DEVELOPMENT OF MODELING AND SIMULATION FOR MAGNETIC PARTICLE INSPECTION USING FINITE ELEMENTS

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ABSTRACT. Magnetic particle inspection (MPI) is a widely used inspection method for aerospace applications with inspection development essentially limited to empirical knowledge and experience-based approaches. Better quantitative understanding of the MPI technique and factors that affect its sensitivity and reliability would contribute not only to reductions in inspection design cost and time but also improvement of analysis of experimental data. We employed a finite element method (FEM) for numerical calculation because this is known to be suitable for complicated geometric objects such as the part shapes encountered in aviation components and defects of concern. Magnetic particles are usually soft magnetic materials and sensitive to the magnetic field distribution around them. They are easily attracted toward a high magnetic field gradient. Selection of magnetic field source, which produces a magnetic field gradient large enough to detect a small defect in the sample, is an important factor in magnetic particle inspection. The magnetic field gradient and magnetic force at the sites of defects having different widths and depths have been calculated. The simulated results can be used to assist in understanding the behavior of magnetic particles around a defect.

INTRODUCTION

A number of nondestructive evaluation (NDE) techniques have been developed for evaluating defects; surface discontinuities, voids, surface flaws, and cracks on the surface or in the body of materials [1-8]. Properly applied NDE techniques will prevent operational failures of the mechanical parts by locating critical defects. Metallic materials are widely evaluated in NDE applications due to their common usage in industry. Different NDE techniques should be used depending on whether the metallic materials are magnetic or non-magnetic. For magnetic metallic materials such as steel, eddy current [1,2], magnetic flux leakage [3,4], magnetic Barkhausen noise [5,6], and magnetic particle inspection [7,8] techniques can be employed. Among these techniques, the magnetic particle inspection (MPI) and the magnetic flux leakage (MFL) are popular due to their inexpensive and simple procedures. Both techniques depend on the distortion of magnetic flux lines caused by a defect on the surface or sub-surface of a ferromagnetic material. The difference between the techniques is the method of detecting defects. The MPI technique uses fine magnetic particles, dry iron powder or wet magnetic particles suspended in a liquid medium, to identify the defect while the MFL technique employs a magnetometer to measure the magnetic leakage field occurring around the defect. Easy distribution of magnetic particles on a test sample makes the MPI technique suitable for samples with large surface areas while the MFL technique may be appropriate for detecting defects in...
the areas where access would be difficult for visualization such as inside surfaces of pipelines.

The magnetic field generator and the magnetic particles are essential components of the MPI method. The magnetic field strength should be large enough to magnetize the sample so that the magnetic particles can interact with the leakage fields. The magnetic force, which drags the magnetic particles to the defect sites, is proportional to the product of the magnetic field and the magnetic field gradient. The distortion of the magnetic field is greatest when the direction of the magnetic field is perpendicular to the axis of a defect, which maximizes the magnitude of the magnetic field gradient. Magnetic fields can be generated either by a direct contact of current source to the test material using prods (not recommended for aerospace components) or by using current coils such as a solenoid or a yoke. The magnetic properties of the magnetic particles are a very important factor in MPI testing. A simple analytical model for the calculation of the magnetic leakage field of surface-breaking cracks and an estimation of the magnetic force on the magnetic particle were studied with an assumption of constant permeability [9]. Computational advances enabled the numerical simulations of MPI for a complicated geometry [10]. In this paper we report the use of the finite element method (FEM) numerical simulations of MPI for defects with various sizes. The simulated results can provide indications of the expected behavior of magnetic particles around a defect.

