Stopping of relativistic heavy ions; the pair production and bremsstrahlung channels

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Abstract. The contribution of electron-positron pair creation and bremsstrahlung to the energy loss of relativistic heavy ions penetrating matter is examined. At extreme projectile energies pair production and bremsstrahlung will be the dominant sources of energy loss, and already at energies of a few hundred GeV/u, the pair-production contribution will be measurable, that is, at the level of several per cent. An analytical formula for the average energy loss due to pair production is presented for the case when atomic screening is neglected along with a simple estimate to account for screening in an approximate manner. The energy loss due to bremsstrahlung is also evaluated. Furthermore, we provide a formula which determines the projectile energy where pair production becomes a more important energy-loss channel than atomic excitation and ionization and we estimate the energy where bremsstrahlung finally takes the lead.

1. INTRODUCTION

When a heavy ion penetrates matter it looses energy through the interaction with target constituents. Over a wide energy range, whose upper end extends far into the relativistic regime, loss of energy appears mainly as a result of excitation and ionization of target electrons. For a non-perturbative theory of the stopping of relativistic heavy ions due to such interactions the reader is referred to a previous publication, [1].

As the projectile energy is raised to very high values, radiative processes become increasingly important energy-loss channels. In particular the creation of electron-positron pairs and the emission of bremsstrahlung prove important. At sufficiently high energies these processes will account for the major part of the energy loss of a heavy ion. Dramatically different estimates of the pair-production contribution to the stopping have been published over the years [2, 3, 4] and the importance of the bremsstrahlung channel has been severely overestimated in various papers [2, 5]. We shall discuss the processes below.

The result of a detailed analytical calculation of the pair-production contribution to the stopping is presented in Section 2. A closed formula is obtained. As an application of this formula the pair-production contribution to the energy loss is determined for the measurements carried out recently at CERN for lead ions of energy 160 GeV/u [6]. Pair production accounts for the reported deviations from ionization theory, at least for the heavier target materials. Moreover a simple formula is produced to determine the projectile energy where pair production becomes a more important energy-loss channel than atomic excitation and ionization.

An estimate of the energy loss due to emission of bremsstrahlung is presented in Section 3. Consideration of the lengths characterizing the emission process points to the origin of errors committed in previous evaluations and allows for a simple estimate based, as that by Ahlen [2], on classical electrodynamics. The result is an estimate which predicts a significantly smaller bremsstrahlung contribution than reported previously, in some cases by orders of magnitude. Compared to bremsstrahlung, pair production is a relatively more important energy-loss channel than reported in, e.g., [2, 5]. Yet, bremsstrahlung will eventually, as the primary energy is raised to extreme values, be the most important energy-loss channel.

2. THE PAIR-PRODUCTION CHANNEL

Let us first establish, by simple means, that pair production will be a more important energy-loss channel than atomic excitation and ionization when the primary energy is raised sufficiently high: In the high-energy limit the cross section for pair production in collisions between a projectile nucleus of charge \( Z_e \) and a stationary target nucleus of charge \( Z_t e \) scales roughly as

\[
\sigma_{pp} \propto Z^2 Z_t^2 \ln^3 \gamma ,
\]

(1)

where \( \gamma \) denotes the energy \( E \) of the projectile in units of its rest mass \( M \), that is, \( \gamma \equiv E/Mc^2 \); cf. [7, 8]. Screening of the target nucleus by atomic electrons changes
the asymptotic scaling with the Lorentz factor $\gamma$ of the projectile to $\ln^2 \gamma$, cf. [8]. With energies of the created electron and positron amounting typically to a substantial fraction of $\gamma mc^2$, where $m$ denotes the electron mass, the energy loss to the pair-production channel will eventually, when the primary energy is raised sufficiently high, beat the ionization energy loss, which becomes $\gamma$-independent due to the combination of the density effect and the effect of finite nuclear size.

To obtain a formula for the energy-loss rate due to pair production we utilize the differential cross section for a point nucleus versus zeroth moment of the probability distribution), the dominance of region I already in the cross section implies that it is safe solely to consider pairs of energy beyond $\tau$ in the evaluation of the energy-loss rate. Hence our computation of the energy loss due to pair production is based on Racah’s differential production cross section for region I. The cross section is differential in two variables, the total energy of the pair, $\varepsilon$ in Racah’s notation, and the relative energy of one of the created particles in the pair, $\omega/\varepsilon$ in Racah’s notation. To obtain the energy-loss rate we multiply Racah’s expression (for region I) with the pair energy $\varepsilon$ and integrate over the two variables. The integration is performed along the lines given by Racah — and, in particular, in the same order. Expansion is performed in the parameter $\delta \equiv \varepsilon/\gamma mc^2$. Details will be given elsewhere [10].

