Sivers Effect and Transverse Single Spin Asymmetries in Drell-Yan Processes

M. Anselmino*, U. D’Alesio† and F. Murgia‡

*Dipartimento di Fisica Teorica, Università di Torino and
INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy
†INFN, Sezione di Cagliari and Dipartimento di Fisica, Università di Cagliari,
C.P. 170, I-09042 Monserrato (CA), Italy

Abstract. Sivers asymmetry, adopted to explain transverse single spin asymmetries (SSA) observed in inclusive pion production, p↑ p → πX and ¯p↑ p → πX, is used here to compute SSA in Drell-Yan processes; in this case, by considering the differential cross section in the lepton-pair invariant mass, rapidity and transverse momentum, other mechanisms which may originate SSA cannot contribute. Estimates for RHIC experiments are given.

Single spin asymmetries in high energy inclusive processes are a unique testing ground for QCD; they cannot originate from the simple spin pQCD dynamics – dominated by helicity conservation – but need some non perturbative chiral-symmetry breaking in the large distance physics.

Among the best known transverse single spin asymmetries (SSA) let us mention: i) the large polarization of Λ’s and other hyperons produced in pN → Λ↑ X; ii) the large asymmetry $A_N = \frac{d\sigma^\uparrow}{d\sigma^\downarrow} + d\sigma^\downarrow$ observed in $p^\uparrow p \rightarrow \pi X$ and $\bar{p}^\uparrow p \rightarrow \pi X$ processes; iii) the similar azimuthal asymmetry observed in $\ell^\uparrow p \rightarrow \ell \pi X$.

Several models [1] to explain the data within QCD dynamics can be found in the literature; here we focus on a phenomenological approach based on the generalization of the factorization theorem with the inclusion of parton intrinsic motion $k_{\perp}$. The cross section for a generic process $AB \rightarrow CX$ then reads:

$$d\sigma = \sum_{a,b,c} \hat{f}_{a/A}(x_a, k_{\perp a}) \otimes \hat{f}_{b/B}(x_b, k_{\perp b}) \otimes d\hat{\sigma}^{abc} (\ldots) \otimes \hat{D}_{c/C}(z, k_{\perp C}),$$

(1)

where the $\hat{f}$’s ($\hat{D}$’s) are the $k_{\perp}$ dependent parton distributions (fragmentation functions).

Even if Eq. (1) is not formally proven in general, it has been shown that intrinsic $k_{\perp}$’s are indeed necessary in order to be able to explain, within pQCD and the factorization scheme, data on (moderately) large $p_T$ production of pions and photons [2].

When dealing with polarized processes the introduction of $k_{\perp}$ dependences opens up the way to many possible spin effects; these can be summarized, at leading twist, by new polarized distribution functions and fragmentation functions,

$$\Delta^N f_{q/p} = \hat{f}_{q/p}(x, k_{\perp}) - \hat{f}_{q/p}(x, -k_{\perp}),$$

(2)

$$\Delta^N f_{q'/p} = \hat{f}_{q'/p}(x, k_{\perp}) - \hat{f}_{q'/p}(x, -k_{\perp}),$$

(3)
\[ \Delta^N D_{h|q^1} \equiv \hat{D}_{h|q^1}(z, k_\perp) - \hat{D}_{h|q^1}(z, -k_\perp) = \hat{D}_{h|q^1}(z, k_\perp) - \hat{D}_{h|q^1}(z, -k_\perp), \quad (4) \]
\[ \Delta^N D_{h^1|q} \equiv \hat{D}_{h^1|q}(z, k_\perp) - \hat{D}_{h^1|q}(z, -k_\perp) = \hat{D}_{h^1|q}(z, k_\perp) - \hat{D}_{h^1|q}(z, -k_\perp), \quad (5) \]

which have a clear meaning if one pays attention to the arrows denoting the polarized 
particles. All the above functions vanish when \( k_\perp = 0 \) and are naively \( T \)-odd. The ones in 
eqs. (3) and (4) are chiral-odd, while the other two are chiral-even. The fragmentation in 
Eq. (4) is the Collins function \([3]\), while the distribution in Eq. (2) was first introduced by 
Sivers \([4]\). Some of the above functions have been widely used for a phenomenological 
description of the observed SSA \([5]\).