FINITE ELEMENT NUMERICAL SIMULATIONS

Equations for an Axisymmetric Geometry

We simulated a test sample in the shape of a cylindrical tube by solving Maxwell’s equations in a cylindrical coordinate system \((r, \theta, z)\). For axisymmetric geometry, the Maxwell’s equation for Ampère’s law under DC mode is as follows:

\[
\frac{1}{\mu} \left( \frac{\partial^2 \vec{A}}{\partial r^2} + \frac{1}{r} \frac{\partial \vec{A}}{\partial r} + \frac{\partial^2 \vec{A}}{\partial z^2} - \frac{\vec{A}}{r^2} \right) = -\vec{J}_s
\]

(1)

where \(J_s\) and \(\vec{A}\) are the source current density and the vector potential respectively. Asymptotic boundary conditions are applied on the outer surface of the domain. Using the Ritz method one can show that the solution of the Maxwell Equation (1) is equivalent to minimizing the energy function described as [11]:

\[
F = \iint \left\{ \frac{1}{2\mu} \left( \frac{\partial \vec{A}}{\partial r} \right)^2 + \frac{\partial \vec{A}}{\partial r} \cdot \vec{A} \right\} - \vec{J}_s \cdot \vec{A} \right\} r dr dz .
\]

(2)

Using the vector potential \(\vec{A}\) obtained from Equation (2), the magnetic flux density \(B\), the magnetic field intensity \(H\), the magnetic field gradient \(dB/dr\), and the magnetic potential energy \(W\) were computed.

Simulation of Magnetic Flux Leakage Field

For the simulation of a sample using the finite element method (FEM), we modeled a test for investigating the behavior of magnetic particles when a solenoidal current field is applied. The test sample was assumed to be cylindrical in shape and the shape of the defect
on the surface in the form of a groove. The cross section of the cylindrical sample and the defect is shown in Figure 1. The length and wall thickness of the sample were chosen to be 16 cm and 1 cm, respectively. The defect is located at the center of the sample. The defect size was varied during the batch of simulation tests. The distance between the outer boundary and the test sample was set to be sufficiently large to satisfy boundary conditions. Asymptotic boundary conditions were applied to the outer surface. The B-H curve of the test sample showed a nonlinear behavior as shown in Figure 2. The magnetic force that attracts magnetic particles is proportional to the magnitude of the magnetic field gradient [12]. Therefore, the distribution of the magnetic field gradient around a defect is an important factor in magnetic particle inspection. The magnetic field gradient versus depth (d in Figure 1) and width (w in Figure 1) are shown in Figure 3.

In the absence of defects the calculations suggested that the magnetic flux density $B$ under a solenoidal current field increases linearly as a probe moves from the center of the z-axis to the outside of the cylinder. However, the change of $B$-field along the radial direction ($dB / dr$) is higher at the center of the defect than outside of the defect. This result
FIGURE 3. Simulation of magnetic field gradient (dB/dr); (a) dB/dr vs. defect depths (defect width=1mm, current density=2.0A/mm$^2$), (b) dB/dr vs. defect widths (defect depth=5mm, current density=1.0A/mm$^2$).

is crucial for the magnetic flux leakage (MFL) test. According to our calculations, as the depth of the defect increases as shown in Figure 3(a), the magnitude of the magnetic field gradient should increase. The peak-to-peak value of magnetic field gradient decreased as the width of the defect increased as shown in Figure 3(b). This implies that the magnetic force should become weaker as the defect becomes wider.

**Magnetic Force on a Magnetic Particle**

For magnetic particles to adhere to a defect, the magnetic force generated by an applied current source should be large enough to drag magnetic particles into the defect. Magnetic force on a saturated magnetic particle can be described by the equation

$$F_m \propto -\nabla (\vec{H} \cdot \vec{M}) = -K\nabla (\vec{H} \cdot \vec{H})$$

(3)

where $\vec{M}$ is the magnetization vector of the magnetic particle, and $K$ is a constant which contains information of the magnetic property of the magnetic particle such as magnetic susceptibility and the volume of the magnetic particle.
For the cylindrical coordinate system, the magnetic field vector $\vec{H}$ can be decomposed into the radial component, $H_r$, and the component along the $z$ direction $H_z$. From Equation (3), the magnetic force components along the $r$- and $z$- directions can be written as:

$$
F_r = -K' \left( H_r \frac{\partial H_z}{\partial r} + H_z \frac{\partial H_r}{\partial r} \right), \\
F_z = -K' \left( H_z \frac{\partial H_z}{\partial z} + H_r \frac{\partial H_z}{\partial z} \right),
$$

(4)

where $K' = 2K$. The quantities $A_r = F_r / K'$ and $A_z = F_z / K'$, which are proportional to the magnetic force components, can provide some information on the behavior of a magnetic particle around a defect.

