Nucleon Electromagnetic Form Factors and Densities

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Abstract. We review data for nucleon electromagnetic form factors, emphasizing recent measurements of $G_E/G_M$ that use recoil or target polarization to minimize systematic errors and model dependence. The data are parametrized in terms of densities that are consistent with the Lorentz contraction of the Breit frame and with pQCD. The dramatic linear decrease in $G_{Ep}/G_{Mp}$ for $1 \leq Q^2 \leq 6 \text{(GeV}/c)^2$ demonstrates that the charge is broader than the magnetization of the proton. High-precision recoil polarization measurements of $G_{En}$ show clearly the positive core and negative surface charge of the neutron. Combining these measurements, we display spatial densities for $u$ and $d$ quarks in nucleons.

INTRODUCTION

The electromagnetic structure of nucleons provides fundamental tests of the QCD confinement mechanism, as calculated on the lattice or interpreted with the aid of models. From elastic electron scattering one obtains the Sachs electric and magnetic form factors, which are closely related to the charge and magnetization densities. Dramatic improvements in the quality of these measurements have recently been achieved by using beams that combine high polarization with high intensity and energy together with either polarized targets or measurements of recoil polarization. In this paper we review the current status of nucleon elastic form factors, emphasizing recent polarization measurements, and analyze these data using a model that permits visualization of the underlying charge and magnetization densities.

Matrix elements of the nucleon electromagnetic current operator $J^\mu$ take the form

$$\langle N(p', s')|J^\mu|N(p, s)\rangle = \bar{u}(p', s')e\Gamma^\mu u(p, s)$$

where the vertex function

$$\Gamma^\mu = F_1(Q^2)\gamma^\mu + \kappa F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}$$

features Dirac and Pauli form factors, $F_1$ and $F_2$, whose dependence upon the spacelike invariant four-momentum transfer $Q^2 = q^2 - \omega^2$ probes the nucleon structure. The interpretation is simplest in the nucleon Breit frame in which a nucleon approaches with initial momentum $-\vec{q}_B/2$, receives three-momentum transfer $\vec{q}_B$ without energy transfer, and departs with final momentum $\vec{q}_B/2$ where $q_B^2 = Q^2/(1 + \tau)$ with $\tau = Q^2/4m^2$. In the Breit frame for a particular value of $Q^2$, the current separates into electric and
magnetic contributions [1]

\[ \bar{u}(p', s') \gamma^\mu u(p, s) = \chi_s^\dagger \left( G_E + \frac{i \bar{\sigma} \times \bar{q}_B}{2m} G_M \right) \chi_s \]  

(3)

where \( \chi_s \) is a two-component Pauli spinor and where the Sachs form factors are given by

\[ G_E = F_1 - \tau \kappa F_2 \quad G_M = F_1 + \kappa F_2 \]  

(4)

Early experiments with modest \( Q^2 \) suggested that

\[ G_{E,p} \approx \frac{G_{M,p}}{\mu_p} \approx \frac{G_{M,n}}{\mu_n} \approx G_D \]  

(5)

where \( G_D(Q^2) = (1 + Q^2/\Lambda^2)^{-2} \) with \( \Lambda^2 = 0.71 \text{ (GeV/c)}^2 \) is known as the dipole form factor [2, 3].

**FORM FACTORS FROM POLARIZATION MEASUREMENTS**

In the one-photon exchange approximation, the differential cross section for elastic scattering of an electron beam from a stationary nucleon target is given by

\[ \frac{d\sigma}{d\Omega} = \frac{\sigma_{NS}}{\epsilon(1 + \tau)} \left( \tau G_M^2 + \epsilon G_E^2 \right) \]  

(6)

where \( \epsilon = (1 + (1 + \tau)2\tan^2 \theta_e/2)^{-1} \) is the transverse polarization of the virtual photon for electron scattering angle \( \theta_e \) and \( \sigma_{NS} \) is the cross section for a structureless Dirac target. Thus, the traditional Rosenbluth technique separates the electric and magnetic form factors by varying \( \epsilon \), but extraction of \( G_E \) becomes extremely difficult at large \( Q^2 \) because the magnetic contribution becomes increasingly dominant and because it is difficult to control the kinematic variation and radiative corrections with sufficient accuracy when both the form factors and the kinematic coefficients vary rapidly over the acceptance. Alternatively, the electromagnetic ratio

\[ g = \frac{G_E}{G_M} = -\sqrt{\frac{\tau(1 + \epsilon)}{2\epsilon} \frac{P_x'}{P_z'}} \]  