Despite its successful phenomenology, the Sivers function was always a matter of 
discussions and its very existence rather controversial; in fact in Ref. \([3]\) a proof of 
its vanishing was given, based on time-reversal invariance. Ways out based on initial 
state interactions or non standard time-reversal properties \([6]\) were discussed. Very 
recently a series of papers \([7]\) have resurrected Sivers asymmetry in its full rights: a 
quark-diquark model calculation has given an explanation of the HERMES azimuthal 
asymmetry different from the Collins effect and has shown that initial state interactions 
can give rise to SSA in Drell-Yan processes. Moreover Collins recognized that \( i) \) such a 
new mechanism is compatible with factorization and is due to the Sivers asymmetry \((2), \)
\( ii) \) his original proof of the vanishing of \( \Delta^N f_{q^1/p^5} \) is incorrect.

Some issues concerning factorizability and universality of these effects are still open 
to debate; however, we feel now confident to use Sivers effects – and equally all func-
tions in Eqs. (2)-(5) – in SSA phenomenology. The natural process to test the Sivers 
asymmetry is Drell-Yan where there cannot be any effect in fragmentation processes 
and, by suitably integrating over some final configurations, other possible effects van-
ish. SSA in Drell-Yan processes are particularly important now, as ongoing or imminent 
experiments at RHIC will be able to measure them.

Let us consider a Drell-Yan process, that is the production of \( \ell^+\ell^- \) pairs in the 
collision of two hadrons \( A \) and \( B \): the difference between the single transverse spin 
dependent cross sections \( d\sigma^\uparrow \) for \( A^+B \to \ell^+\ell^-X \) and \( d\sigma^\downarrow \) for \( A^\downarrow B \to \ell^+\ell^-X \) from 
the Sivers asymmetry of Eq. (2), is

\[
d\sigma^\uparrow - d\sigma^\downarrow = \sum_{ab} \int [dx_a d^2 k_{\perp a} dx_b d^2 k_{\perp b}] \Delta^N f_{a/A^1} (x_a, k_{\perp a}) \hat{f}_{b/B} (x_b, k_{\perp b}) d\hat{\sigma}^{ab} = \hat{\sigma}^{\ell^+\ell^-}. \quad (6)\]

We consider the differential cross section in the variables \( M^2 = (p_a + p_b)^2 \), \( y \) and 
\( q_r \), that is the squared invariant mass, the rapidity and the transverse momentum of the 
lepton pair. Notice that we do not look at the angular distribution of the lepton pair 
production plane, which is integrated over.

We take the hadron \( A \) as moving along the positive \( z \)-axis, in the \( A-B \) c.m. frame and 
measure the transverse polarization of hadron \( A, \hat{P}_A \), along the \( y \)-axis.

In the kinematical regions such that: \( q_r^2 \ll M^2 \ll M_Z^2 \) and \( k_{\perp a, b}^2 \approx q_r^2 \), the asymmetry 
becomes

\[
A_N = \frac{\sum_q e_q^2 \int d^2 k_{\perp q} d^2 k_{\perp \bar{q}} \delta^2 (k_{\perp q} + k_{\perp \bar{q}} - q_r) \Delta^N f_{q/A^1} (x_q, k_{\perp q}) \hat{f}_{\bar{q}/B} (x_{\bar{q}}, k_{\perp \bar{q}})}{2 \sum_q e_q^2 \int d^2 k_{\perp q} d^2 k_{\perp \bar{q}} \delta^2 (k_{\perp q} + k_{\perp \bar{q}} - q_r) \hat{f}_{q/A} (x_q, k_{\perp q}) \hat{f}_{\bar{q}/B} (x_{\bar{q}}, k_{\perp \bar{q}})}, \quad (7)\]
where \( x_q \sim \frac{M}{\sqrt{s}} e^y \) and \( \bar{x}_q \sim \frac{M}{\sqrt{s}} e^{-y} \), with \( a, b = q, \bar{q} \) and \( q = u, \bar{u}, d, \bar{d}, s, \bar{s} \).