The values of $A_z$ and $A_r$ are plotted in Figure 4. The geometry of the sample for this simulation was the same as shown in Figure 1. The size of a defect was assigned as 1-mm width and 5-mm depth. The applied current density is $2 \times 10^6$ A/m$^2$. Figure 4(a) shows that the magnetic leakage field from the defect induces a magnetic force that makes the magnetic particle around the defect region move to the center of the defect. If there is no defect in the test material, the magnetic field from the solenoid coil will make the magnetic particle pull out of the center position of the $z$-axis. Figure 4(b) shows that the quantity proportional to the radial component of magnetic force increases as the magnetic particle moves toward the bottom of the defect. It suggests how the magnetic force attracts and retains magnetic particles at the defect against their weight.

**FEM algorithm for Magnetic Particle Inspection**

The next step is the description of physical behavior of the magnetic particles at the defect. We assume that the magnetic particles are uniformly sprayed on the surface of a sample. The magnetic energy of a material with nonlinear permeability can be described as

$$
W = \int_0^L \int_0^W \int_0^L B(h) dh \ dV,
$$

(5)
where \( B(h) \) represents the magnetic flux density as a function of magnetic field \( h \), and \( H \) is the magnetic field in a small volume \( dV \). For the calculation of energy at the \( i^{th} \) element of meshes, it is computed approximately as

\[
W_i = \left( \sum_{k=1}^{n_i} B(h_k) \Delta h_k \right) V_i, \quad \text{such that} \quad h_k = \sum_{k=i}^{i} \Delta h_k \quad \text{and} \quad H = \sum_{k=i}^{i} \Delta h_k,
\]

where \( H \) and \( V_i \) are the magnetic field and volume of the \( i^{th} \) element, and the sum given in Equation (6) is the approximate value of the integral in the parenthesis of the Equation (5). The \( B(H) \) curve is obtained from the user-defined nonlinear permeability data. The magnetic force on the magnetic particles with a volume \( V \) and a susceptibility \( \chi(H) \) at magnetic field of \( H \), is given by [12]:

\[
F_r = -\frac{dW}{dr} = -\mu_0 \chi(H) V \left( H \cdot \frac{dH_z}{dr} + H_z \frac{dH_z}{dr} \right) = \mu_0 \chi(H) V \cdot A,
\]

Figure 5 shows a simulation result of magnetic potential energy and the corresponding magnetic force. The light bars in Figure 5(b) represent the magnetic forces at the bottom surface of defect holes and the dark bars in Figure 5(b) show the forces at 1 mm above the flaw bottoms. The solid lines are the logarithmic interpolation curves of magnetic energy and forces, respectively. The magnetic forces are proportional to the magnetic potential energy as shown in Figure 5.

We assumed that the magnetic field intensity \( H \) and the susceptibility \( \chi(H) \) of magnetic particles are constant in the area of each finite element. Therefore, from Equation (7), the volume of magnetic particles in the \( i^{th} \) finite element retained inside the defect is given by

\[
V_i = \frac{1}{\mu_0 \chi(H)} \left( \frac{dW}{dr} \right) \approx \frac{1}{\mu_0 \chi(H)} \left( \frac{\Delta W}{H \Delta H_x + H_z \Delta H_z} \right) = \frac{1}{\mu_0 \chi(H)} \frac{\Delta W}{H \cdot \Delta H_x}.
\]
Let $R_i$ be the radius of the center position of the $i^{th}$ finite element in the defect. Then, the approximated volume of the $i^{th}$ finite element is the product of $2\pi R_i$ and the cross-sectional area $A_i$ of the element. Note this is an axially symmetric problem. It is assumed that the wet magnetic particles are uniformly distributed on the surface of the test material. Therefore, the magnetic particles are uniformly accumulated inside the defect region. From Equation (8), the cross-sectional area of the magnetic particles at the $i^{th}$ finite element is

$$A_i = \frac{1}{2\pi R_i \mu_0 \chi(H)} \frac{\Delta W}{H \cdot \Delta H}. \quad (9)$$

