APPLICATION OF LINEARIZED INVERSE SCATTERING METHODS TO THE MATERIAL WITH FLAT MEASUREMENT SURFACE

K. Nakahata, M. Onishi, and M. Kitahara
Department of Civil Engineering, Tohoku University
Aoba-yama06, Aoba-ku, Sendai, Miyagi 980-8579, Japan

ABSTRACT. The linearized inverse scattering methods based on Born and Kirchhoff approximations are utilized for the flaw reconstruction in an elastic material with flat surfaces. The flaw shape can be reconstructed through the 2-D FFT of scattering amplitudes in the K-space. The scattering amplitudes are extracted from the measured waveforms by data processing with the reference waveforms from a small circular hole.

INTRODUCTION

In the ultrasonic nondestructive evaluation, two dimensional elastodynamic inverse scattering methods have been investigated to estimate the size, shape and orientation of flaws[1, 2]. In these methods, Born and Kirchhoff approximations were introduced into unknown displacements to linearize the far-field integral representation. For practical use of these methods, we here consider the shape reconstruction of flaws in an elastic material with flat surfaces. In this case a transducer moves linearly, therefore the backscattering waveforms are acquired at points on one side. The flaw shape can be reconstructed by performing the 2-D fast Fourier transform of the backscattering amplitudes from flaws in the K-space[3], which consists of the wave numbers and observation angles. In the ultrasonic measurement, it is important to extract the scattering amplitude in the elastic material from measured waveforms. Here, we adopt a data processing based on a measurement model in the linear system[4]. The data processing requires the reference waveforms from a small circular hole and the processed waveforms are fed into the Born and Kirchhoff inversions. A specimen with an interior cavity is prepared and the shape of the cavity is reconstructed from the measured backscattering waveforms.

The review for the inverse scattering methods has been given by Langenberg[3]. The Born inversion method has been studied by Rose et al.[5, 6] and the Kirchhoff inversion method has been treated in detail by Cohen and Bleistein[7, 8]. A unified approach of the Born and Kirchhoff inversions has been shown by Schmerr et al.[9]. A classification method of flaws is proposed in Ref.[10] by using the Born and Kirchhoff inversions. The linearized inverse scattering analysis for the plate by using SH wave has been reported by Hara and Hirose[11].
LINEARIZED INVERSE SCATTERING METHODS

We consider the two-dimensional isotropic elastic material $D \setminus D'$ with flaws $D'$ whose surface is $S$ as shown in Fig. 1. The elastic modulus and mass density are denoted by $C_{ijkl}$ and $\rho$ for the host matrix $D \setminus D'$ and by $C_{ijkl} + \delta C_{ijkl}$ and $\rho + \delta \rho$ for the flaws $D'$. Here we adopt the L-L pulse-echo method for the ultrasonic measurement, where the incident longitudinal wave is transmitted from a transducer and the backscattered longitudinal wave is received at the same transducer position $y$. The incident wave $u^i$ is now assumed to be a time harmonic plane wave

$$u^i(x) = -u^0 \hat{y} e^{-i k_L \hat{y} \cdot x}$$

where $u^0$ is the amplitude, $\hat{y}$ is the unit vector pointing to the transducer position $y$ and $k_L$ is the longitudinal wave number. In Equation (1), the time variation with the angular frequency $\omega$, exp$(-i \omega t)$, is omitted for convenience. The incident wave $u^i$ is sent to the flaws and the scattered wave $u^s$ is generated by the flaws. Then the scattered wave $u^s$ at the far field can be written as

$$u^s_m(y) = A_m^L(k_L, \hat{y})D(k_L | y|) + A_m^T(k_T, \hat{y})D(k_T | y|)$$

where $D(z) = \sqrt{2/(\pi z)} e^{-z^2/(4z)}$. In Equation (2), $A_m^L$ and $A_m^T$ are scattering amplitudes for longitudinal and transverse waves, respectively, and we use the longitudinal scattering amplitude $A_m^L$ in the actual measurement. In this study, two inversion methods based on the elastodynamic Born and Kirchhoff approximations are applied to reconstruct the shape of flaws. The details of the linearization process for two inversions have been given in Refs. [1, 2], therefore we briefly summarize the final inversion forms in the following section.