It is convenient to express the energy-loss rate due to pair production as

$$-\frac{dE}{dx} = \pi Z^2 Z'^2 \frac{\alpha^2}{\varepsilon} N \gamma mc^2 \Lambda,$$

where $N$ is the atomic density of the target. To lowest order in $\delta$, the dimensionless quantity $\Lambda$ then reads

$$\Lambda_0 = \frac{19}{9} (l_0 - 3 \ln 2 - \frac{11}{6}) = \frac{19}{9} (\ln \frac{\gamma}{4} - \frac{11}{6}).$$

In an expansion of the form $\Lambda = \Lambda_0 + \Lambda_1 + \ldots$ the first correction to (6) assumes the value

$$\Lambda_1 = \delta \left[ \frac{4178}{81\pi^2} - \frac{21}{27} - \frac{248}{27\pi^2} l_0 \\ + \frac{1}{\gamma^2} \left( \frac{28}{9} l_0 - \frac{446}{27} \right) \ln \delta^2 + \frac{14}{9\pi^2} (\ln \delta^2)^2 \right].$$

With our previous choice for $\varepsilon$, the quantity $\delta$ will be of order $1/\sqrt{\gamma}$.

Screening of the target nucleus by atomic electrons is important at high energies. In this region higher-order corrections similar to (7) are small and there is no need to worry about screening corrections to such terms. For the lowest-order term screening may be accounted for approximately by applying, at all energies, the following expression in place of (6):

$$\frac{19}{9} \ln \left( \frac{183Z_t^{1/3}}{1 + 4e^{1/6} \frac{183Z_t^{1/3}}{\gamma} \gamma} \right) \equiv \Lambda_{\text{screen}}.$$
Furthermore, pair production on atomic electrons may be accounted for very roughly by multiplying the rates obtained so far by the factor \((1 + 1/Z_t)\).

The results reported above compare well with various results obtained more than thirty years ago for muon stopping. For a critical review of the early literature the reader is referred to [11]. In particular, our analytical result for bare target nuclei compares quite well with similar results obtained numerically by Mando and Ronchi [12], while our simple and approximate procedure for inclusion of screening produces losses which are close to those considered most accurate and reliable by Wright [11]. Our results also compare well to those displayed in [13]. As to the newer literature, the situation is (with the exception of [13]) quite different. The result for the ratio of pair production to bremsstrahlung losses given in [2] (and cited in [5]) is in general much too small, the error being caused by an overestimate of the bremsstrahlung losses (see below). In [3] relative energy losses \(E^{-1}dE/dx\) due to pair creation are claimed to grow in proportion to \(\gamma\) at high energies. In [4], which as [3] is based on the virtual photon method, a formula for the high-energy asymptote of the stopping is given; it has the same structure as ours but the absolute values are about a factor of 4 lower. A detailed discussion of the comparison with earlier works will be published elsewhere [10].

Measurements of the energy loss of lead ions of energy 160 GeV/u (\(\gamma = 168\)) in various solid targets are reported in [6]. The measured losses are compared to predictions for the ionization energy loss. In the main, good agreement with non-perturbative theory is obtained when the finite size of the projectile nucleus is accounted for. Yet small deviations are observed. The deviations increase with the target atomic number. It is convenient to express the ionization energy loss as

\[
\frac{dE}{dx} = \frac{4\pi Z^2 e^4}{m v^2} N Z_t L ,
\]

where \(v\) is the projectile speed (essentially equal to \(c\)). The dimensionless stopping number \(L\) is in general a complicated function of target properties, but at extreme energies the combined action of the Fermi density effect and the finite nuclear size causes \(L\) to approach the remarkably simple value [1]

\[
L \to \ln(1.62c/R_0) \equiv L_\infty , \quad \gamma \to \infty .
\]

Here \(R\) is the radius of the projectile nucleus and \(\omega_{pl} = \sqrt{4\pi N Z_e e^2/m}\) the plasma frequency of the target. For the cases considered in [6], the asymptotic values determined from (10) are only 0.2–0.1 lower than the exact values from ionization theory which are around 14.

The deviations from ionization theory found in the CERN experiment are displayed in Table 1. Also shown are our predictions for the energy loss due to pair production. When converted to a stopping number by dividing out the front factor appearing in Eq. (9) pair production scales linearly with \(Z_t\) (up to screening corrections). Clearly, pair production is of just the right magnitude to explain the deviations observed. For the heavier target materials the theory values are very close to the experimental numbers, for the lighter materials the experiment overshoots the rather small theory values (but remember, we are talking about corrections to the total energy loss of only 1–2 % for the two lightest targets).

Since the asymptotic result (10) applies well at energies where pair production becomes a competitive energy-loss channel and since corrections like (7) are negligible here it is simple to produce a formula for the energy beyond which pair creation produces a higher energy loss than target excitation and ionization. Equating the expressions (5) and (9), and substituting \(\Lambda_0^{\text{screen}}\) for \(\Lambda\) and \(L_\infty\) for \(L\), we get

\[
\gamma = \frac{4}{Z_t \alpha^2} \frac{L_\infty}{\Lambda_0^{\text{screen}}} .
\]

Since \(\Lambda_0^{\text{screen}}\) itself depends on \(\gamma\) the equation will have to be solved by iteration. A simpler expression is obtained by substituting the high-energy "full-screening" value \(10^2 \ln(183/Z_t^{(3)})\) for \(\Lambda_0^{\text{screen}}\). For Pb on Pb, a \(\gamma\)-value of 1733 results by iteration whereas use of the full-screening value yields 1513. For lighter target materials the difference is relatively smaller (since the absolute value for cross over increases as \(Z_t\) decreases).

