Large Amplitude Alfvén Waves
In Open And Closed Coronal Structures

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Abstract. The time-dependent response of the corona in a spherical shell between 1.8 and 16 $R_s$ to injection of low-frequency Alfvén waves at the inner boundary is considered in the MHD, isothermal and axisymmetric framework, without approximation for the wave-wind coupling. The magnetic field is the sum of an external dipole field assumed to be produced by the sun and of the field induced by the plasma motion in the spherical shell. Due to Alfvén wave injection, the wind and magnetic structure change, leading to an increased overexpansion of the high-latitudes flows and fields. Some of the factors which affect these changes: dissipation, and latitudinal distribution of the waves are explored, and the quantitative relation between wind speed and wave amplitude are discussed. We conclude that Alfvén waves alone lead ultimately to the disappearance of the slow wind, and that other factors, such as transverse structures and compressive waves, are necessary to explain the observed structure of the solar wind.

INTRODUCTION

It is well-known that one-fluid models have limitations when dealing with the heating problem (see the review by Hollweg in the present proceedings). Nevertheless, we believe that the dynamics of the corona, and in particular its response to perturbations, can be adequately studied in the framework of MHD, as soon as we assume that it is approximately (or fully as done here) isothermal.

In fluid models of the solar wind, an additional simplifying assumption, namely that waves are pure Alfvén waves following the WKB prediction is frequently made, together with a damping length, to be fixed ad hoc. Such an assumption allows to represent the effect of the waves by a time-average pressure term (the wave pressure). However, in doing so, one misses the effects of other wave modes which are often strongly coupled with the Alfvén mode, as we will see here. Why are other coupled modes important? The basic reason is that the mean flow velocity is not solution of the primitive equations with time-averaged coefficients, because the auto-correlation of the fluctuations is not zero in general: this is the origin of the Alfvén wave pressure [1]. Other types of fluctuations also contribute, as found for instance in coronal hole simulations [2], and may lead to surprising results, as in the particular case of upward propagating acoustic fluctuations, where the wave pressure term appears with a negative sign, so that the wind gets decelerated [3].

These preliminary remarks may sound irrelevant, as in fact models based on averaged Alfvén wave-pressure terms are able to predict the important properties of the wind like the factor two contrast between fast and slow wind [4]. However, in recent time-dependent simulations which do not rely on the pure Alfvén wave WKB approximation [5], we find that the equatorial wind is also strongly accelerated by the waves, so that the contrast between fast and slow wind is not large enough to meet the factor two, when extrapolated to large distances. In this previous work, we proposed two possible explanations of the discrepancy: either the latitudinal distribution of wave injection (which includes the closed field region in our work, not in Usmanov et al's work) was too wide, or the Reynolds number is too low in our simulations.

In this previous work [5], we observed other long-time phenomena related to flows in the closed magnetic structures, but we choose here to concentrate on the issue of the overall wind structure due to Alfvén...
waves, and in particular on the overexpansion of high-latitudes. We find that to cope with this question, compressive fluctuations cannot be neglected, so that it is difficult to trust pure Alfvén wave simulations. We conjecture that compressive fluctuations could disappear in presence of transverse quasi-equilibrium structures, but these also could not be described in the WKB limit.

**METHOD**

We consider the corona between 1.8 and 16 R_s. The parameters are as follows [5]: temperature is uniformly at 1.6 MK; the unperturbed wind has $\beta$ at the inner radius equal to 0.017 at the poles, 0.086 at the equator. The magnetic field is the sum of a fixed dipole field and of the field induced by the flow. Boundaries are open, only upward propagating waves are specified, not fields. The wave amplitude is 140 km/s, compatible with the upper limit by Esser et al. [6]; the wave period is 18 minutes; eventually also 32 min and 1 hour 24 min.

**FIGURE 1.** Wave injection conditions versus latitude at the inner boundary (1.8 R_s). Top: radial velocity profile before injection. Mid panel: $z_\phi = u_\phi \text{sign}(B_\phi)b_\phi/\sqrt{\rho}$; bottom: $u_\phi$; from start up to 16.8 hrs.

The injection of Alfvén waves in the simplest case is illustrated in Fig.1. We start from a relaxed wind with the usual properties of the Pneuman and Kopp [7] solution: at the inner boundary, there is a large region around the equator with a quasi-stagnation region (with albeit some small inflows). We excite outward propagating Alfvén waves via the incoming Alfvén characteristics, while there is no perturbation introduced via the incoming slow and fast characteristics [8]. This is done in a latitudinal range including a part of the stagnant region (mid panel). The poles are not excited, because this would be incompatible with the axisymmetric hypothesis, and a small region around the equator is excluded too, as this would also introduce singularities. In our basic run, waves are in phase in each hemisphere. The resulting azimuthal velocity (bottom panel) has an approximately uniform amplitude outside of the stagnant region, but is strongly varying within the stagnant region, due to interference there between waves propagating upward and downward along loops of closed field lines.

Note that the code has explicit dissipation terms: viscosity $\nu$, as well as artificial dissipation $\sigma$ for density, and filters. The overall dissipation is mainly due to $\sigma$ and $\nu$ [5].