**Born Inversion**

We define $\Gamma(x)$ as the characteristic function of the flaws $D'$ which has a unit value inside of the flaws $D'$

$$\Gamma(x) = \begin{cases} 
1 & \text{for } x \in D' \\
0 & \text{for } x \in D \setminus D'. 
\end{cases}$$

(3)

For cavities, the elastic modulus $\delta C_{ijkl}$ and mass density $\delta \rho$ are set to be $-C_{ijkl}$ and $-\rho$, then the characteristic function $\Gamma(x)$ is reconstructed as the inverse Fourier transform of the scattering

![Figure 1. Wave fields in the host matrix $D \setminus D'$, and the flaws $D'$.](image)
amplitude $A_m^L(k_L, \hat{y})$:

$$\Gamma(x) = -\frac{2i}{\pi^2} \int_0^{2\pi} \int_0^\infty A(k_L, \hat{y}) \frac{\partial^2 \phi_x}{\partial k_L^2} e^{2i k_L x} dk_L d\hat{y}$$  \hspace{1cm} \text{(4)}$$

where $A(k_L, \hat{y}) = A_m^L(k_L, \hat{y}) \hat{y}_m$.

**Kirchhoff Inversion**

The singular function $\gamma(x)$ is defined to represent the boundary of flaws and it has values only on the flaw surface

$$\int_D \gamma(x) dV = \int_S dS.$$  \hspace{1cm} \text{(5)}$$

For cavities, $\gamma(x)$ is reconstructed from the scattering amplitude $A(k_L, \hat{y})$ by the inverse Fourier transform

$$\gamma(x) = -\frac{2i}{\pi^2} \int_0^{2\pi} \int_0^\infty A(k_L, \hat{y}) \frac{\partial^2 \phi_x}{\partial k_L^2} e^{2i k_L x} dk_L d\hat{y}.$$  \hspace{1cm} \text{(6)}$$

**FAST INVERSION TECHNIQUE**

Equations (4) and (6) show that the shape functions $\Gamma$ and $\gamma$ can be reconstructed by integrating the scattering amplitude $A(k_L, \hat{y})$ in the K-space. The K-space consists of the polar coordinates $(k_L, \hat{y})$, where $k_L$ is the longitudinal wave number and $\hat{y}$ is the observation direction as shown in Fig. 2. Two dimensional fast Fourier transform (2-D FFT) is introduced into the inversion algorithm to evaluate the integration in Equations (4) and (6). In order to apply the 2-D FFT for Equations (4) and (6), we introduce the rectangular coordinates $(k_1, k_2)$ for the K-space. Here, the observation angle $\theta$ is defined between the $k_1$-axis and observation direction $\hat{y}$:

$$k_L \hat{y} = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} k_L \cos \theta \\ k_L \sin \theta \end{pmatrix}.$$  \hspace{1cm} \text{(7)}$$

Introducing rectangular coordinates $(k_1, k_2)$, the Born inversion in Equation (4) reduces to

$$\Gamma(x_1, x_2) = \frac{1}{\pi^2} \int_0^{2\pi} \int_0^\infty A_1(k_1, k_2) e^{2i(k_1 x_1 + k_2 x_2)} dk_1 dk_2$$  \hspace{1cm} \text{(8)}$$

**FIGURE 2.** Scattering amplitudes $A(k_L, \hat{y})(\bigcirc)$ and $A(k_1, k_2)(\bullet)$ in the K-space.
where we set

\[ \tilde{\mathcal{A}}(k_1, k_2) = \frac{-2i}{u^2(k_1^2 + k_2^2)} \mathcal{A}(k_1, k_2). \]  

(9)

The Kirchhoff inversion in Equation (6) becomes

\[ \gamma(x_1, x_2) = \frac{1}{\pi^2} \int \int \tilde{\mathcal{A}}(k_1, k_2) e^{i2k_1x_1 + i2k_2x_2} dk_1 dk_2 \]  

