A new exospheric model of the solar wind acceleration: the transsonic solutions

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Abstract. This paper presents basic issues for the solar wind acceleration with a collisionless model when the base of the wind is sufficiently low for the potential energy of the protons to have a maximum, thereby producing a transsonic wind. Using a formulation in terms of the particle invariants of motion, we study the existence of different categories of ion orbits and the consequences on the wind acceleration. We also study how a suprathermal tail in the electrons velocity distribution enhances the wind acceleration and makes the electron temperature increase within a few solar radii.

INTRODUCTION

The exospheric approach of the solar wind acceleration is based on the assumption that the Coulomb collisional mean free path of the particles in the corona is much greater than the density scale height above a given altitude called the exobase. The exobase is therefore usually defined as the altitude where these two characteristic quantities become equal. In these collisionless models, the interplanetary electrostatic field, which pulls electrons inward and accelerates protons outward in order to preserve zero electric charge and current, is calculated self-consistently [2], [5].

In previous models [6], the altitude of the exobase (> 5R⊙) was so high that the gravitational attraction on protons was everywhere smaller than the outward electric force, so that the total potential (electric+gravitational) of the protons was decreasing monotonically. This high exobase altitude may be adequate to model the slow solar wind emanating from equatorial regions where the density is high enough. This is not so, however, for the fast solar wind, which emanates from the coronal holes where the exobase is located deeper in the corona. In that case, the gravitational force is stronger than the electric one at the base of the wind so that the total potential energy of the protons is attractive out to some distance where the two forces balance each other. Farther out, the outward electric force dominates. That means that the total potential for the protons is not monotonic, presenting a maximum at a certain distance from the exobase [2]. The existence of this maximum has important consequences on the physics of the wind acceleration (see [8] in this issue) in governing the ion orbits, and is fundamentally associated to the production of a transsonic wind.

Since the electrostatic potential is determined by the balance of electric charge and current, it is rather sensitive to the velocity distribution function (VDF) of the electrons. Following Scudder’s pioneer work [10], recent exospheric models have modelled the electron VDFs with generalized Lorentzian ("kappa") functions in order to allow for the possibility of electron suprathermal tails. Such non-thermal VDFs enable us to obtain increased terminal bulk speeds [6].

In the present paper, we present some basic issues emerging from an exospheric model allowing both a non-monotonic potential and non-maxwellian electron VDFs (see [4] in this issue). We examine in particular how the wind properties vary with the non thermal character of the electrons VDF and set the basis for an improved exospheric description of the solar wind acceleration, which maps particle orbits in terms of the invariants of motion.

NON-MONOTONIC PROTON POTENTIAL ENERGY

As the plasma is assumed to be collisionless, the total energy and the magnetic moment of the particles are conserved, i.e.

\[ E = \frac{1}{2}mv^2 + m\phi_g + Ze\phi_E = cst \]  

(1)

\[ \mu = \frac{mv^2}{2B} = cst \]  

(2)

where \( v \) is the velocity of the particle of mass \( m \), \( Ze \) its charge, \( \phi_g(r) = -M_⊙G/r \) the gravitational potential
and $\phi_e(r)$ the interplanetary electrostatic potential. The total potential energy for the protons is then $\Psi(r) = m\phi_e(r) + e\phi_e(r)$, which reaches a maximum at some radial distance $r_{\text{max}}$. Below this altitude, the gravitational force is larger than the electrostatic one and the total potential is attractive. The opposite is true above $r_{\text{max}}$, forcing all the protons present at these altitudes to escape from the system. Whereas in previous models [6] all protons were escaping, the existence of a maximum allows other kinds of proton trajectories to exist below $r_{\text{max}}$. There are now protons (called “ballistic”) which do not have enough energy to escape from the gravitational well of the Sun and are therefore returning towards it. Another kind of non-escaping protons are the “trapped” ones which do not have enough energy to escape, but whose inclination to the magnetic field lines is large enough that they are reflected by the magnetic mirror force before returning back to the exobase $r_0$. Note that there are no particles coming from infinity.

Although previous exospheric models with non-maxwellian electron VDFs at the exobase [6] assumed a monotonic potential energy for the protons, the method of treating non-monotonic potentials for the solar wind had already been developed by Jockers [2] who did not allow for suprathermal electrons.

