Three-dimensional MHD modeling of the solar corona and solar wind

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Abstract. A global MHD model is developed to reproduce Ulysses observations during its fast latitude transition in 1994-1995. The governing polytropic single-fluid MHD equations are solved for a steady coronal outflow. The model includes Alfvén wave momentum and energy addition into open field regions. We combine a solution for a tilted dipole magnetic field in the inner computational region (1-20 $R_\odot$) with a three-dimensional solution in the outer region which extends to 1 AU. The inner region solution is essentially the same as in [1], but obtained with a different numerical algorithm and rotated to match the inclination inferred for the solar dipole from observations during the Ulysses transversal. The steady solution in the outer region is constructed by a marching-along-radius method and accounts for solar rotation. We show that the simulated variations of plasma and magnetic field parameters and in particular the extension of slow wind belt agree fairly well with the Ulysses observations.

INTRODUCTION

Ulysses observations during its first fast latitude scan in 1994-1995 revealed a bimodal solar wind with a sharp transition from a uniform fast wind at high latitudes to a relatively slow wind around solar equator [2]. The magnetic field was dominantly outward (inward) in the northern (southern) hemisphere and in the fast wind showed no prominent dependence on latitude [3]. Those observations were taken just prior solar activity minimum when the dominant component of the solar source magnetic field was a dipole inclined by about 10° to the solar rotation axis [4]. The lack of a significant latitudinal gradient in the radial magnetic field implies that magnetic field is transported to lower latitudes by non-radial coronal expansion.

The large-scale structure of the expanding solar corona is determined largely by the pattern of magnetic fields on the solar photosphere. A number of simulation studies have tried to match the Ulysses observations by solving the equations of magnetohydrodynamics (MHD) with the photospheric field as boundary condition [5, 6, 7, 8, 9, 10]. To produce a fast wind and to get a reasonable agreement with typical coronal data, however, an additional source of momentum must be incorporated into the models. There are a number of candidates for this role, one of which is Alfvén waves. The ability of the waves to bring models into agreement with observations both near the Sun and at large distances was recognized three decades ago [11, 12] and was extensively exploited in one-dimensional models with the flow tube geometry prescribed more or less arbitrarily ab initio, e.g., [13, 14]. It appears to be very attractive and natural to combine the wave acceleration mechanism with two- and three-dimensional approaches to solar corona and solar wind MHD modeling in which the flow geometry is determined self-consistently.

Usmanov et al. [1] developed an axisymmetric model in which a steady coronal outflow was simulated in a dipolar magnetic field. In that work, the WKB Alfvén waves were explicitly invoked as a means of heating and accelerating the solar wind flow and a steady-state two-dimensional solution was obtained. The dipole field strength and the amplitude of Alfvén waves at 1 $R_\odot$ (solar radius) were chosen to obtain a good fit to Ulysses data. A self-consistent solution was constructed by applying a time-relaxation technique in the region near the Sun. In the outer computational region, a marching-along-radius numerical algorithm was used. The solution formed a bimodal structure of fast and slow wind, as observed, and the computed parameters were generally consistent with Ulysses data and with typical parameters at the coronal base.

Although the bimodality is already present in the models without waves — faster and more tenuous wind from polar regions and slower and denser wind above the streamer in the heliospheric plasma sheet, the contrast is relatively small to match the pronounced latitudinal variation observed by Ulysses [1]. The addition of waves increases the velocity of the faster wind and lowers its den-
sity to observed values, but has a relatively small effect on the slower wind belt, where the divergence of flow tubes is much higher, plasma is denser, and the plasma beta is much higher than unity. Note also that in [1] the Alfvén wave energy drops to zero (i.e., has no acceleration effect) at the neutral sheet, which is embedded in the plasma sheet where the magnetic field is essentially zero.

The two-dimensional study [1] neglected all gradients in the azimuthal direction and the north-south symmetry was enforced by assuming the solar magnetic field to be a dipole perpendicular to the solar equatorial plane. The dipole assumption led to a heliospheric current sheet (HCS) that was aligned with the equatorial plane, while the actual HCS deviated slightly from that plane [15]. To account for that effect the model curves in [1] were shifted artificially by 15° north and south, emulating a relatively wide belt of slow wind near solar equator.

In the present study, we relax the assumptions of axial and north-south symmetry to account for that HCS warping more consistently. The basic idea is to use essentially the same solution for the inner region as in [1], but to transform it to match the observed position of the solar dipole, and than extend that tilted-dipole solution to the Earth’s orbit through the outer region II using a three-dimensional model. By taking into account the solar rotation in the outer region, we incorporate into our model the interaction between faster and slower solar wind streams and the azimuthal component of magnetic field. To solve the problem in the inner region, we applied a newer TVD (Total Variation Diminishing) Lax-Friedrichs algorithm with the Woodward limiter [16].