(10)

where

\[ \tilde{\mathcal{A}}(k_1, k_2) = \frac{-2}{u^2 \sqrt{k_1^2 + k_2^2}} \mathcal{A}(k_1, k_2). \]  

(11)

For convenience, we use the unified expression for Equations (8) and (10)

\[ G_\sigma(x_1, x_2) = \frac{1}{\pi^2} \int \int \tilde{\mathcal{A}}_\sigma(k_1, k_2) e^{i2(k_1x_1 + k_2x_2)} dk_1 dk_2 \]  

(12)

where

\[ G_\sigma = \begin{cases} \Gamma & (\sigma = B_{\mathrm{BN}}) \\ \gamma & (\sigma = K_{\mathrm{Kirchhoff}}) \end{cases} \]

(13)

for \( \sigma = \text{Born and Kirchhoff inversions} \). The values of scattering amplitude \( \mathcal{A}(k_1, \hat{y}) \) are distributed in the \( K \)-space as shown with the black dot (•) in Fig.2. However, the data \( \mathcal{A}(k_1, k_2) \) in the rectangular coordinates are necessary for Equation (12). Here, we obtain the data \( \mathcal{A}(k_1, k_2) \) in the rectangular coordinates as indicated with (○) in Fig.2 by the interpolation of scattering amplitude \( \mathcal{A}(k_1, \hat{y}) \). Introducing new variables \( k_1 = \pi k_1 \) and \( k_2 = \pi k_2 \), we can rewrite Equation (12) as

\[ G_\sigma(x_1, x_2) = \int \int \tilde{\mathcal{A}}_\sigma(k_1, k_2) e^{i2k_1x_1 + i2k_2x_2} dk_1 dk_2. \]  

(14)

From the values of \( \tilde{\mathcal{A}}_\sigma(m_1, m_2) \) at \( N_1 \times N_2 \) points in the \( K \)-space, the discrete Fourier transform (DFT) of Equation (14) is given by

\[ G_\sigma(n_1, n_2) = \frac{1}{N_1N_2} \sum_{m_1=0}^{N_1-1} \sum_{m_2=0}^{N_2-1} \tilde{\mathcal{A}}_\sigma(m_1, m_2) W_{N_1}^{m_1m_1} W_{N_2}^{m_2m_2} \]

(0 \leq n_1 \leq N_1 - 1, \ 0 \leq n_2 \leq N_2 - 1)  

(15)

where \( W_{N_1} = \exp(2\pi i/N_1) \) and \( W_{N_2} = \exp(2\pi i/N_2) \). Calculation of Equation (15) can be performed by the 2-D FFT. For the purpose of fast calculation, \( 2^N (N = 7) \) is adopted here for \( N_1 \) and \( N_2 \).

Fig.3 shows the process of the shape reconstruction from the scattering amplitude \( \tilde{\mathcal{A}}_\sigma(k_1, k_2) \) in the \( K \)-space. The low frequency components of scattering amplitude \( \tilde{\mathcal{A}}_\sigma(k_1, k_2) \) are arranged at the center, and the high frequency components are arranged at the corner of the \( K \)-space as shown in Fig.3(a). Before performing the 2-D FFT, the scattering amplitudes are rearranged to take account of the Nyquist wave number (Fig.3(b)). Then the 2-D FFT is performed (Fig.3(c)) and the transformed data are shifted back to the original arrangement (Fig.3(d)).
EXPERIMENTAL SETUP AND FLAT ALUMINUM SPECIMENS

The experimental setup is shown in Fig.4. The aluminum specimen with flat surface is immersed in water. The scattered waveform from the flaw is measured by the L-L pulse echo method and the waveform is recorded on the digital-oscilloscope as the time-averaged data. The data in time domain are transformed to the frequency domain. The data in frequency domain are used as the input for the linearized inverse methods. In this study we use the immersion type transducer whose center frequency is 1.0MHz and the wavelength in aluminum is about 6mm at this 1.0MHz.

