Mechanisms of halo formation

Alexei V. Fedotov

Brookhaven National Laboratory, Upton, NY 11973

Abstract.
Uncontrolled beam loss leads to excessive radioactivation in high-intensity machines. At the same time, it strongly affects performance of high-energy accelerators and colliders. For the well controlled beam, the loss is typically associated with the low density halo surrounding beam core. There are many mechanisms which contribute to halo formation. Some of them are more important for linear accelerators while other are more relevant to circular machines. In order to minimize uncontrolled beam loss or improve performance of an accelerator, it is very important to understand what are the sources of halo formation, as well as which of them can have significant contribution for a specific machine of interest. In this paper, we overview various mechanisms of halo formation. We then specifically discuss which effects are expected to be dominant in linear accelerator and which effects dominate in rings, concentrating on high-intensity machines.

BEAM LOSS AND BEAM HALO

The latest designs of high-current accelerators set strict requirements on beam-induced radioactivation of the vacuum chamber. Guided by such restrictions, significant efforts were made to understand various mechanisms which can contribute to beam loss. Because losses at a very low level became of concern, more detailed understanding of beam tails (low density region outside the dense beam core) was necessary. Naturally, such a low density region surrounding the dense core was referred to as beam "halo". At the same time, similar effects were studied using standard concepts of emittance growth and beam tails.

On the academic side, there is a tendency to come up with a precise definition of beam halo. The attempts were made to distinguish "halo" from "tails" of beam distribution. Such definitions become problematic as soon as one wants to describe various mechanisms of halo formation which may contribute to a beam loss. In fact, as far as the radioactivation is concerned, it does not matter whether the loss comes from beam tails or halo. From practical point of view, beam halo can be regarded based on its application:

1) If the concern is beam loss, then beam halo is just some number of particles of any origin which lie in the low-density region of the beam distribution far away from the dense core. Of course, the behavior of such particles will be very different depending on their origin.

2) The next step is to understand the mechanisms of halo formation. The structure of beam halo and its characteristics are different depending on the mechanism. The parameters of the driving mechanism may be enhanced to provide a clear signatures of halo formation. Although, in such cases with enhanced parameters, "halo" may be at a relatively high level of beam distribution, it is an essential stage towards understanding of halo dynamics. At this point, the attention is devoted to studies of halo formation before the beam loss actually occurs, typically by an observation of emittance growth or beam profiles.

Not surprisingly, there are many mechanisms which contribute to halo formation:

in high-current linear accelerators: 1) various sources of machine nonlinearities and misalignments 2) rms mismatch 3) space-charge coupling resonances 4) space-charge induced structure resonances 5) Coulomb scattering within the beam and on the residual gas 6) collective instabilities, etc.
in high-current circular accelerators: 7) injection, foil stripping and extraction 8) rf noise 9) machine nonlinearities 10) rms mismatch 11) space-charge coupling resonances 12) space-charge induced structure resonances 13) imperfection lattice resonances 14) gas scattering 15) collective instabilities 16) e-cloud effects 17) numerous "project-specific" effects ranging from different painting schemes for multi-turn injection to a closed orbit oscillation at extraction, etc.

including short bunches: 18) transverse-longitudinal coupling 19) effects from synchrotron motion, etc.
including high-energy accelerators: 20) intrabeam scattering 21) instabilities relevant for high-energies, etc.

---

Work supported by the SNS through UT-Battelle, LLC, under contract DE-AC05-00OR22725 for the U.S. Department of Energy.
including colliders: 22) beam-beam driven halo 23) etc., etc., etc.
And, yes, one needs to consider both the transverse and longitudinal halo, especially for short bunches typical for high-intensity linacs.
Many mechanisms listed under "linear accelerators" were specifically repeated for rings to emphasize that some effects in high-current linear accelerators are also relevant for high-current rings. However, due to a significant difference in the regimes of the tune depression, the time scale (growth rates) of the space-charge driven effects becomes very different. In this paper, we limit our discussion to generic mechanisms relevant for both linacs and rings. In addition, we briefly address some ring-specific and project-specific mechanisms, using an example of the SNS ring.

BASIC MECHANISMS COMMON FOR LINACS AND RINGS

Intrinsic incoherent resonances

The oscillating space-charge force can lead to a class of resonances where individual particles inside the beam can get into a resonance with an oscillating beam mode. These resonances are referred to as intrinsic or incoherent resonances of the individual particles. Such a parametric resonance mechanism was suggested as one of the dominant effects for halo generation in linacs [1]. Some literature on this subject can be found, for example, in Ref. [2]. In recent years, existence of self-consistent three-dimensional computer codes allowed systematic study of this mechanism for realistic beam distributions [3],[4],[5]. Such resonances between the motion of the individual particles and collective beam oscillations are governed by the rms beam mismatch. For example, in the case of a uniform density beam envelope, which is oscillating with the frequency \( \Omega \) and the mismatch parameter \( \mu \), the equation of motion for the individual particle has the form

\[
x'' + q^2 x = \mu \frac{2\kappa}{a_0^2} x \cos \theta,
\]

