A FORMAL APPROACH TO INCLUDE MULTIPLE SCATTERING IN THE ESTIMATION OF ULTRASONIC BACKSCATTERED SIGNALS

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ABSTRACT. An ultrasonic wave propagating through a microscopically inhomogeneous medium, such as polycrystalline materials, is subject to scattering at the grain boundaries. The fraction of energy removed from the incident wave is responsible for important phenomenon like attenuation, dispersion, and background “noise” associated with a given ultrasonic inspection system. Since the backscattered signals tend to mask the signals from small and subtle defects, the estimation of probability of detection of such defects requires the quantitative description of these signals. Although considerable attention has been given to the understanding of mean propagation characteristics of an ultrasonic beam, until recently there have been relatively little efforts devoted towards rigorous treatments of backscattered signals. In this research, we attempt to include some degree of multiple scattering in the calculation of the backscattered signals by developing a formalism that relates mean wave propagation characteristics to the noise.

INTRODUCTION

When an ultrasonic wave propagates through a polycrystalline aggregate, it is scattered at the grain boundaries. As a result, the ultrasonic beam suffers attenuation and exhibits dispersion of the phase velocity. Additionally, in an ultrasonic inspection system, the backscattered signals tend to mask the signals from small and subtle defects affecting the probability of detection of such defects. Reliable ultrasonic testing of such materials thus require the quantitative knowledge of these scattered signals. Furthermore, since the magnitude of the scattered power depends on the size, shape, and orientation of the constituent grains, the measured scattered power can be used as a tool to perform material characterization.

The propagation of elastic waves in randomly oriented, equiaxed polycrystals has received considerable attention, with most recent contributions for the cubic
TABLE 1. Material Properties

<table>
<thead>
<tr>
<th>Materials</th>
<th>(c_{11} \text{ N/m}^2)</th>
<th>(c_{12} \text{ N/m}^2)</th>
<th>(c_{44} \text{ N/m}^2)</th>
<th>(\rho \text{ kg/m}^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>2.16 x 10^{11}</td>
<td>1.45 x 10^{11}</td>
<td>1.29 x 10^{11}</td>
<td>7.86 x 10^{3}</td>
</tr>
</tbody>
</table>

materials being made by Hirsekorn [1,2], Stanke and Kino [3,4], Beltzer and Brauner [5], Ahmed and Thompson [6], and Turner [7]. Until recently, there has been fewer attempts to develop rigorous expressions for backscattered signals. Margetan et. al. [8] formulated backscattered power using independent scatterer approximation. More recently, Rose [9] has developed a general formalism, based on Auld’s [10] electro-mechanical reciprocity relations. Since this formalism is basically intractable, he then proceeded with the relevant calculations using Born and the single scattering approximations. We have, in the past [11], employed Rose’s formalism to calculate the backscattered power due to preferentially oriented spherical and nonspherical grains. In this paper, we describe a formalism that accounts for some degree of multiple scattering in the calculation of the backscattered signals. Preliminary computed results for the case of randomly oriented equiaxed grains are presented. The material properties of the chosen medium are listed in Table 1.

THEORY

Formal Approach

The displacement field \(u_i\) associated with time-harmonic elastic wave with angular frequency \(\omega\) in a medium is given by

\[
[c_{ijkl}(\mathbf{r})u_{k,l}(\mathbf{r})]_{ij} + \rho(\mathbf{r})\omega^2 u_i(\mathbf{r}) = 0,
\]

where \(c_{ijkl}(\mathbf{r})\) represent the elastic constants and \(\rho(\mathbf{r})\) represent the density at a point \(\mathbf{r}\) in the medium. We decompose the elastic constants and the displacement field in the following way

\[
c_{ijkl}(\mathbf{r}) = c_{ijkl}^0(\omega) + \delta c_{ijkl}(\mathbf{r}),
\]

\[
u_i(\mathbf{r}) = \langle u_i \rangle + \delta u_i(\mathbf{r}),
\]

where \(c_{ijkl}^0(\omega)\) are frequency dependent “expected” elastic constants prescribed by some “suitable” theory and \(\delta u_i(\mathbf{r})\) is the fluctuation of the displacement relative to the mean value \(\langle u_i \rangle\) in a statistically independent homogeneous medium. Obviously, the expected field \(\langle u_i \rangle\) satisfies the following wave equation in a constant density medium.