On the other hand the SSA generated by the distribution function in Eq. (3) would lead to a contribution of the kind [8]

\[
\sum_q h_{1q}(x_q, \mathbf{k}_{\perp q}) \otimes \Delta^N f_{q/q}^{\perp}(x_q, \mathbf{k}_{\perp q}) \otimes d\Delta \hat{\sigma}^{q\bar{q} \to e^+ e^-} ,
\]

where \( h_{1q} \) is the transversity of quark \( q \) (inside hadron \( A \)) and \( d\Delta \hat{\sigma} \) is the double transverse spin asymmetry \( d\hat{\sigma}^{++} - d\hat{\sigma}^{\perp \perp} \). Such an elementary asymmetry has a \( \cos 2\phi \) dependence [8], where \( \phi \) is the angle between the transverse polarization direction and the normal to the \( \ell^+ \ell^- \) plane; when integrating over all final angular distributions of the \( \ell^+ \ell^- \) pair – as we do – the contribution of Eq. (8) vanishes.

Analogously, other mechanisms [9], based on higher twist quark-gluon correlation functions, lead to expressions of \( A_N \) vanishing upon integration over the leptonic angles.

In order to give numerical estimates, we introduce here a simple model for the Sivers asymmetry (2), and for the unpolarized distributions, which is similar to the one introduced for the polarizing fragmentation function (see Eq. 5) in Ref. [10].

Let us start from the most general expression for the number density of unpolarized quarks \( q \), inside a proton with transverse polarization \( \mathbf{P} \) and three-momentum \( \mathbf{p} \). One has

\[
f_{q/p}^{\perp}(x, \mathbf{k}_{\perp}) = \hat{f}_{q/p}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/p}^{\perp}(x, k_{\perp}) \hat{\mathbf{P}} \cdot \hat{\mathbf{p}} \times \mathbf{k}_{\perp} \,.
\]

In our configuration one simply has \( \hat{\mathbf{P}} \cdot \hat{\mathbf{p}} \times \mathbf{k}_{\perp} = (\mathbf{k}_{\perp})_x = \cos \phi_{k_{\perp}} \).

We consider simple factorized and Gaussian forms (see also [11]):

\[
\hat{f}_{q/p}(x, k_{\perp}) = f_{q/p}(x) g(k_{\perp}) = f_{q/p}(x) \frac{\beta^2}{\pi} e^{-\beta^2 k_{\perp}^2} ;
\]

by imposing the positivity bound \( |\Delta^N f_{q/p}^{\perp}(x, k_{\perp})| \leq 2 \hat{f}_{q/p}^{\perp}(x, k_{\perp}) \), we can write

\[
\Delta^N f_{q/p}^{\perp}(x, k_{\perp}) = 2 \mathcal{N}_q(x) f_{q/p}(x) \frac{\beta^2}{\pi} \sqrt{2e(\alpha^2 - \beta^2)} k_{\perp} e^{-\alpha^2 k_{\perp}^2} ,
\]

with

\[
\mathcal{N}_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)(a_q + b_q)}{a_q b_q} , \quad |N_q| \leq 1 .
\]

Inserting the above choice of \( \Delta^N f(x, \mathbf{k}_{\perp}) \) and \( \hat{f}(x, \mathbf{k}_{\perp}) \) into Eq. (7) one can perform analytical integrations; assuming \( \beta \) independent of \( x \) (see below) one gets

\[
A_N(M, y, \mathbf{q}_r) = \mathcal{D}(q_r, \phi_{q_r}) \cdot \mathcal{A}(M, y)
\]

\[
= 2 \sqrt{\frac{r^2}{(1+r)^2}} \left( \frac{2e^\frac{1-r}{r}}{\frac{1-r}{r}} \right)^{1/2} \beta q_r \cos \phi_{q_r} \exp \left[ -\frac{1}{2} \frac{1-r}{1+r} \beta^2 q_r^2 \right]
\]

\[
\times \frac{1}{2} \sqrt{\sum_q \frac{e^2 \Delta^N f_{q/p}^{\perp}(x_q) f_{q/p}(x_q)}{2 \sum_q e^2 f_{q/p}^{\perp}(x_q) f_{q/p}(x_q)}} .
\]
where $\phi_{q_T}$ is the azimuthal angle of $q_T$ and $r = \beta^2 / \alpha^2 < 1$. $\mathcal{O}(q_T)$ has a maximum when $q_T = q_T^M = \sqrt{(1+r)/(1-r)} / \beta$, where its value is $\mathcal{O}(q_T^M) = \mathcal{O}_M = [2r/(1+r)]^{3/2}$. Notice that only the position of the maximum depends on the parameter $\beta$.