### Table 1. Pair-production contribution to the energy loss of lead ions at \(\gamma = 168\) compared to the deviations from ionization theory measured at CERN. The table lists corrections to the stopping number \(L\).

<table>
<thead>
<tr>
<th>(Z_t)</th>
<th>PP theory* with screening†</th>
<th>experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.066</td>
<td>0.064</td>
</tr>
<tr>
<td>14</td>
<td>0.141</td>
<td>0.135</td>
</tr>
<tr>
<td>29</td>
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<tr>
<td>50</td>
<td>0.479</td>
<td>0.449</td>
</tr>
<tr>
<td>82</td>
<td>0.780</td>
<td>0.722</td>
</tr>
</tbody>
</table>

* corresponding to Eqs. (5–7) but including factor \(1 + 1/Z_t\)
† as previous column but with \(\Lambda_0\) replaced by \(\Lambda_0^{\text{screen}}\)

### 3. THE BREMSSTRAHLUNG CHANNEL

Emission of bremsstrahlung will contribute substantially to the moderation of a heavy projectile at high energies. Formulas for the corresponding energy-loss rate are given in [2] and [5]. However, if applied to the CERN experiment discussed above both lead for the heaviest target to bremsstrahlung losses far beyond the experimental
values listed in the last column of Table 1, the former [2] by more than an order of magnitude (in part due to a "full-screening" approximation) and the latter [5] by a factor of 4. The estimate in [2] is based on classical electrodynamics [14], the quantum formula of [5] is taken from [8] (electron impact) with inclusion of a factor of $Z^4$ and the substitution $m \rightarrow M$.

The primary cause for overestimation in [2] and [5] is inclusion of processes with too high momentum transfer $Q$: As explained in [14] the maximum transfer $Q_{\text{max}}$ in Coulomb collisions has a value of $Q_{\text{max}} \approx 2Mc$. This corresponds to distances of order the Compton wavelength of the projectile. However, for any heavy ion this length is orders of magnitude smaller than any nuclear radius. Effectively, bremsstrahlung losses due to coherent action of the constituents of both the projectile and the target nucleus are limited by a maximum momentum transfer of order $Q_{\text{max}} \approx \hbar/R$, where $R$ is the nuclear radius (or, rather, the sum of the nuclear radii).

Following the classical electrodynamics approach in [14] but substituting the value $\hbar/R$ for $Q_{\text{max}}$ we arrive at the following estimate for bremsstrahlung losses:

$$- \frac{dE_{\text{BS}}}{dx} = \frac{16}{3} Z^4 \alpha m c^2 \kappa N \gamma mc^2 \ln \left(1 + 2 \frac{\hbar/Mc}{R}\right).$$  \hspace{1cm} (12)

Atomic screening will only influence the emission of very soft quanta and is hence neglected. For the CERN experiment Eq. (12) leads to an energy loss for the lead target of only 3% of the observed deviation from ionization theory (last column in Table 1) and hence our estimate for this particular case is at least two orders of magnitude smaller than those of [2] and [5].

As the energy is increased, bremsstrahlung will eventually dominate the energy loss. Equating the expressions (5) and (12), and substituting $\Lambda_0^{\text{screen}}$ for $\Lambda$, we get the following expression for the $\gamma$-value beyond which bremsstrahlung is the major energy-loss mode:

$$\gamma = \frac{R}{\hbar/Mc} \left[ \exp \left( \frac{3\pi \Lambda_0^{\text{screen}}}{16} \frac{\alpha M}{Z^3 m} \right) - 1 \right].$$  \hspace{1cm} (13)

The cross-over happens at such high $\gamma$ that it is fair to substitute $\frac{\gamma}{\bar{\gamma}} \ln(183/Z^3_0)$ for $\Lambda_0^{\text{screen}}$. For Pb on Pb Eq. (13) then leads to $\gamma = 2.8 \times 10^4$ (compared to $2.7 \times 10^4$ if the full-screening approximation is avoided).

4. CONCLUSION

Let us conclude the discussion by showing a figure which displays all the energy-loss channels, Figure 1. The increasing importance of the radiative channels with increasing projectile energy is clearly visible. Each of the channels has an energy region where it dominates, start-

![Figure 1](image.png)

**FIGURE 1.** Relative energy-loss rate as a function of $\gamma \equiv E/Mc^2$ for bare lead ions in a lead target. The dotted curve shows the ionization contribution, the chained curve displays the pair-production channel (thin curve obtains by neglect of screening), the dashed curve is the bremsstrahlung loss, and the full-drawn curve shows the sum of all contributions.

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