(7)

can be obtained by comparing the components of the nucleon recoil polarization along the momentum transfer direction, denoted by \( \hat{z} \), and in the \( \hat{\epsilon} \) direction transverse to \( \hat{z} \) in the scattering plane [4, 5]. For the proton, both components can be measured simultaneously using a polarimeter in the focal plane of a magnetic spectrometer, thereby minimizing systematic uncertainties due to beam polarization, analyzing power, and kinematic parameters. The systematic uncertainty due to precession of the proton spin in the magnetic spectrometer is usually much smaller than the uncertainties in comparing the cross sections obtained with different kinematical conditions and acceptances needed for the Rosenbluth method.
Figure 1 compares recent high-precision measurements of the proton electromagnetic ratio performed at Jefferson Laboratory (JLab) [6, 7] with earlier Rosenbluth data obtained at SLAC [8, 9]. I had expected the new JLab experiments to confirm the SLAC findings that $G_{Ep} \approx G_{Mp}/\mu_p$, merely improving their precision, but instead we found the surprisingly strong linear decrease shown in Fig. 1. The systematic uncertainties, primarily due to spin precession in the magnetic spectrometer, are shown by the hatched region and were substantially reduced in the second experiment at larger $Q^2$ where the deviation from unity is strongest. A recent re-analysis of the SLAC data failed to discover any systematic correction which could account for this disagreement [10] and I am unaware of any plausible mechanism which could produce a failure of the one-photon approximation at this level. Hopefully, a recent super Rosenbluth experiment [11] designed to minimize systematic uncertainties will help clarify this discrepancy. Until those results become available, I will rely on the recoil polarization measurements and discard the cross section data for $Q^2 > 1$ (GeV/c)$^2$ with larger, and possibly seriously underestimated, systematic uncertainties.

![FIGURE 1. Recent $G_E/G_M$ data for the proton (left) and neutron (right). The hatched region indicates the systematic uncertainty in JLab recoil polarization measurements for the proton. The proton data are from: SLAC E140 [8], SLAC NE11 [9], MIT [12], MAMI [13], and JLab [6, 7]. The neutron data are from: MAMI $d(\vec{e}, e'\vec{n})$ [14], MAMI $^3$He$(\vec{e}, e' n)$ [15, 16], NIKHEF $d(\vec{e}, e' n)$ [17], JLab $d(\vec{e}, e' n)$ [18], and JLab $d(\vec{e}, e'\vec{n})$ [19]. See text for explanation of $G_{En}$ curves.]

For the neutron one must use quasifree scattering from a neutron in a nuclear target and correct for the effects of Fermi motion, meson-exchange currents, and final-state interactions also. Both recoil and target polarization for quasifree knockout give similar PWIA formulas for the form factor ratio, but the systematic errors and the nuclear physics corrections are appreciably different. Therefore, confident extraction of $G_{En}/G_{Mn}$ benefits from comparison of data for both recoil and target polarization. Data from recent experiments of these types are also shown in Figure 1 and additional data from Mainz on $d(\vec{e}, e'\vec{n})$ at $Q^2 = 0.6$ and 0.8 and from JLab on $d(\vec{e}, e' n)$ at $Q^2 = 1.0$ (GeV/c)$^2$ are expected soon. Details of the high-precision JLab experiment on $d(\vec{e}, e'\vec{n})$ at $Q^2 = 0.45, 1.15$, and 1.47 (GeV/c)$^2$ may be found in Ref. [19]; corrections for nuclear physics and acceptance averaging have not yet been applied but are expected to be small. Several two-parameter fits based upon the Galster parametrization are shown also. Our fit (solid) to these data and additional data from Refs. [20, 21] remains rather
close to the original Galster fit (dotted) to electron-deuteron elastic scattering data for $Q^2 < 0.7$ (GeV/c)$^2$ [22] despite the rather larger model dependence of the Rosenbluth method. The highest curve shows a fit by Schmieden [23] to a subset of the polarization data for $Q^2 < 0.7$ (GeV/c)$^2$. The dashed line shows the commonly quoted Platchkov analysis of more recent elastic scattering data (not shown) that used the Paris potential, but other realistic interactions produce variations greater than the spread between the highest and lowest curves in this figure [24]; thus, the model dependence of the Rosenbluth method is at least this large while the model dependence of the recoil polarization method for $Q^2 > 0.5$ (GeV/c)$^2$ is less than 10% [25]. Therefore, polarization techniques provide much more accurate measurements of $G_{En}$ than the Rosenbluth method and show that $G_{En}$ for $Q^2 < 1.5$ (GeV/c)$^2$ is substantially larger than the common Platchkov parametrization, although it remains compatible with the Galster parameterization.