For the estimation of the magnetic particle volume in the defect region, the cross-sectional area given by Equation (9) was calculated for each finite element in the defect region. If each of the cross-sectional areas obtained from Equation (9) is smaller than the area of the corresponding finite element, then the value of the cross-sectional area of the retained magnetic particles ($A_i$) is recorded to the database of the finite element. At this step the simulation loop is terminated. The total sum of the recorded areas ($\Sigma A_i$) of magnetic particles in the finite elements around a defect indicates the estimated volume of accumulated magnetic particles in the defect.

If any of the areas of the retained magnetic particles is larger than or equal to the area of the corresponding finite element, then the material property of this finite element is changed from air to magnetic particles. That is, the magnetic particles are fully occupied in the area of the finite element region. The next step is rebuilding and solving a new MPI problem given by Equation (2). This simulation loop is continued until the termination rule is satisfied. This is a generic algorithm to update material properties of finite elements by an interaction between the magnetic particles and the updated magnetic field conditions ($H$, $\Delta H$, and $\Delta W$).

Table 1 shows that the volume of the magnetic particles increases proportionally as the depth of the defect increases when the size of defect width is fixed. For the sensitivity analysis of defect width, the defect geometry was set at a fixed depth of 5-mm and variable widths of 1 mm, 3 mm, and 5 mm. The stack heights of magnetic particles inside defects were 1.175 mm, 0.748 mm, and 0.648 mm, respectively. Therefore, the height of the stack of particles inside the defect proportionally decreased as the width of defect increased.

**Table 1.** Simulation Results of FEM algorithm for MPI.

<table>
<thead>
<tr>
<th>Defect Geometry (Depth x Width)</th>
<th>Cross-Section Area of Magnetic Particles inside Defect</th>
<th>Sim. Loop 1</th>
<th>Sim. Loop 2</th>
<th>Sim. Loop 3</th>
<th>Sim. Loop 4</th>
<th>Sim. Loop 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm x 1 mm</td>
<td>0.194 mm$^2$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>3 mm x 1 mm</td>
<td>0.416 mm$^2$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>5 mm x 1 mm</td>
<td>1.000 mm$^2$</td>
<td>1.175 mm$^2$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>7 mm x 1 mm</td>
<td>1.000 mm$^2$</td>
<td>2.000 mm$^2$</td>
<td>3.000 mm$^2$</td>
<td>3.208 mm$^2$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>9 mm x 1 mm</td>
<td>1.000 mm$^2$</td>
<td>2.000 mm$^2$</td>
<td>3.000 mm$^2$</td>
<td>4.000 mm$^2$</td>
<td>4.936 mm$^2$</td>
<td>N/A</td>
</tr>
<tr>
<td>5 mm x 3 mm</td>
<td>2.243 mm$^2$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>5 mm x 5 mm</td>
<td>3.239 mm$^2$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

† The current field is fixed at $2.0 \times 10^6$ A/m$^2$.
‡ “Sim.” means simulation.
CONCLUSIONS

Using the finite element method, the magnetic flux density, the magnetic field gradient, and the magnetic force on magnetic powder particles at the site of a defect were calculated. The calculation showed that the magnetic flux leakage field at the defect created a magnetic force, which attracted and retained the magnetic particles at the defect location. The magnetic particle inspection technique is more sensitive to the defect geometry than the magnetic flux leakage measurement technique, from which method it is difficult to predict the geometry of a defect. Reduction of the inspection design cost, time, and improvement of analysis of experimental data can be achieved by the use of FEM simulations combined with careful incorporation of MPI parameters such as magnetic field source, magnitude of the applied current, defect size, position of defect, and magnetic properties of both sample and magnetic particles.

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