\[
c_{ijkl}^0 \left[\langle u_{k,l} \rangle_{ij} \right]_{ij} + \rho \omega^2 \langle u_i \rangle = 0.
\]

Application of the decomposition specified in equations (2) and (3) to equation 1 yields the following equation for the fluctuation field \(\delta u_i\)

\[
c_{ijkl}^0 \delta u_{k,l} + \rho \omega^2 \delta u_i = - \left[ \delta c_{ijkl} \left( \frac{\partial u_k}{\partial x_l} + \frac{\partial u_k}{\partial x_l} \right) \right].
\]

Eqn. (5) is untractable since the source term on the right hand side also contains the unknown \(\delta u_i\). One can, however, seek an iterative solution. This is the
approach we follow here. In an unbounded region, the first order iterative solution can then be written as

$$\delta u_i(\vec{r}) = - \int G_{im}(\vec{r} - \vec{r}') \left[ \delta c_{mjk}(\vec{r}') \langle u_k \rangle_{\vec{r}'} \right] d^3\vec{r}' .$$  \hspace{1cm} (6)$$

where $G_{im}(\vec{r} - \vec{r}')$ is a Green’s function taken from the work of Lifshits and Parkhamovski [12] and the primed subscripts indicate differentiation with respect to the primed coordinate system. Applying Green’s divergence theorem to the foregoing equation yields, after some manipulation,

$$\delta u_i(\vec{r}) = \int G_{im}(\vec{r} - \vec{r}') \delta c_{mjk}(\vec{r}') e^{ik_0\vec{r}'} d^3\vec{r}'. \hspace{1cm} (7)$$

For a plane wave of the form $\langle u_i \rangle = \hat{u}_i e^{-ik\vec{r}}$, the scattered displacement field at the observation point $\vec{r}$ becomes

$$\delta u_i(\vec{r}) = -ik\hat{u}_k \hat{k}_i \int G_{im}(\vec{r} - \vec{r}') \delta c_{mjk}(\vec{r}') e^{-ik\vec{r}'} d^3\vec{r}'. \hspace{1cm} (8)$$

The expected scattered power at $\vec{r}$ in a statistically homoeogeneous medium is then given by

$$\langle \delta u_i \delta u^*_p \rangle = \hat{u}_k \hat{u}_p \hat{k}_i \hat{k}_j \langle \delta c_{mjk}(\vec{r}') \delta c^*_{mjk}(\vec{r}') \rangle \int G_{im}(\vec{r} - \vec{r}') \int G^*_{pq}(\vec{r} - \vec{r}''') W(\vec{r}'' - \vec{r}''') e^{-ik\vec{r}'} d^3\vec{r}'. \hspace{1cm} (9)$$

Here $W(\vec{r}'' - \vec{r}'''$) represents the probability that two points, placed randomly in the material and separated by a displacement $\vec{r}'' - \vec{r}'''$, fall in the same crystallite and the superscript * represent complex conjugate.