The field-interpolated central difference approach suggested in [17] is used to maintain the $\nabla \cdot \mathbf{B} = 0$ constraint.

### MODEL FORMULATION

The governing MHD equations for a single-fluid, polytropic flow driven by thermal and Alfvén wave pressure gradients, including solar rotation are

\[
\frac{d\rho}{dt} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)
\]

\[
\frac{d(\rho \mathbf{v})}{dt} + \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + \left( P + \frac{\varepsilon}{2} + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{1}{4\pi} \rho \mathbf{B} \mathbf{B} \right]
+ \rho \left[ \frac{GM_\odot}{r^2} \mathbf{r} + 2 \Omega \times \mathbf{v} + \Omega \times (\Omega \times \mathbf{r}) \right] = 0, \quad (2)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (3)
\]

\[
\frac{d}{dt} \left[ \frac{\rho}{2} (v^2 - |\mathbf{r} \times \mathbf{r}|^2) + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi} \frac{\rho GM_\odot}{r} + \varepsilon \right]
+ \nabla \cdot \left\{ \left[ \frac{\rho}{2} (v^2 - |\mathbf{r} \times \mathbf{r}|^2) + \frac{P}{\gamma - 1} + \frac{\rho GM_\odot}{r} \right] \mathbf{v} 
+ \frac{B}{4\pi} \times (\mathbf{v} \times \mathbf{B}) + \left( \frac{3}{2} \frac{\mathbf{v} + \mathbf{V}_A}{L} \mathbf{E} \right) \right\} = 0, \quad (4)
\]

\[
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot [ (\mathbf{v} + \mathbf{V}_A) \varepsilon ] = - \frac{\varepsilon}{2} \nabla \cdot \mathbf{v} - |\mathbf{v} + \mathbf{V}_A| \frac{\varepsilon}{L}, \quad (5)
\]

where the dependent variables $\rho$, $\mathbf{v}$, $\mathbf{B}$, $P$, and $\varepsilon$ are the plasma density, the flow velocity in the frame rotating with the Sun, the magnetic field, thermal pressure, and the Alfvén wave pressure, respectively. $M_\odot$ is the solar mass, $\Omega$ the solar angular velocity vector, $\gamma$ the polytropic index, $t$ the time, $r$ the heliospheric distance, $G$ the gravitational constant, $\mathbf{r}$ a unit vector in the radial direction, $\mathbf{V}_A = B/(4\pi\rho)^{1/2}$ the velocity of outward propagating Alfvén waves, and $L$ the unit matrix. The Alfvén wave effects are incorporated into the governing equations in the WKB limit and it is assumed that the waves are damped by a mechanism that may be characterized by a dissipation length $L$.

As in [1], we separate the computational domain into two regions so that the flow in the outer region would be super-sonic and super-Alfvénic: the inner region I (1-20 $R_\odot$), where the equations (1–5) are solved by the time-relaxation method, i.e., the governing equations are integrated in time up to a steady state, and the outer region II (20 $R_\odot$-1 AU) where the solution is constructed by forward integration along the hyperbolic radial coordinate [18].

### The governing equations: region I

In this region, we solve a two-dimensional problem of axisymmetric flow in the dipole field and neglect the solar rotation. The equations (1–5) can be then rewritten in component form as

\[
\frac{\partial}{\partial t} (r^2 \rho) = - \frac{\partial}{\partial r} (r^2 \rho u_r) - \frac{r}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho u_\theta), \quad (6)
\]

\[
\frac{\partial}{\partial t} (r^2 \rho u_r) = - \frac{\partial}{\partial r} \left[ r^2 \rho \left( u_r^2 + \frac{P}{\rho} + \frac{\varepsilon}{2} + \frac{B^2}{8\pi \rho} + \frac{B^2}{r^2} \right) \right]
- \frac{r}{\sin \theta} \frac{\partial}{\partial \theta} \left[ \rho \sin \theta \left( u_r u_\theta - B_r B_\theta \right) \right]
+ \rho r \left( u_r^2 - \frac{GM_\odot}{r} + \frac{2P}{\rho} + \frac{\varepsilon}{2} + \frac{B^2}{8\pi \rho} \right), \quad (7)
\]