Polarization measurements of nucleon electromagnetic ratios and selected cross section data for the form factors relative to $G_D$ are compared in Figs. 2-3 with representative calculations. The chiral soliton model of Holzwarth (dotted lines) predicted the linear behavior of $G_{Ep}/G_{Mp}$ but fails to reproduce neutron form factors [26, 27]. The light-cone diquark model (long dashes) needs only 5 parameters to obtain a reasonable fit for modest $Q^2$ [28], but except for $G_{En}$ its form factors fall too rapidly for $Q^2 > 1$ (GeV/c)$^2$. The point-form spectator approximation (PFSA) using pointlike constituent quarks and a Goldstone boson exchange interaction fitted to spectroscopic data successfully describes a wider range of $Q^2$ (short dashes) without fitting additional parameters to the form factors [29]. Finally, a light-front calculation using one-gluon exchange and constituent-quark form factors fitted to $Q^2 < 1$ (GeV/c)$^2$ provides a good fit (dash-dot) up to about 4 (GeV/c)$^2$ [30]. However, none of the available theoretical calculations provides a truly quantitative description for all four form factors over a wide range of $Q^2$. The differences between these models are largest for $G_{En}$, which is especially sensitive to small mixed-symmetry and deformed components of the nucleon wave function. Clearly it will be very important to extend the $G_{En}$ data to larger $Q^2$ — a proposal to measure $^3\bar{H}_e(\bar{e},e'n)$ up to 3.4 (GeV/c)$^2$ has been approved at JLab [31] and proposals for higher $Q^2$ are under development.

Figures 2-3 also show fits made by Lomon [32] using an extension of the Gari-Krümpelmann model [33] that interpolates between vector meson dominance (VMD) at low $Q^2$ and perturbative QCD (pQCD) at high $Q^2$. This type of parametrization is very useful for nuclear physics calculations, but offers no insight into the spatial distributions of charge and magnetization within nucleons. In the next section we offer an alternative phenomenology in terms of spatial densities.

**Fitted Densities**

**Relativistic Inversion**

Although rigorous comparisons between theory and experiment must be made at the level of form factors, for many it would seem desirable to extract charge and magnetization densities from the corresponding form factors because our intuition is
FIGURE 2. Polarization data for electromagnetic ratios are compared with representative calculations: chiral soliton (dotted) [26, 27], light-cone diquark (long dashes) [28], PFSA (short dashes) [29], light-front OGE with constituent form factors (dash-dot) [30]. The data have the same legend as Fig. 1.

FIGURE 3. Selected form factor data are compared with representative calculations: chiral soliton (dotted) [26, 27], light-cone diquark (long dashes) [28], PFSA (short dashes) [29], light-front OGE with constituent form factors (dash-dot) [30].

usually stronger in space than in momentum transfer. Intrinsic charge and magnetic form factors, \( \tilde{\rho}_{ch}(k) \) and \( \tilde{\rho}_{m}(k) \), may be defined in terms of the Sachs form factors by

\[
\tilde{\rho}_{ch}(k) = G_{E}(Q^{2})(1 + \tau)^{\lambda_{E}} \quad \mu \tilde{\rho}_{m}(k) = G_{M}(Q^{2})(1 + \tau)^{\lambda_{M}} \tag{8}
\]

where the intrinsic spatial frequency \( k \) is related to the invariant momentum transfer \( Q \) by the Breit-frame boost \( k^{2} = \frac{Q^{2}}{1 + \tau} \) and where the model-dependent exponents, \( \lambda_{E} \) and \( \lambda_{M} \) will be discussed shortly. Due to the Lorentz contraction of spatial distributions in the Breit frame, a measurement with Breit-frame momentum transfer \( q_{B} = Q \) probes a reduced spatial frequency \( k \) in the rest frame. In fact, the intrinsic frequencies accessible
to elastic scattering with spacelike momentum transfer are limited to \( k < 2m \) such that the asymptotic Sachs form factors in the limit \( Q^2 \to \infty \) are determined by the intrinsic form factors in the immediate vicinity of the limiting frequency \( k_m = 2m \). This limitation can be understood as a consequence of relativistic position fluctuations, known as zitterbewegung, that smooth out radial variations on scales smaller than the Compton wavelength.