**ALFVÉN WAVES INCREASE OVEREXPANSION**

Fig.2 shows, in the case of a non-monochromatic wave (three frequencies) with modulation of the phase with latitude, how waves injected in a large portion of the meridian propagate into the corona. The run starts with the unperturbed quasi-stationary wind, which shows a (mildly) slow equatorial wind with a current sheet where the density is slightly larger, and a (mildly) faster wind at high latitudes. Once injected, waves first propagate along the magnetic field lines without modifying them too much (note that the magnetic field lines are about parallel to the flow lines). The situation after 8.4 hrs is shown in the left panel. In a second phase (after 33.5 hrs injection, right panel), the flow lines are strongly bent towards the equator, so that the waves, which were previously excluded from this region, invade now almost completely the current sheet. As a consequence, the wind speed, which increased progressively at first well outside the equatorial region, is at the end substantially accelerated, even in the ecliptic.
FIGURE 2. Wave field (azimuthal component of the velocity) propagating in the accelerating region between 1.8 and 16 $R_s$. Low dissipation, three frequencies, with modulation of the phase. Left panel: after 8.4 hrs injection; right panel: after 33.5 hrs injection. Lines are selected lines starting at a given set of latitudes (same for left and right panels). Bold lines are flowlines, thin lines are magnetic field lines.

Fig.3 shows, in the case of a monochromatic wave with uniform phase as in Fig.1, the wind speed at three stages: unperturbed wind, after 8.4 hours injection and after 33.5 hrs injection. The progressive closure of the near-equatorial field lines due to the overexpansion of the high latitudes is again observed, together with the progressive acceleration of the global flow.

FIGURE 3. Monochromatic waves, uniform phases. Radial velocity field at three stages: unperturbed wind, transient and developed (8.4 hrs and 33.5 hrs). Lines: same as in Fig.2.

Due to the progressive closure of the near-equatorial field lines, and to the associated invasion of the ecliptic by the wave trains (Fig.2), the wind speed is not increased only at high latitudes, but also at the equator. As a result, the contrast between the high-latitude and the equatorial wind is not much increased, compared with the stationary Pneuman and Kopp wind [7], and finally it is far, at the outlet, from the factor two observed by Ulysses. We examine now whether this situation is generic or not, by varying some of the parameters of the numerical simulations.

**SAVE THE SLOW WIND?**

Diffusion leads to radial momentum transfer between fast and equatorial wind, and hence could be responsible for a large part for the acceleration of the slow wind [5]. To check this idea, we compare in Fig.4 the outlet radial velocity (16 $R_s$), obtained with two viscosities differing by a factor three. Profiles are drawn between 16.8 and 33.5 injection time, illustrating the evolution.

![Figure 4](image_url)

FIGURE 4. Radial velocity profiles versus latitude at outlet. Low Reynolds (top) and large Reynolds (bottom). Low profiles (below 400 km/s): unperturbed wind. High profiles: from 16.8 hrs up to 33.5 hrs after injection start. Note velocity is always increasing with time at the equator.

We find that decreasing the viscosity has two effects. First, the fast wind speed increases substantially. Second, the slow wind region becomes thinner, and its speed increases even more than that of the fast wind. Finally, the fast/slow wind contrast is
not increased, but decreased, and it seems plausible that in the limit of infinite Reynolds number, the slow wind would completely disappear.

A direct way to prevent accelerating the equatorial wind is to restrict waves to the open regions. This is illustrated in Fig. 5. The top and mid panels show the azimuthal velocity fluctuations, with respectively the previous, wide injection range, and the smaller domain, restricted to open regions. The bottom panel shows the radial velocity profile for reference, the lower curve showing the unperturbed velocity profile, the other curves showing the perturbed profiles generated by the waves when injected in the small domain.

Fig. 6 allows to compare propagation in the two cases, after 33.5 hrs injection. One sees immediately that the second injection mode (bottom) preserves, as expected, a large equatorial region from waves, as the overexpansion remains limited. Let us examine again the propagation with the previous injection mode where waves travel on the boundary of the closed region (top figure): one sees that, although wavetrains propagate at very different speeds and angle inside and outside the stagnant region, the wave packets outside remain connected with the wavepackets inside, so that wavetrains outside the closed region remain close to it, which ultimately lead to the overexpansion observed.

This description is of course valid only after long times. On the contrary, when the injection zone is restricted to the open region, the waves do not follow closely the stagnation region, and the unperturbed region remains large.

Fig. 7 shows several radial velocity profiles at outlet, during the period between 16.8 and 33.5 hrs injection. One sees that the equatorial speed is increasing at a much slower rate when the injection region is limited to open lines. However, there is no
indication that the process relaxes, and that the overexpansion stops increasing. In fact, with limited injection, the process is slowed down, but does not seem to be suppressed.

FAST WIND PROPERTIES

We now examine whether we can predict the wind speed from the wave amplitude. We consider for that a particular case, namely a simulation with three frequencies and a large injection region.