\[
\frac{\partial}{\partial t} (r^2 \rho u_\theta) = - \frac{\partial}{\partial r} \left[ r^2 \rho \left( u_r u_\theta - B_r B_\theta \right) \right]
+ \frac{\partial}{\partial \theta} \left[ \frac{1}{r} \rho \left( u_r u_\theta - B_r B_\theta \right) \right]. \quad (8)
\]
\[-r \frac{\partial}{\partial \theta} \left[ \rho \left( \frac{u_r^2 + P}{\rho} + \frac{\gamma P}{\gamma - 1} + \frac{B_r^2 - B_\theta^2}{8\pi \rho} \right) \right] \]
\[-r \rho \left[ u_r u_\theta - \frac{B_r B_\theta}{4\pi \rho} + \cot \theta \left( \frac{u_\theta u_\theta}{4\pi \rho} - \frac{B_\theta^2}{4\pi \rho} \right) \right], \quad (8) \]
\[\frac{\partial}{\partial t}(r B_r) = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta}[\sin \theta (u_r B_\theta - u_\theta B_r)], \quad (9)\]
\[\frac{\partial}{\partial t}(r B_\theta) = -\frac{\partial}{\partial r}[r(u_r B_\theta - u_\theta B_r)], \quad (10)\]
\[\frac{\partial}{\partial t} \left[ r^2 \left( \frac{\rho u_r^2}{2} + \frac{P}{\gamma - 1} + \frac{B_r^2}{8\pi} + \mathcal{E} \right) \right] \]
\[= -r \frac{\partial}{\partial r} \left[ r^2 \left( u_r \left( \frac{\rho u_r^2}{2} + \frac{\gamma P}{\gamma - 1} \right) \right) \right. \]
\[+ \frac{B_r}{4\pi \rho} (u_r B_\theta - u_\theta B_r) + \left( \frac{3}{2} u_r + V_A r \right) \mathcal{E} \right] \}
\[= -\frac{\rho u_r}{\rho} \nabla \cdot \mathbf{u} - |\mathbf{u} + \mathbf{V}_A| \mathcal{E} \] \quad (12)

where \( \mathbf{u} = (u_r, u_\theta) \) is the velocity vector in the inertial reference, \( B_r \) and \( B_\theta \) are the magnetic field components, \( u^2 = u_r^2 + u_\theta^2 \) and \( B^2 = B_r^2 + B_\theta^2 \).

The governing equations: region II

In region II, we assume the absence of a steady electric field \([1, 18]\). Consequently, plasma is flowing along the magnetic field in the rotating frame and the equations for tangential magnetic field components may be excluded from the governing set by using the relations: \( B_r = u_r B_r / u_r \) and \( B_\theta = u_\theta B_\theta / u_r \), where \( u_\phi = u_\theta - w \) is the azimuthal velocity in the rotating frame and \( w = \Omega r \sin \theta \). Using the divergence-free condition instead of the induction equation (3) for the magnetic field we have

\[\frac{\partial}{\partial t}(r^2 \rho u_r) = -\frac{r}{\sin \theta} \frac{\partial}{\partial \theta}(\sin \theta \rho u_\theta) - \frac{r}{\sin \theta} \frac{\partial}{\partial \phi}(\rho u_\phi), \quad (13)\]
1 $R_\odot$ is assumed to be 35 km s$^{-1}$, close the upper limit value inferred by Hassler et al. [19]. The strength of the dipole field at the coronal base was chosen to be 12 G. The plasma temperature and density in the initial state at $1 R_\odot$ are $1.8 \times 10^6$ K and 7.5 $\times 10^6$ particles cm$^{-3}$, respectively, and the dissipation length for Alfven waves is $L = 80 R_\odot$. Note that the values above were selected to optimize the fit to Ulysses data.

**Simulation Results**

We start from an initial state with radial flow in a dipolar magnetic field [1] and integrate the region I equations (6-12) in time until a steady state is achieved. The computations are performed on a grid with the angular spacing of 2$^\circ$ and the radial step which increases linearly from 0.02 $R_\odot$ at 1 $R_\odot$ to 0.4 $R_\odot$ at 20 $R_\odot$. The boundary conditions are similar to those in [1]. Once the axisymmetric solution is obtained, it is transformed into a three-dimensional distribution that matches approximately the orientation of solar dipole during the first fast latitude scan of Ulysses (September 1994 - July 1995). The dipole orientation is computed from the expansion coefficients for the photospheric field inferred at the Wilcox Solar Observatory (WSO) from the line-of-sight boundary condition and the source surface at 2.5 $R_\odot$ [4]. Although the dipole orientation changed slightly during the Ulysses transition, for the entire interval we used the dipole parameters from solar rotation 1887 (at the beginning of the transition, September-October 1994), when the dipole axis deviated by 9.7$^\circ$ from the rotation axis and its azimuth in the northern hemisphere was 330$^\circ$.

The boundary between regions I and II is placed in the supersonic and super-Alfvenic flow, so a solution in region II depends only on the flow characteristics on the boundary. The transformed solution at the upper radial level in region I is used to initialize integration of equations (13-19) along radius through region II to 1 AU.