Using a quark cluster model Licht and Pagnamenta [34] originally proposed to use \( \lambda_E = \lambda_M = 1 \), but these choices do not conform with pQCD scaling unless one imposes upon both form factors the somewhat artificial constraint \( \tilde{\rho}(k_m) = 0 \). Mitra and Kumari [35] then demonstrated that a more symmetric version of the quark cluster model that is also applicable to inelastic transitions suggests \( \lambda_E = \lambda_M = 2 \) and is compatible with pQCD scaling without constraining \( \tilde{\rho}(k_m) \). For the present work we employ \( \lambda_E = \lambda_M = 2 \) and refer to Ref. [36] for a more comprehensive analysis. Although we cannot claim that \( \rho_{ch}(r) \) and \( \rho_m(r) \) are the true charge and magnetization densities in the nucleon rest frame because the boost operator for a composite system depends upon the interactions among its constituents, this model can be used to fit the form factor data using an intuitively appealing spatial representation that is consistent with relativity and with pQCD.

### Fitting Procedures

The model dependence of the fitted form factor can be minimized by expanding the density in a complete set of radial basis functions, such that

\[
\rho(r) = \sum_n a_n f_n(r) \implies \tilde{\rho}(k) = \sum_n a_n \tilde{f}_n(k)
\]

where

\[
\tilde{f}_n(k) = \int_0^\infty dr \, r^2 j_0(kr) f_n(r)
\]

represents basis functions in momentum space. The expansion coefficients, \( a_n \), are fitted to form factor data subject to several minimally restrictive constraints. An arbitrarily large number of terms can be included by using a penalty function to constrain high-frequency contributions with an envelope of the form

\[
k > k_{\text{max}} \implies |\tilde{\rho}(k)| < |\tilde{\rho}(k_{\text{max}})| \left(\frac{k_{\text{max}}}{k}\right)^4
\]

where \( k_{\text{max}} \) is the largest frequency for which experimental data are available. This condition ensures that fitted density does not have an unphysical cusp at the origin but does permit the density sufficient flexibility to estimate the uncertainty due to the absence of data for \( k > k_{\text{max}} \). In addition, one constrains the density for very large radii. Details of these procedures can be found in Ref. [36] and references cited therein.

Analyses of this type are often described as model independent because a complete basis can reproduce any physically reasonable density; if a sufficient number of terms are included in the fitting procedure the dependence of the fitted density upon the assumptions of the model is minimized. By contrast, simple parametrizations like the
Galster model severely constrain the shape of the fitted density. As shown in Ref. [36], virtually identical results are obtained using either the Laguerre-Gaussian expansion (LGE) or Fourier-Bessel expansion (FBE).

**Results**

We fit all four nucleon electromagnetic form factors using a data selection that emphasizes recent polarization methods where available. For $G_{Mp}$ and $G_{Mn}$ we employ the highest quality cross section data in each range of $Q^2$. For $G_{Ep}$ we use the recoil polarization data from Refs. [6, 7, 13, 12] and chose cross section data from Refs. [37, 38] for low $Q^2$ but omitted the higher $Q^2$ Rosenbluth data from Refs. [8, 9]. For $G_{En}$ we use recoil and target polarization data corrected for nuclear physics effects and use the results of an analysis of the deuteron quadrupole form factor by Schiavilla and Sick [20]; Rosenbluth data for elastic or quasielastic scattering from deuterium were omitted. We also include the measurement of $\langle r^2 \rangle_n$ by Kopecky et al. [21] using the energy dependence for the transmission of thermal neutrons through liquid $^{208}$Pb. A more complete review of these selections and omissions can be found in Ref. [36].

Figure 4 shows fits to the form factor data using the LGE model with $\lambda = 2$. Where data are available the widths of the error bands are governed by the statistical quality of data while for large $Q^2$ the growth of these uncertainties is limited by the large-$k$ constraint specified by Eq. (11). These fits are generally very good, but in the $G_{Mn}$ data there remain appreciable systematic differences between data sets that probably reflect errors in the efficiency calibration for some of the experiments.

![Figure 4](image.png)

**FIGURE 4.** LGE fits to selected data for nucleon form factors using $\lambda_E = \lambda_M = 2$. For $G_{En}$ we also show a Galster fit.