We prepared a aluminum specimen as shown in Fig.5. The flaw model is a two dimensional rectangular cavity whose height is 5mm and width is 10mm. In this case the transducer moves linearly on the side of the object surface, therefore the backscattering waveforms are acquired at the restricted measurement points.
DATA PROCESSING FOR INVERSION

The longitudinal scattering amplitudes $A_L^L(\omega, \hat{y}) \cdot \hat{y}$ in the elastic material are required for the shape reconstruction from Equations (4) and (6). To extract the waveform in the solid, the following data processing is adopted here. Since the entire measurement process is modeled as linear system[4], the output-voltage $V_{out}(\omega)$ in the frequency domain is related to the input-voltage $V_{in}(\omega)$ in the following product form (see, Fig.6(a))

$$V_{out}(\omega) = V_{in}(\omega) C_0 B_1(\omega) T_1(\omega) P_1(\omega) r_1(\omega) [A_{out}^L(\omega, \hat{y}) \cdot \hat{y}] B_2(\omega) T_2(\omega) P_2(\omega) \sigma_2(\omega)$$

(16)

where $B_\beta(\omega)$, $T_\beta(\omega)$, $P_\beta(\omega)$ and $\sigma_\beta(\omega)$ ($\beta = 1$(transmitting pass), 2(receiving pass)) are the property of transducer, transmission, propagation and material attenuation, respectively. In Equation(16), $[A_{out}^L(\omega, \hat{y}) \cdot \hat{y}]$ represents the component of scattering amplitude from flaws pointing to the transducer direction $\hat{y}$, and $C_0$ is the coefficient of measurement system. The reference output-voltage $V_{ref}(\omega)$ is measured through the pass shown in Fig.6(b)

$$V_{ref}(\omega) = V_{in}(\omega) C_0 B_1(\omega) T_1(\omega) P_1(\omega) r_1(\omega) [A_{ref}^L(\omega, \hat{y}) \cdot \hat{y}] B_2(\omega) T_2(\omega) P_2(\omega) \sigma_2(\omega)$$

(17)

where $A_{ref}^L(\omega, \hat{y}) \cdot \hat{y}$ is the reference scattering from a small circular hole in Fig.6(b). The circular hole plays roles of the target and reflector in ultrasonic measurement. It is to be remarked that the measured reference output-voltage in Equation(17) is the same as the measured output-voltage in Equation(16) except for the term $A_{ref}^L(\omega, \hat{y}) \cdot \hat{y}$. Therefore the scattering amplitude in the elastic material is obtained from

$$[A_{out}^L(\omega, \hat{y}) \cdot \hat{y}] = [A_{ref}^L(\omega, \hat{y}) \cdot \hat{y}] \frac{V_{out}(\omega)}{V_{ref}(\omega)}$$

(18)

Since the shape of a small hole is known(see, Fig.6(c)), we can calculate the scattering amplitude $[A_{ref}^L(\omega, \hat{y}) \cdot \hat{y}]$ by using the BEM and far-field integral representation of the scattered wave.

SHAPE RECONSTRUCTION RESULTS

Ultrasonic measurement is performed in water tank and the transducer moves linearly in water with the 10° increment of the measurement angle. The frequency range used for the inversions is from 0.1MHz to 1.7MHz. Fig.7 shows the results of shape reconstructions from the upper side measurement. In this figure, the left part represents the reconstruction of $\Gamma(x)$. 

FIGURE 6. Measured output-voltage $V_{out}$ from flaws and $V_{ref}$ from a small circular hole.
by Born inversion and the right part $\gamma(x)$ by Kirchhoff inversion. The illuminated side of flaw is reconstructed in this Fig.7. The shape reconstructions from both upper and lower side measurements are shown in Fig.8. From this figure, it is understood that the whole shape of flaw is clearly reconstructed. Fig.9 is the result of shape reconstruction from the upper side measurement. In this measurement, the scattered waves reflected at the lower side are also utilized for the shape reconstruction.

**FIGURE 7.** Shape reconstruction from the upper side measurement.

**FIGURE 8.** Shape reconstruction from the upper and lower side measurements.

**FIGURE 9.** Shape reconstruction from the upper side measurement using reflected waves at lower side.
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