**Simplified Calculation**

We have simplified the evaluation of backscattered signals indicated by equation (9) by considering a polycrystal with randomly oriented and weakly scattering equiaxed grains. This allows us, at low frequencies, to employ the independent scatterer approximation. Following the approach of Gubernatis et. al. [13], the scattered field at $\vec{r}$ when $|\vec{r} - \vec{r}'|$ is large, is written as

$$\delta u_i(\vec{r}) = \frac{e^{i\sigma r}}{r} A_i + \frac{e^{i\beta r}}{r} B_i, \hspace{1cm} (10)$$

where the vectors $A_i$ and $B_i$ are called the scattering amplitudes and $\sigma$ and $\beta$ are the longitudinal and the transverse wave numbers, respectively, in the average attenuative medium. We choose this average medium to be described by the unified theory of Stanke and Kino [3]. That is the complex $c_{ijkl}(\omega)$ is given by

$$c_{ijkl}^o = \delta_{ijkl} + i(\Delta_{ijkl}) + \epsilon^2 \left[ (\Delta_{ijkl} \Delta_{0kjl}) \right] \int_{\infty} G_{0\gamma}(\vec{r}) e^{i\vec{k}\vec{r}} d^3\vec{r}. \hspace{1cm} (11)$$
where $\bar{c}_{ijkl}$ represent the Voigt averaged elastic constants. After some required manipulation, the expected backscattered power for a longitudinal wave propagating in the z-direction can be written as

$$\langle A_3 A_5^* \rangle = \frac{\alpha^4 (\alpha^*)^4}{(4\pi \rho \omega^2)^2} \left( \delta c_{3333} \delta c_{333}^* \right) \left[ \int_{\text{grain}} e^{2i\omega r \cdot d^3 r} \right] \left[ \int_{\text{grain}} e^{-2i\alpha^* \cdot r} d^3 r \right]. \quad (12)$$

For spherical grains of radius $a$, the expected backscattered power can be easily evaluated to give

$$\langle A_3 A_5^* \rangle = \frac{4\pi}{(4\pi \rho \omega^2)^2} \left( \delta c_{3333} \delta c_{333}^* \right) \left[ \frac{\sin(2\alpha a) - 2a\alpha \cos(2\alpha)}{(2\alpha)^3} \right] \left[ \frac{\sin(2\alpha a) - 2a\alpha \cos(2\alpha)}{(2\alpha)^3} \right]^*. \quad (13)$$

If $N$ is the number of scatterers per unit volume, the forgoing equation may be rewritten in terms of the backscatter coefficient $\eta = N(|A_3|^2)^{1/2}$ as follows

$$\eta = 3 \left( \frac{\alpha^4 (\alpha^*)^4}{(4\pi \rho \omega^2)^2} \left( \delta c_{3333} \delta c_{333}^* \right) \right) \left[ \frac{\sin(2\alpha a) - 2a\alpha \cos(2\alpha)}{(2\alpha)^3} \right] \left[ \frac{\sin(2\alpha a) - 2a\alpha \cos(2\alpha)}{(2\alpha)^3} \right]^* \right)^{1/2}, \quad (14)$$

where $k_{ad} = \omega/\sqrt{(\delta c_{3333}/\rho)}$. 

\textbf{FIGURE 1.} Frequency dependence of normalized backscatter coefficient.
RESULTS

We have calculated the backscattered coefficient for ultrasonic wave propagation in polycrystalline iron with randomly oriented equiaxed grains. The material properties of the chosen medium are listed in Table 1. Fig. 1 shows the frequency dependence of the computed backscatter coefficients using 1) the averaged elastic constants extracted from the unified theory of Stanke and Kino[3] and 2) the Voigt averaged elastic constants. At low frequencies both the representations of the average medium yield the same result. This is to be expected since the incident beam is not significantly scattered as it propagates in the medium at these frequencies. As the normalized frequency $2k_0a$ increases, effects of some degree of multiple scattering begin to appear. Some of the early scattered signals due to the grains are further scattered back to the observation point, causing an increase in the backscatter coefficient.

CONCLUDING REMARKS

The simple scheme to account for some degree of multiple scattering in the computation of backscatter coefficient shows promise. The main result described here appear to be intuitive. It must be reiterated that the formalism referred to in this paper does not restrict the calculation to be performed using independent scatterer approximation. We are currently in the process of performing more detailed evaluation of the formalism.

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REFERENCES