Figure 1 shows variations of velocity and density with latitude at $\phi = 0$ for various heliocentric distances from $1 R_\odot$ to 1 AU ($\sim 215 R_\odot$). A slower wind around the equator turns into a uniform fast wind at higher latitudes. The flow structure is not symmetric about the equator in the meridional plane and the slow wind pattern at 1 AU is shifted towards the equator due to solar rotation. The slow wind speed at the Earth’s orbit is less than 400 km s$^{-1}$ at minimum, jumps to $\sim 700$ km s$^{-1}$ by $\sim 15^\circ$, and then slowly increases towards the pole. The number density is $\sim 3$ cm$^{-3}$ in the fast wind and reaches $\sim 20$ cm$^{-3}$ near the equator.

Contour maps of the flow parameters in the heliographic coordinates are presented in Figure 2. The left three panels show the radial magnetic field $B_r$, radial velocity $u_r$, and number density $n$ at the coronal base.
FIGURE 2. Contour plots of the radial magnetic field, the radial velocity and the number density in the heliographic coordinates at 1 $R_\odot$ (left panels) and at 1 AU (right panels). Ulysses’ trajectory is shown on the right panels by dotted lines. The stagnation region (of no mass outflow) is hatched on the left middle plot. Negative levels of $B_r$ are shown by dashed lines.

(1 $R_\odot$). The $B_r$ distribution at 1 $R_\odot$ is dipolar and is kept as boundary condition during the relaxation process. The map of $u_r$ at 1 $R_\odot$, where the hatching highlights the region without outflow ($u_r = 0$), provides a view of solar wind sources at the coronal base: the wind is streaming out of the polar regions extending down by $\sim 30^\circ$ from the dipole axis. In these regions which can be regarded as “coronal holes,” the outflow speed is $\sim 16$ km s$^{-1}$ (see also Figure 1), while $n \sim 7.5 \times 10^7$ cm$^{-3}$ which is markedly lower than in the stagnation belt nearby where $n$ sharply increases to $\sim 1.5 \times 10^8$ cm$^{-3}$. The distribution of plasma temperature (not shown) is similar to that of density; the temperature changes from $1.8 \times 10^8$ K in polar regions to $2.0 \times 10^8$ K near the equator.

The computed solar wind parameters at 1 AU are presented on the right plots in Figure 2. The computed wind is clearly bimodal: no prominent variations both in plasma and magnetic field parameters except for a narrow equatorial belt where the flow is relatively dense and slow and $B_r$ changes its sign. The trajectory of Ulysses in the heliographic coordinates during solar rotations 1887–1898 is superimposed on the plots and consists of slightly inclined lines depicting its travel from south to north polar regions.

A direct comparison of Ulysses observations with the model results is shown in Figure 3. The daily Ulysses data were scaled to 1 AU assuming that $B_r$ and $n$ to fall off with radial distance as $r^2$, the azimuthal magnetic field $B_\phi$ as $r^{-1}$, and the temperature $T$ as $r^{-2(\gamma-1)}$ with $\gamma = 1.46$. It is seen from Figure 3 that our attempt to account for the HCS warping by inclining the solar dipole with respect to the solar rotation axis has provided fairly good agreement between the model and the Ulysses observations, including the latitudinal extension of the slower wind belt. Note that the normalization of Ulysses data to 1 AU is not a necessary condition for comparison with the model because the latter can be easily extended to $\sim 2.3$ AU to cover the range of heliospheric distances scanned by Ulysses. Our decision in favor of the normalization was made mainly to eliminate relatively easy-to-follow radial variations and emphasize the latitudinal structure of the solar wind flow. The computed temperature is somewhat higher than that observed, but could be adjusted length $L$: larger $L$ would provide less wave dissipation and ultimately lower temperatures. The electron temperature in the distant solar wind is larger than the proton temperature, so that the single-fluid temperature used in our simulation can be higher than the proton temperature observed by Ulysses and presented in Figure 3.
results from latitudinal oscillations of a relatively narrow region of slow velocities (see Figure 1) due to warping of the heliospheric current sheet and the solar rotation. To reproduce the slow wind observations in more detail, we plan to implement a fully three-dimensional model which would incorporate higher-than-dipolar harmonics of solar magnetic field into simulation.

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FIGURE 3. The model output (dashed lines) versus Ulysses data from the first fast latitude scan in 1994-1995 (solid lines). The Ulysses data are normalized to conditions at 1 AU.

SUMMARY

We have produced a simulation of the inner heliosphere during solar activity minimum as determined by boundary conditions at the coronal base and compared output from the model with Ulysses observations in 1994-1995. The bimodality of solar wind with a rapid change in flow parameters with latitude is reproduced along with the observed extension of the slower wind belt in latitude. In the present tilted-dipole model, this extension