All the parameters of the model have been fixed in a complementary analysis of unpolarized inclusive particle production and pion SSA [13]. As a result of this study we have: $\beta = 1.25 \text{(GeV/c)}^{-1}$ ($\langle k_{T}^{2} \rangle^{1/2} = 0.8 \text{GeV/c}$, independent of $x$), and

$$
\begin{align*}
N_u &= 0.5 \quad a_u = 2.0 \quad b_u = 0.3, \\
N_d &= -1.0 \quad a_d = 1.5 \quad b_d = 0.2, \quad r \simeq 0.7. \quad (14)
\end{align*}
$$

For the unpolarized partonic distributions, $f_{q/p}(x)$, we adopt the GRV94 set [12].

One further uncertainty concerns the sign of the asymmetry: as noticed by Collins and checked by Brodsky [7], the Sivers asymmetry has opposite signs in Drell-Yan and SIDIS, respectively related to $s$-channel and $t$-channel elementary reactions. As in $p - p$ interactions we expect that large $x_F$ regions are dominated by $t$-channel quark processes, we think that the Sivers function extracted from $p - p$ data should be opposite to that contributing to D-Y processes. Our numerical estimates will then be given with the same parameters as in Eq. (14), changing the signs of $N_u$ and $N_d$. Given these considerations, even a simple comparison of the sign of our estimates with data might be significant.

In Fig. 1 we show $A_N$ at $\sqrt{s} = 200 \text{ GeV}$ as a function of $y$ averaged over two kinematical ranges $6 \leq M \leq 10 \text{ GeV}$ and $10 \leq M \leq 20 \text{ GeV}$ (on the left) and as a function of $M$ averaged over the ranges $|y| < 2$ and $0 < y < 2$ (on the right). We have fixed $q_T = q_T^M (\simeq 1.9 \text{ GeV/c})$, and $\phi_{q_T} = 0$, which maximizes the $q_T$-dependent part of the asymmetry; on the other hand $A_N$ is reduced by a factor of 50% at $q_T^1 \simeq 0.6 \text{ GeV/c}$.

We can also consider the asymmetry averaged over $q_T$ up to a value of $q_T = q_T^1$ (integrating over $\phi_{q_T}$ in the range $[0, \pi/2]$ only, otherwise one would get zero). In our simple model (for a full account of this study see [11]), for $q_T^1 \geq 1.7 \text{ GeV/c}$ we would get $\langle A_N \rangle \simeq 0.4 A_N(q_T^M)$ (for $q_T^1 = 0.6 \text{ GeV/c}$ $\langle A_N \rangle \simeq 0.2 A_N(q_T^M)$).

Our numerical estimates show that $A_N$ can be well measurable within RHIC expected statistical accuracy. The actual values depend on the assumed functional form of the Sivers function and its role with valence quarks only.

Transverse single spin phenomenology, within QCD dynamics and the factorization scheme, is a rich and interesting subject. It combines simple pQCD spin dynamics with new long distance properties of quark distribution and fragmentation; the experimental measurements are relatively easy and clear, many have been and many more will be performed in the near future, both at nucleon-nucleon and lepton-nucleon facilities.

Very recently a large transverse SSA (contrary to naive expectations) has been observed at $\sqrt{s} = 200 \text{ GeV}$ (at RHIC) in $p^1 p \to \pi X$ processes [14]; a reasonable agreement with these preliminary data has been found in our approach.

We have presented here the explicit formalism for computing single transverse spin asymmetries in Drell-Yan processes, within a generalized QCD factorization theorem formulated with $k_{T}$ dependent distribution functions. Simple Gaussian forms have been assumed and available data from other processes have been exploited, in order to give estimates for single spin effects in D-Y production at RHIC, which should be of interest for the incoming measurements. Again, sizable and measurable values have been found.
FIGURE 1. Single spin asymmetry $A_N$ for the Drell-Yan process, at RHIC energies, $\sqrt{s} = 200$ GeV, as a function of $y$ and averaged over $M$ (left) and as a function of $M$ and averaged over $y$ (right), (see text).

ACKNOWLEDGMENTS

One of us (U.D.) would like to thank the organizers for their kind invitation to a fruitful and interesting Symposium. U.D. and F.M. thank COFINANZIAMENTO MURST-PRIN for partial support.

REFERENCES

14. Rakness, G., these proceedings; Bland, L.C., these proceedings.