When dealing with the Vlasov equation, it is convenient to use the total energy $E$ and the magnetic moment $\mu$ as primary coordinates. The velocity distribution function can then be written as $f(E, \mu)$ as well as its moments. This choice is very convenient because it removes the spatial dependence of the VDF and only the region of integration (over which the moments are calculated) in $E - \mu$ space changes. The basic problem is the accessibility of the different particle populations in the $E - \mu$ space, described in [3] for an arbitrary potential energy structure.

The conservation laws (1) and (2) determine the region where the function $f$ is defined as:

$$v^2_1 \geq 0 \Rightarrow E \geq \mu B(r) + \Psi(r)$$  \hspace{1cm} (3)

where $B(r)$ is the magnetic field which is assumed to be radial. The relation (3) defines the line $v^2_1 = 0$ for each altitude $r$; the distribution function $f$ is defined only above this line. Note that the slope of this line is just the amplitude of the magnetic field $B$. Figure 1 shows the regions of integration for the different particle populations. The function $f$ is defined at the exobase $r_0$ only above the corresponding line $v^2_1(0)$. Escaping protons are present at all altitudes including $r_{\text{max}}$. This means that they are defined above all $v^2_1 = 0$ lines, a region which corresponds to the upper part of the figure. The filled region corresponds to ballistic protons which cannot escape from $r_{\text{max}}$ or lower altitudes. Trapped protons can exist in the region filled of circles as they are not present at $r_0$, nor at $r_{\text{max}}$.

![FIGURE 1. Accessibility of the different particle populations in $E - \mu$ space.](image)

Note that if all $v^2_1 = 0$ lines are located above the intersection point $(E^*, \mu^*)$ between the lines (0) and (m) (with $E^* = \mu^* B_0 + \Psi_0$), there are no trapped protons. We can therefore show that there are no trapped protons at altitudes where:

$$\Psi(r) \geq (\Psi_{\text{max}} - \Psi_0) \frac{r_{\text{max}}^2}{r^2} - \frac{r_0^2 - r^2}{r_{\text{max}}^2} + \Psi_0$$  \hspace{1cm} (4)

As we will see below, there are some cases in which the inequality (4) holds for all $r$ so that no trapped protons exist.

The situation presented in figure 1 is not complete, since only the influence of two altitudes ($r_1$ and $r_2$) is shown. Actually we have to take into account the influence of all altitudes between the exobase $r_0$ and $r_{\text{max}}$. In order to simplify the calculations, we make the following assumption. All particles which can be present at both $r_0$ and $r_{\text{max}}$ are considered as escaping. i.e. we consider that the region labelled $\varepsilon$ in figure 1 corresponds to escaping particles at $r_{\text{max}}$, instead of ballistic which do not overcome $r_2$. This important approximation is used by Jockers as the resulting error in the particle flux is in general less than 1% [2]. However, the errors due to this approximation may be much greater with suprathermal electrons. The relation (4) is valid only with this approximation.

In this paper, this assumption has been made for the protons for two reasons: the induced error is low enough with the chosen parameters and the simplicity of the method enables us to deduce basic aspects of the acceleration physics. The expressions given by Jockers [2] are therefore used for the protons, assuming a maxwellian VDF. An equivalent approach is to perform the integrations of the VDF in the velocity space [5]. This is leading to the same results as our approximative model (see [4] in this issue).
SUPRATHERMAL ELECTRONS

For the electrons, the situation is much simpler since their total potential energy \( \approx -e\Phi(r) \) is monotonically increasing with distance. The attractive potential allows the three kinds of particle orbits (escaping, ballistic and trapped) to be present at all altitudes (see [8] in this issue).

In the solar wind, the electron distributions are observed to have important high velocity tails and the most convenient way to fit them is with kappa functions [7], which play an important role in accelerating the wind by a strong filtration mechanism [10].

The electrons VDFs in the corona might also be non-maxwellian. The fundamental reason is that fast electrons collide much less frequently than slow ones because of their greater free path, so that they cannot relax to the equilibrium maxwellian distribution. This is why electron VDFs could have suprathermal tails, although the core distribution should be closer to a maxwellian. Recent exospheric models have therefore used kappa VDFs at the exobase [6] given by the expression

\[
f_{\kappa e} = \frac{n_{e0}}{\left(\pi kv_{th}^2\right)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \left(1 + \frac{v^2}{k^2 v_{th}^2}\right)^{-\frac{\kappa+1}{2}}
\]

where the thermal speed is defined by \( v_{th} = \left(\frac{2\pi - 3 k_{B} T_{e}}{m_{e}}\right)^{1/2} \). This function tends to a maxwellian when \( \kappa \to \infty \).

