Sensitivity of Cosmic Ray Modulation to an Outer Scale of Turbulence

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Abstract. We develop an ab initio modulation model, in which the diffusion tensor is specified by particle transport theory based on observed properties of the turbulence, and the spatial variation is determined by turbulence transport models that specify how turbulence properties vary throughout the heliosphere. Using a recent perpendicular diffusion treatment based upon the Taylor-Green-Kubo equation we study the sensitivity of solar wind modulation to an outer scale of turbulence, the "ultrascale", which is very poorly understood. For larger values of the ultrascale (approximately hundreds of correlation lengths) we find that the radial profile of modulated cosmic ray protons compares well with the observational data from Voyager and IMP for both polarities.

INTRODUCTION

In modeling the solar modulation of cosmic rays, the diffusion tensor and its spatial variation are usually treated as free parameters that can be adjusted to fit observational data. This has been recognised not to be a satisfactory procedure, since the diffusion tensor in fact is not free, but rather is governed by the interplanetary turbulent magnetic field. Thus we develop an ab initio theory [1] using a recent treatment of perpendicular diffusion based on Taylor-Green-kubo formalism [2], which is one of several available theoretical or numerical formulations of perpendicular diffusion [3, 4, 5]. The perpendicular diffusion in two-component slab + 2D turbulence depends critically upon an outer scale of turbulence called the “ultrascale” [6, 7] which has been studied very little.

Another important factor which is essential for ab initio theory is the radial variation of both the parallel and perpendicular diffusion coefficient which are strongly dependent on radial variation of the ordinary correlation length. However, the latter is very poorly understood which can be attributed to the uncertain impact of pickup ion-driven turbulence [8, 9] and to the difficulties in measuring the parallel correlation length in the outer heliosphere.

Here, we explore the sensitivity of modulation to ultrascale by direct numerical solution of transport equations by introducing a new perpendicular diffusion formalism as described earlier.

DESCRIPTION OF THE ULTRASCALE

Recently, a nonperturbative approach has been developed for a two-component model (slab plus 2D turbulence) to better understand the diffusion of magnetic field lines [6, 9]. In the 2D plus slab model of magnetic turbulence we assume \( B = B_0 + b (x,y,z) \), where \( B_0 = B_0 \hat{z} \), \( b \perp \hat{z} \), and \( b = b_{2D}(x,y) + b_{slab}(z) \). The magnetic field line diffusion coefficients are found to be a nonlinear combination of magnetic field line wandering for slab turbulence with \( l_{slab} \), as the parallel correlation length and the same associated with 2D fluctuations with ultrascale \( \bar{l} \) as an outer scale weighted by the 2D magnetic fluctuations. The field line diffusion coefficient in composite turbulence, \( D_{\perp} \), is related to the separate slab and 2D diffusion coefficients, \( D_{slab}^{\perp} \) and \( D_{2D}^{\perp} \), as follows [6, 10]:

\[
D_{\perp} = D_{slab}^{\perp} + (D_{2D}^{\perp})^2 / D_{\perp}^0,
\]

where \( D_{slab}^{\perp} = b_{slab}^2 / l_{slab}^2 / 2B_0^2 \), \( D_{2D}^{\perp} = b_{2D}^2 / B_0^2 \). Here, variance of slab magnetic fluctuation \( b_{slab}^2 = \langle b_{slab}(z) \rangle^2 \), variance of 2D magnetic fluctuation \( b_{2D}^2 = \langle b_{2D}(x,y) \rangle^2 \), \( k_{\perp}^2 = k_x^2 + k_y^2 \), and \( k = (k_x, k_y, k_z) \) denotes wavevector.

Physically the ultrascale is governed by large scale solar wind fluctuations. It can be considered as the length obtained from the ratio of mean square magnetic flux to mean magnetic fluctuation energy and is, therefore, a measure of the mean size of poloidal (2D) flux structures which could be as large as the order of the system size [7]. On the other hand it is not clear as to how much fluctuation energy resides in ultrascale fluctuations though it is known that substantial amount of energy resides in spatial scales between ion inertial scale...
FIGURE 1. Results from an ab initio modulation model for \( l_{\text{sab}} \propto r^{-0.3} \), and various ultrascales \( \tilde{l} = 10 \, l_{\text{sab}}, 50 \, l_{\text{sab}}, 500 \, l_{\text{sab}}, \) and \( 2500 \, l_{\text{sab}} \). Panels display model predictions for 1 AU spectrum at equatorial plane. Red (blue) is used for results for negative (positive) solar polarity.

FIGURE 2. Panels display model predictions for radial profile for 200 MeV particles at equatorial plane for the same four values of \( \tilde{l} \) as in Fig. 1. Red / diamond (blue / star) used for both model and observational results for negative (positive) solar polarity. Radial profile data [11] come from Voyager and IMP.
and the observed correlation scale. It seems very little has been done to describe these fluctuations intuitively, or in terms of mathematics of field line random walk. Nevertheless one can see from the definition of $D_{2D}^\perp$ and $D_{2D}^\parallel$ that $D_{2D}^\perp \propto (b_{2D}/B_0)$ whereas $D_{2D}^\parallel \propto (b_{2D}/B_0)^2$, and hence for small amplitude magnetic field fluctuations which means $b/B_0 \to 0$, the correct model is to include the ultrascale for 2D diffusion coefficient, and not to consider simply the slab or quasi linear approach which has smaller correlation length.

