction both with and without experimental error.
FIGURE 1 ATR reconstruction (thermal conductivity).

FIGURE 2 ART reconstruction (heat capacity).
We were also able to use this technique to image heat capacity instead of thermal conductivity. This is shown in Figure 2. Again, good agreement between the actual and predicted values is seen.

CONCLUSIONS

In this work, a tomographic reconstruction algorithm was developed for thermal property analysis of structures. The method was developed to directly account for heat propagation via diffusion without the need for pseudo-wave analogies. Local heating at several points along the perimeter of the specimen serves as the excitation and temperature-time profiles are generated for preselected receiver locations. Peak contrast time (PCT) a thermal analog to acoustic transit time is then used as the basis for the tomographic reconstruction. For this effort a diffusion modified version of the algebraic reconstruction technique (ART) was developed as the reconstruction algorithm. To illustrate the potential utility of the approach, synthetic data were generated and used as inputs into the reconstruction algorithm. Target values and the reconstruction results for thermal conductivity were found to be in good agreement with one another.

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REFERENCES