Figure 5 compares the four fitted densities. For the proton we find that the charge is distributed over a larger volume than the magnetization. The difference between rms radii is not large, 0.883(14) for charge versus 0.851(26) fm for magnetization, but the large difference in interior densities reflects the strong decrease in $G_{Ep}/G_{Mp}$ for
\( Q^2 > 1 \text{ (GeV/c)}^2 \). The magnetization density for the neutron is very similar to that for the proton, but closer examination shows that its distribution is slightly wider. For the purposes of comparing shapes, the neutron charge density is shown scaled to the interior magnetization. Despite the limited range of \( Q^2 \) and larger uncertainties in the \( G_{En} \) data, the neutron charge density is determined with useful precision.

**FIGURE 5.** Nucleon electromagnetic densities are shown on the left and quark densities on the right using \( \lambda_E = \lambda_M = 2 \).

The neutron charge density features a positive interior and negative surface. In the meson-baryon picture these characteristics are explained in terms of quantum fluctuations of the type \( n \leftrightarrow p\pi^- \) in which the light negative meson is found at larger radius than the heavier positive core. Alternatively, in the quark model these features arise from incomplete cancellation between \( u \) and \( d \) quark distributions that are similar but not identical in shape. Using a symmetric two-flavor quark model of the nucleon charge densities

\[
\rho_p(r) = \frac{4}{3} u(r) - \frac{1}{3} d(r) \quad \rho_n(r) = -\frac{2}{3} u(r) + \frac{2}{3} d(r)
\]

one can obtain the quark densities

\[
u(r) = \rho_p(r) + \frac{1}{2} \rho_n(r) \quad d(r) = \rho_p(r) + 2 \rho_n(r)
\]

where \( u(r) \) is the radial distribution for an up quark in the proton or a down quark in the neutron while \( d(r) \) is the distribution for a down quark in the proton or an up quark in the neutron. These quark densities are also displayed in Fig. 5 weighted by \( r^2 \) to emphasize the surface region. We find that the \( u \) distribution is slightly broader than the \( d \) distribution, which is consistent with the repulsive color hyperfine interaction between like quarks needed to explain the \( N - \Delta \) mass splitting. The slightly negative \( d(r) \) near 1 fm suggests a \( \bar{d} \) contribution from the pion cloud. The secondary lobes near 1.4 fm appear to be robust features of the data — elimination of these features seriously
degrades fits to data for $Q^2 \sim 1 \text{(GeV}/c)^2$ — and might arise from mixed symmetry or $\ell = 2$ admixtures with larger radii than the dominant $S$-state configuration.

CONCLUSIONS

The advent of highly polarized electron beams with large currents and high energy coupled with advances in recoil polarimetry and polarized targets permit much more precise measurements of the nucleon electromagnetic form factor ratio, $G_E/G_M$, than was possible with the Rosenbluth technique at large $Q^2$. Despite the fact that Rosenbluth data suggested $\mu_p G_{Ep}/G_{Mp} \approx 1$ for $Q^2 < 6 \text{(GeV}/c)^2$, recoil polarization measurements at Jefferson Laboratory show a strong, nearly linear, decrease for $Q^2 > 1 \text{(GeV}/c)^2$ that demonstrates that the charge density is significantly broader than the magnetization density of the proton. Similarly, recoil and target polarization measurements show that $G_{En}$ is substantially larger than the commonly quoted Platchkov analysis of deuteron elastic scattering using the Paris potential yet remains surprisingly close to the original Galster parametrization despite the prohibitively large model dependence of Rosenbluth separations for $G_{En}$.

We have developed a phenomenological model of these form factors in terms of spatial densities that is consistent with pQCD at large $Q^2$ and with the Lorentz contraction of the Breit frame relative to the rest frame. The model dependence of the fitted densities is minimized by using an expansion in a complete set of basis functions with minimally restrictive constraints upon the behavior for either large frequency or large radius. The flexibility of the fitted form factor for frequencies beyond the measured momentum transfer provides an estimate of the incompleteness error in the extracted density. The error envelopes for the magnetization densities and the proton charge density are quite narrow, but the uncertainty in the neutron charge density is significantly larger because the data are still limited to $Q^2 < 1.5 \text{(GeV}/c)^2$ and are not as precise as for the other form factors. Nevertheless, the precision is already quite useful and will improve when the next generation of $^3\text{He}(e',e'n)$ experiments reaches about 3.4 $(\text{GeV}/c)^2$ [31].

We find that the neutron and proton magnetizations densities are quite similar but that the proton charge density is significantly broader. The neutron charge density results from incomplete cancellation between $u$ and $d$ quark densities with slightly different shapes that leaves a positive core surrounded by negative surface charge. By comparing the fitted neutron and proton charge densities, we find that the distribution of like quarks in the nucleon is broader than the distribution of the unlike quark.

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