MODELS AND STRATEGY

The modulation of galactic cosmic rays is described by Parker’s transport equation [12]. For our two dimensional model we describe the anisotropic diffusion by the radial diffusion coefficient $\kappa_r = \kappa_0 \cos^2 \psi + \kappa_0^\theta \sin \psi$ and by $\kappa_0^\theta$, the diffusion coefficient perpendicular to the mean magnetic field in the polar direction. $\kappa_r$ and $\kappa_\theta^\theta$ denote respectively the diffusion coefficients parallel and perpendicular to the mean background magnetic field in the $r-\phi$ plane. Here $r$ and $\theta$ are heliocentric radial distance and colatitude (polar angle) respectively, and $\psi$ is the spiral angle. In our two dimensional model $\kappa_r^\theta$ acts only in radial direction, and the model simulates the effect of a wavvy current sheet [13] by using an averaged drift field with only an $r$ and $\theta$ component. We define $\kappa_r^\theta$ as [2]

$$\kappa_r^\theta = \frac{V R_L}{3} \frac{1}{\Omega \tau} \left( \frac{\Omega^2 \tau^2}{1 + \Omega^2 \tau^2} \right),$$

where $\Omega = 2 R_L/3 D_\perp$. Here $R_L$ is the particle Larmor radius, $V$ is the particle speed, and $\Omega = V/R_L$ is the angular gyrofrequency. We also have introduced the timescale $\tau$ which is the effective timescale for the decorrelation of the particle trajectories. In this report we set $\kappa_\theta^\theta$ as 5 times $\kappa_\theta^\theta$ at high latitudes, in accord with Ulysses observations of the magnetic field variance and anisotropy at high latitudes [14]. Because the particle drifts are sensitive to the sign of the magnetic field, inclusion of drift terms leads to different predictions for different solar magnetic polarities. For drift we use the form $\kappa_\perp = V R_L/3$. The motivation for this weak scattering form is given in [13]. The tilt angle $\alpha$ of the wave current sheet is set at $20^\circ$, a value appropriate for moderate solar modulation conditions. The solar wind velocity is radial. The solar wind speed is 400 km/sec in the equatorial plane and increases to 800 km/sec in the polar regions.

FIGURE 3. Latitudinal mean free path $\lambda_{\theta \theta} = 3 \kappa_{\perp}^\theta / V$ versus radial distance is displayed for the same four values of $\tilde{l}$ as in Fig. 1 at high latitude ($80^\circ$) for 200 MeV protons.

NUMERICAL RESULTS

We employ Direct Numerical Simulation of Parker’s transport equation by varying only $l/l_{slab}$. We study its impact on particle intensity, mean free path, and radial gradient in our attempt to build an ab initio model of modulation. In Fig. 1 the panels show the predicted energy spectrum for positive and negative polarity at 1 AU in the equatorial plane for various ultrascales. For the same ultrascale Fig. 2 shows radial profiles in the equatorial plane for 200 MeV protons. In all cases $l_{slab}$ (and hence $l$) is taken to scale as $r^{-0.3}$, in accord with theoretical predictions for pickup ion driven turbulence [15].

The 1 AU spectrum (Fig. 1) indicates modulation becomes stronger for larger ultrascale in case of positive polarity. Similar trend does not exist for negative polarity. The spectra for the two polarities sometimes cross, as in Fig. 1 (cf [16]). For larger ultrascale as in Fig. 1 (d), the intensity for negative polarity is larger than that in positive polarity which does not happen for smaller ultrascale scans. Fig. 2 clearly indicates the radial profile in negative polarity does not vary much in comparison with the observed data when different ultrascales are considered, whereas the gradient in positive polarity is very sensitive to ultrascale. As a result the radial profile is a good approximation to the observed data for moderately large ultrascale scans.

The results presented in Figures 1 and 2 can be understood with reference to the effect of the ultrascale upon the latitudinal diffusion coefficient at high latitude. As shown in Figure 3, $\lambda_{\theta \theta}$ becomes smaller for larger ultrascale, leading to increased modulation. The effect on modulation is especially prominent during positive solar polarity, because positive charges enter the heliosphere at high latitudes and drift out along the current sheet.
Their access to the equatorial plane is thus impeded by a smaller diffusion coefficient at high latitudes. During negative solar polarity, positive charges drift in along the current sheet and are comparatively insensitive to conditions at high latitudes, consistent with our simulation.

CONCLUSIONS

We describe here an ab initio approach to modulation modeling, in which parameters governing diffusion tensor are computed from available theories of solar wind turbulence based upon particle transport, and the variation of turbulence properties through the heliosphere is determined by turbulence transport models. We have focused on one important but poorly understood aspect of ab initio modeling: the size of the turbulence ultrascale which determines the rate of field line random walk for 2D turbulence modes.

We conclude that modulation is quite sensitive to the ultrascale, particularly during positive solar magnetic polarity (for positive charges). A larger ultrascale leads to reduced latitudinal diffusion at high latitudes, and thus greater modulation observed in the equatorial region. For moderately large ultrascale (hundreds of correlation lengths), we obtain qualitatively reasonable agreement with observed radial gradients, and a spectrum crossover similar to that reported in observations [16].

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