Let us examine how quantities vary with distance, at a given time (33.5 hours injection), and at a given latitude, 39° from south pole. Fig. 8 shows several profiles. First one (a) shows three radial velocities: the unperturbed one, the instantaneous one (after 33.5 hrs injection), and a background profile \( U_1^r \), which is empirically computed from the unperturbed profile \( U^r \), by the following formula:

\[
U_1^r = U^r + A \frac{\langle u_\phi^2 \rangle}{(v_{ac} c_s)^{1/2}}
\]  

where \( v_a \) is the Alfvén speed, \( c_s \) the sound speed, \( A \) is equal to 1 and \( \langle u_\phi^2 \rangle \) is the variance of the azimuthal velocity along the radial line (panel c). As shown in the top panel, the predicted background profile \( U_1^r \) fits well the real background profile, that is, the profile obtained by erasing the peaks from the instantaneous wind. Note that the Alfvén speed varies by a large factor in the distance range, so that relation (1) should have some physical significance.

Panel (d) shows another fit, which relates the instantaneous amplitude of the fluctuating wind, \( \delta u_\phi = u_\phi - U_1^r \), to an expression involving the instantaneous wave amplitude \( u_\phi \):

\[
\delta u_\phi = 0.5 \frac{u_\phi^2}{v_a}
\]  

Inspection by eye shows that errors in the fit come mainly from errors in the estimation of the background velocity \( U_1^r \), which in turn lead to errors in \( \delta u_\phi \). The relation (2) is almost the one expected for sound waves driven by Alfvén waves propagating parallel to the mean field (except for a factor \( 1/(1-\beta) \), see panel (b)). One can check indeed in panel (e) that the radial velocity fluctuations are related to the density fluctuations by the expected linear relation:

\[
\frac{\delta \rho}{\rho} = \frac{\delta u_\phi}{v_a}
\]  

Note that the density fluctuation \( \delta \rho \) is again computed as for \( \delta u_\phi \) by taking the difference between the instantaneous value and a background density. The three relations (1) to (3) are found to be quite general: they are valid at all latitudes not too close to the equator, with only some variations around unity for the value of the coefficient \( A \) in (1).

DISCUSSION

We studied in this work the propagation of monochromatic (or three frequencies) Alfvén waves in an isothermal, axisymmetric solar wind with external dipole field, and the resulting effect on the wind structure. The simulations show large compressive fluctuations which are difficult to reproduce via a
usual WKB-type model, hence confirming that fully
time-dependent simulations are necessary.

In summary, the injection of Alfvén waves
increases much the overexpansion of high latitudes.
The overexpansion is a slow process, which needs first
that the overall radial speed be substantially larger than
the unperturbed bulk speed. When this is achieved, the
fast streams reach even the ecliptic, so that the slow
wind practically disappears. The rate at which this is
achieved depends on the Reynolds number and the
latitudinal distribution of the wave injection. The
process is faster when the Reynolds number increases,
but slower when wave injection is restricted to initially
open regions. Nevertheless, the slow wind seems also
to ultimately disappear at long times, even in the latter
case.

Although it is not excluded that by playing with
injection (intermittency?) one could stop the
overexpansion towards equator, it thus seems likely
that some additional ingredient is necessary to recover
the slow wind. The solution could come from the
equator being a direct source of mass flux, which
might be possible by considering a multipolar external
field, and/or a direct injection of compressive waves.

Another result concerns the quantitative relations
between the background fast wind, the instantaneous
fast wind, the wave amplitude, and the compressive
fluctuations. Relation (1) which predicts the
background wind from the mean Alfvén amplitude, the
Alfvén and sound speeds remains to be explained
theoretically. On the other hand, relations (2) and (3)
between Alfvén and radial (compressive) fluctuations
are easily interpreted: they are a clear signature of
quasi-parallel propagation of Alfvén waves. This holds
also when injecting an oblique distribution of
wavevectors via a fast phase modulation with latitude
(see Fig.2). This is because, in spite of large transverse
Alfvén speed gradients which in principle trigger
deviations from parallel propagation, expansion
restores rapidly quasi-radial wavectors by stretching
wavefronts transversally [9]; this effect is visible in
Fig.2 by comparing wavefronts angle fluctuations at
small and large radial distance.

In the present simulations, in the 10 to 15 R range,
both the relative rms wave amplitude δB/B and the
relative density fluctuation are around 0.5 (Fig. 8b, c
and e): this is compatible with estimations based on
however that Fig. 8 indicates a rapid growth of the
relative density fluctuation with distance. A less
compressible regime has been obtained [8] by
considering small amplitude Alfvén waves and
temperature structures (channels), in which the waves
adopted an oblique, almost incompressible propagation
mode.

It would thus be interesting to investigate at what
conditions one could recover the oblique quasi-
incompressible regime but with large amplitude
waves. This is relevant if we recall that sound waves
decrease the average wind speed, contrary to Alfvén
waves which increase it. Hence, in the regions where
Alfvén waves would adopt an incompressible regime,
the wind would be faster, while in other regions with
more mixed conditions, the wind would be slower.
This offers an alternative way to control the fast/slow
wind speed ratio, resembling, superficially at least, the
situation in the real solar wind [12].

REFERENCES