where \( \kappa \) is the space-charge parameter, and \( q^2 = k^2 - \kappa/\alpha_0^2 \) is the depressed incoherent frequency without the approximation of small space charge. This equation has a primary parametric resonance at \( q = \Omega/2 \). When higher order terms are included, one also gets the non-linear parametric resonances [6]. The halo extent associated with the 1 : 2 parametric resonance (limited due to non-linearity omitted from Eq. [ 1]) was extensively stud-
ied for the envelope modes in high-current linacs. The halo extent associated with this resonance can be large not only for a strong tune depression of the order of \( \eta \sim 0.5 \) (typical for linacs) but also for the tune depression of only a few percent \( \eta \sim 0.97 \) (typical for rings). Examples of halo extent for the 6D stationary distributions as a function of the mismatch parameter are shown in Figs. 2 and 3 for the transverse and longitudinal halo, respectively. For realistic distributions, the halo extent is slightly higher due to an additional redistribution [3]. Such a finite extent is a feature of bounded motion due to the nonlinearity for a single isolated resonance (simplified assumption). In real accelerator, with magnet misalignments, rf noise, interplay of various resonances, etc., one can expect to have some sort of diffusion process which may have impact on the halo extent if the rate is sufficiently high. Also, the extent of parametric halo can be enhanced due to a combination with other effects. The realistic prediction thus rely on computer simulations which can predict emittance growth/halo in the "parametric halo", did not require any resonances with the lattice since the collective oscillation was induced by a mismatch. In rings, however, such collective beam modes may have a fast excitation as a result of both the space-charge and machine resonances. The resonances of the individual particles with such driven collective beam modes can be called "driven incoherent resonances", since the collective modes are first resonantly driven and then incoherent particles are trapped into the resonance with the corresponding collective mode. In such a situation, the incoherent resonances may play an important role in halo formation even in the limit of a weak tune depression [9].

linear accelerators: Space-charge tune depression is typically strong. As a result, the rate of such space-charge driven halo is very fast. This is an important mechanism - both the transverse and longitudinal mismatches should be minimized. Since bunches are typically rather short, it is necessary to consider the effect of the longitudinal-transverse coupling [3].

circular accelerators: Tune depressions are relatively weak (apart from "cooler rings" or specific small scale rings for space-charge studies). In addition, there are such effects as multi-turn injection (phase-mixing), redistribution, etc. - in most practical situation such a halo has little chance to develop. However, the "driven halo" is possible [9].

Space-charge coupling resonances

Another type of a mechanism common for both high-intensity linacs and rings is the space-charge coupling resonances, which are driven by the space-charge potential itself rather than the field potential of magnets [9].

linear accelerators: Both the symmetric and anti-symmetric resonances can be excited due to a possibility of a very different focusing constants - this results in the "equipartitioning charts", which suggest to avoid opera-
tation near such resonances [5].

**Circular accelerators**: Resonances with the zeroth harmonic become important for beams with unequal emittances. In addition, there is now a possibility of asymmetric resonances with the error harmonic.

### Space-charge structure resonances

Collective modes of beam oscillation resonating with the lattice structure generate substantial emittance growth [10]-[12]. Thus, operation in such a regime should be avoided.

**Linear accelerators**: Since focusing gradients ("tunes") are not limited by imperfection resonances, the intensity can be increased until one hits the structure stopbands. To avoid the largest second-order stopband, the recommendation is typically made to design a zero-current phase-advance below the $90^0$.

**Circular accelerators**: If one can compensate emittance growth associated with the crossing of the imperfection resonances then one gets similar limitations. Otherwise, intensity is typically limited by emittance growth due to the machine imperfection resonances.

### Intrabeam Scattering

**Linear accelerators**: Since time from time various studies suggest importance of this effect for the high-intensity linacs, a rigorous treatment was needed. From the kinematics of Coulomb collisions one can find the extent of halo around the beam core including the space-charge effect. However, the probability of particle to occupy such a shell in linacs is too small. To confirm this rigorously, some simple scaling formulas were derived [13]. For the class of the 6D stationary self-consistent distribution, given by:

$$ f(r, v) = \begin{cases} N(H_0 - H)^n & , H < H_0 \\ 0 & , H > H_0 \end{cases} $$

(2)

where the Hamiltonian includes the space-charge potential and is given by

$$ H(r, v) = m v^2 / 2 + k r^2 / 2 + e \Phi_{\text{cc}}(r), $$

(3)

one finds that the fraction of ions which leave the beam (and form a halo around the bunch) per unit length is

$$ \frac{dP}{cdt} \sim \begin{cases} r_p^2 / \varepsilon_N^3 & , n > 0 \\ (r_p^2 / \varepsilon_N^3) \ln(e_p^2 / r_p a) & , n = 0 \\ (r_p^2 / \varepsilon_N^3) (e_p^2 / r_p a)^n & , -1 < n < 0 \end{cases} $$

(4)

where, for singular distributions with $0 \geq n > -1$, we assume that the Coulomb force between ions is screened at the Debye length $\lambda_D$. Taking, for example, $r_p = 1.5 \times 10^{-13}$ [m], $\varepsilon_N \approx 10^{-6}$ [mm rad] and $a \approx 10^{-5}$ [m], we then have

$$ \frac{dP}{cdt} \sim \begin{cases} 10^{-15} / \text{km} & , n > 0 \\ 10^{-14} / \text{km} & , n = 0 \\ 10^{-11} / \text{km} & , n = -5 \\ 10^{-8} / \text{km} & , n = -9 \end{cases} $$

(5)

which is clearly negligible for linear accelerators.