In our model we use this electron distribution, with the expressions of the moments given by [9]. Note, however, that the value of \( \kappa \) in eq.(5), which represents the non thermal character, remains constant with altitude, which does not seem to agree with recent observations [1]. We use this distribution as a first step to represent suprathermal tails, having in mind that a more realistic VDF could be a sum of two maxwellians [8] (which, however, needs one more free parameter) or a superposition of a maxwellian and a truncated power law. We will return to this point below.

RESULTS AND DISCUSSION

As in previous exospheric models, the electrostatic potential is calculated by imposing the quasi-neutrality and by equalizing the electron and proton fluxes at all altitudes [2],[6]. In order to study the basic physics and since the exobase location is defined from parameters which are not accurately known, we put the exobase at \( r_{0} = 1R_{e} \). This stands as an approximation for a more realistic value which should be slightly higher and has no important qualitative impact on the results. The initial temperature values at the exobase are taken as \( T_{e0} = 10^6 K \) and \( T_{p0} = 2T_{e0} \). The total potential energy for the protons is shown in figure 2 for different values of \( \kappa \) ranging from \( \kappa = 6 \) to \( \kappa = 2.5 \), a case with a conspicuous suprathermal tail. We can see that the value of the maximum of potential increases as \( \kappa \) decreases. This is because with more suprathermal electrons, a stronger electric potential is needed to preserve quasi-neutrality. Note also that \( r_{max} \) increases with increasing \( \kappa \). In any case the potential tends to zero at large distances.

When the electric potential is known, the bulk speed can be calculated at all distances [6]. This is shown in figure 3 for the same values of \( \kappa \) as in fig.2. It can be seen that a high terminal bulk speed (>700km/s) is obtained when the suprathermal electron tail is conspicuous (\( \kappa = 2.5 \)). This is due to the large value of the maximum in ion potential energy (\( \approx 14k_{B} T_{p0} \)), which is transformed into kinetic energy of the escaping protons as they are accelerated above \( r_{max} \). An important remark is that the major part of this high terminal bulk speed is obtained within small distances (\( \approx 10R_{e} \)) which is due to the high acceleration represented by the important slope of the potential after \( r_{max} \).

The large suprathermal tail assumed has another important consequence. It makes the electron temperatures increase considerably up to a maximum (\( \approx 7 \times 10^{6} K \)) within a few solar radii. This maximum in electron temperature is smaller for larger values of \( \kappa \) and disappears as \( \kappa \to \infty \) as shown in fig.4. This heating is a direct consequence of filtration of the non maxwellian VDF by the attracting potential [10]. A problem with the \( \varepsilon \) region approximation is a discontinuity in the calculated proton temperatures obtained just above \( r_{max} \). The same

\[
|\varepsilon| = \frac{1}{\kappa} \left(1 - \frac{1}{\kappa} \right) \left(1 + \frac{v^2}{k^2 v_{th}^2}\right)^{\frac{\kappa+1}{2}}
\]

FIGURE 2. Total proton potential energy for different values of \( \kappa = 2.5, 3.0, 4.0 \) and 6.0. The energy is normalized in \( k_{B} T_{p0} \). The maximum in each case is designated by the diamont.
problem was present in Jockers’ models [2] and could be probably eliminated with the implementation of a generalized model with no assumption regarding the $\varepsilon$ region (fig.1).

The $E - \mu$ formalism enables us to examine whether trapped protons are present. In figure 5 the dashed line represents the right hand side of eq.(4) between $r_0$ and $r_{\text{max}}$ for $\kappa = 2.5$, whereas the full line corresponds to the calculated $\Psi(r)$ values. We can see that the inequality (4) holds for all $r$, which means that there are no trapped protons. This result is very important as it shows that trapped protons are not necessary to produce a wind, when the electron VDFs are highly non-thermal. This no longer holds when the distribution is close to a maxwellian as the two lines intersect in the space $(r_0, r_{\text{max}})$ for the case $\kappa = 6$. Trapped protons are present up to the intersection point and there are only ballistic (and escaping) ones beyond it. The presence or not of trapped particles is an important issue as they are ad hoc populated in fully collisionless models.

Note, finally, that our results suggest that the Kappa velocity distribution may not be adequate to model VDF having suprathermal tails in the corona. Indeed, producing large enough speeds requires irrealistically small values of $\kappa$, which, furthermore, yield an electron temperature increase much larger than observed. Alternatively, more realistic values of $\kappa$ produce an electron temperature increase close to the observed values, but a too small terminal velocity. A future study will include more realistic non thermal distributions.

REFERENCES