**Circular accelerators**: In storage rings, the intrabeam scattering is typically one of the dominant mechanisms of emittance growth and is always considered.

### Collective instabilities

Both in linear and circular accelerators collective instabilities lead to halo formation and thus should be carefully avoided. An example of halo growth due to a collective dipole instability, driven by the transverse impedance, is shown in Figs. 4 and 5, for the evolution of the unstable harmonic of the dipole oscillation and associated beam halo, respectively [14].

### Misalignments, magnet errors and noise

Such effects should be considered in both linacs and rings to estimate realistic emittance growth. By itself, very small misalignments, magnet fields errors or magnet noise may not lead to a significant growth of halo but they can enhance other halo mechanisms. For example, the enhancement of parametric halo (due to a mismatch)
by misalignments was shown in Ref. [15]. Halo generation due to RF noise (see, for example, [16]) or due to a magnet noise [17], was also recently addressed. Understanding of emittance growth, as a result of many effects combined together, requires realistic computer simulations [18].

RING SPECIFIC MECHANISMS

Some ring specific mechanisms are related to the fact that characteristic times in rings are much longer than in linacs. Also, there is a possibility to accumulate effects over successive turns. One of the most important mechanisms of halo growth is due to the crossing of magnet imperfection resonances in the tune-space. The high-intensity aspects of halo growth due to various types of resonance was reviewed in Ref. [9]. Most of halo growth can be avoided by choosing the working point appropriately, and by applying the corresponding resonance correction schemes [19]. An example of halo growth due to several nonlinear resonances and its compensation is shown in Fig. 6.

PROJECT SPECIFIC MECHANISMS

In addition to basic mechanisms described above, each individual project has many other contributions to halo generation. For example, for the SNS ring, the following mechanisms gave significant contribution to beam halo:

Halo growth due to painting bump function

Beam tail distributions of three different bumps for correlated painting are shown in Fig. 7, where we plot the percentage of particles outside a given emittance (in $\pi \text{mm mrad}$). The bump which collapses as a square-root function (red color) performs better than the other two bumps, which decay exponentially with different time constants $\tau (\tau = 0.6: \text{pink color}, \tau = 0.3: \text{blue color})$ [8].

Space-charge redistribution of painted beam

In the case of correlated painting, the beam is painted to a square shape; this results in a high density distribution along the diagonals. The inclusion of space charge leads to rapid azimuthal diffusion which was estimated analytically and confirmed numerically [20]. The radial diffusion was also explored numerically.

"Banana-shape" effect

The extraction kickers of the SNS are not centered with respect to the zero closed orbit. They are offset from the center in order to save on mechanical dimensions when the full-size accumulated beam is extracted. As a result, the longitudinal beam centroid experiences a kick different from the head and tail of the beam, due to the longitudinal current distribution along the bunch. This results in a "banana-shape" distortion along the bunch or oscillation of beam centroid [21]. Figure 8 shows beam halo due to this effect for the case of an old impedance from the extraction kickers, which is a factor of two larger than the present impedance budget.
Effect of wall image charges

Increasing the $b/a$ ratio (where $b$ is the beam pipe radius and $a$ is the final radius of the painted beam) decreases the threshold of the instability caused by the extraction kicker impedance, which leads to strong halo. On the other hand, when the ratio $b/a$ becomes too small the damping of instability becomes more effective but one can get significant halo due to the image effects of a quadrupole mode [14].

Coherent resonances

The halo due to the crossing of the second-order and high-order coherent resonance was systematically studied. The associated space-charge limit was discussed in Refs. [22]-[23].

SUMMARY

In summary, we give some general comments: 1) it is important to keep in mind that there are many mechanisms which contribute to halo formation 2) when several mechanisms are present simultaneously, one gets a complex behavior understanding of which requires realistic computer simulations 3) physics of some mechanisms may be the same in linacs and rings but applications of these mechanisms can be very different (for example, due to different regimes of the tune depression, which provides different growth rate for the space-charge driven effects) 4) some generic mechanisms should not be taken as granted and should be always double checked for the parameters of interest 5) there can be some "machine-specific" dominant mechanisms.

ACKNOWLEDGMENTS

I would like to thank many collaborators with whom I worked on the subject of beam halo in linear and circular accelerators, as well as the SNS Accelerator Physics group for numerous useful discussions on this subject: R.L. Gluckstern, T. Wangler, R. Ryne, S. Kurennoy, N. Pichoff, I. Hofmann, J. Holmes, S. Cousineau, S. Danilov, Y.Y. Lee, J. Wei, D. Raparia, M. Blaskiewicz, G. Parzen, N. Malitsky and many others.

REFERENCES