Abstract
In reservoir characterization, the value of each piece of data does not lie in its isolated use, but rather in the value it adds to the analysis when integrated with other data. Earth scientists from different disciplines have made many efforts to better predict the spatial distribution of petrophysical properties. However, the interpretation of isolated piece of knowledge, without a real integration of disciplines, cannot lead to a quantitative answer in terms of reservoir characterization. In this paper we present a joint inversion scheme for estimating petrophysical properties by integrating both production and 4D seismic related data in geological modeling. Simultaneous history matching of production and 4D seismic related data leads to a better prediction of petrophysical reservoir properties and of production forecast.

Introduction
This paper describes an integrated methodology, based on a non-linear optimization loop, for a quantitative use of 4D seismic data for improved geological modeling and reservoir characterization.

The proposed methodology is made in two sequential phases (Figure 1):

- Phase 1 - Pre-stack seismic data inversion: For each seismic survey, the available angle sub-stacks are jointly inverted using a 3D stratigraphic inversion methodology. This methodology allows the direct estimation of an optimal elastic model in terms of compressional and shear impedances (Ip and Is). These impedances carry information related to static (porosity and permeability) and dynamic (pressure and saturation) properties of the reservoir.
Phase 2 - Production and pre-stack 4D seismic data history matching: The optimal impedance data derived from pre-stack seismic are used, in addition to production data, to constrain and update the geological model. Synthetic production data and 4D compressional/shear impedance data are computed from a multi-phase fluid flow simulator coupled with rock physics model. The reliability of the geological model is improved through the minimization of a weighted, least-squares objective function which measures the mismatch between the forecast of the 4D workflow and the data (production and 4D seismic).

We applied successfully the methodology for a multi-scale reservoir characterization process using a three-dimensional synthetic case including 12 years production data and 3 seismic surveys.

Pre-stack Seismic Data Inversion
Reservoir geophysicists commonly use inversion to obtain compressional and shear impedances from pre-stack seismic amplitude data. Seismic amplitude is appropriate for structural interpretation because it reflects the presence of an interface between two layers of different elastic properties, but also carries information on elastic properties themselves (impedances) that can be extracted from seismic amplitudes by means of stratigraphic inversion. Most algorithms and software solutions rely on the acoustic approximation of the elastic wave equation, which allows to model seismic amplitudes with the convolution model \(^1\). This approximation which is based on Zoeppritz equations, allows the direct estimation of an optimal elastic model in compressional and shear impedances, under the constraint of geological and petrophysical a priori information. In many cases, the a priori model is built from well data only, but can be improved by the addition of velocity data. Pre-stack seismic data inversion allows the direct estimation of an optimal elastic model in terms of compressional and shear impedances. These impedances are used to constrain and update the geological model.

Production and Pre-stack 4D Seismic Data History Matching
The advent of four-dimensional seismic technology brings an additional challenge to industry, in terms of simultaneous integration of production and seismic related data \(^2\)-\(^6\). The purpose of this work is to enhance the characterization of subsurface reservoirs by combining geological, geophysical and reservoir engineering data. This goal is fulfilled by developing a 4D workflow (Figure 2), integrating 4D seismic and production related data, through the application of the inversion framework. The main difficulty is to account for different scales in the proposed 4D workflow: the geological scale, the fluid flow simulation scale and the seismic scale \(^6\). The proposed methodology includes:

- Perturbation of the geostatistical model using the gradual deformation parameterization \(^7\).
  The geostatistical model includes the facies distribution as well as the petrophysical property distributions (porosity and permeability).
- Computation of the flow simulation model by upscaling the fine scale geostatistical model.
- Simulation of the fluid flow model to generate synthetic production results as well as synthetic saturation and pressure distributions.
- Generation of fine scale saturation / pressure distributions by downscaling fluid flow simulation results.
- Simulation of the petro-elastic model to generate synthetic compressional / shear impedances.
- Filtering the synthetic impedances in the bandwidth of the seismic data.
- Upscaling to the seismic data scale.
- Computation of the objective function.
- Updating of the fine scale geostatistical model using classical optimization process.
Geostatistical model perturbation

Most of the success in data integration for reservoir characterization purposes has been obtained with the application of geostatistical techniques. The parameterization of the geostatistical model is fundamental to honor geological information. This integration is carried out within the framework of an iterative scheme, based on an optimization process. It aims at perturbing an initial porosity / permeability field representative of the considered geostatistical model. Ideally, the final porosity / permeability field must honor not only the dynamic data (production and 4D seismic data), but also the geostatistical coherence of the model (average, variogram, etc). The match of the dynamic data is controlled by the objective function that measures the difference between simulated and observed data. The gradual deformation method ensures the preservation of the geostatistical model properties. The gradual deformation parameterization consists in writing a new realization of the porosity and permeability fields, assumed to be of Gaussian type, as a linear combination of independent realizations of a multi-Gaussian random function. The porosity and permeability fields are thus given by:

\[ \phi(\lambda) = \sum_{i=1}^{n} \lambda_i \phi_i \quad \text{and} \quad k(\theta) = \sum_{i=1}^{n} \theta_i k_i \]

- \( \lambda_i, \theta_i \): the coefficient values of the linear combination (deformation parameters);
- \( \phi_i, k_i \): the independent realizations of the random function (porosity and permeability).

In order to preserve the geostatistical properties of the model, the coefficients \( \lambda \) and \( \theta \) must satisfy the following normality constraint:

\[ \sum_{i=1}^{n} \lambda_i^2 = 1 \quad \text{and} \quad \sum_{i=1}^{n} \theta_i^2 = 1 \]

The coefficients \( \lambda \) and \( \theta \) are estimated so that the resulting porosity field \( \phi(\lambda) \) and permeability field \( k(\theta) \) reproduce the dynamic data as well as possible. If we dispose of a \( \phi - k \) correlation law describing the dependency of the permeability compared to the porosity, we just apply the gradual deformation method on the porosity field and then update the permeability field using this correlation law.

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Upscaling
An important issue regarding the role of flow simulation in seismic reservoir monitoring is the simulator grid size. It is always desirable to have the flow simulation grid as fine as possible to capture the details of reservoir heterogeneity and the spatial saturation distribution. For practical considerations, there is always a compromise to find between the desire to capture the details and the availability of computational resources. Consequently, flow simulation results are always spatial averages over large blocks (upscaling). The equivalent permeability calculation (upscaling) of heterogeneous porous media is a subject largely studied by the geologist community, reservoir engineers and more generally physicists of the porous media. The state of the art of the existing techniques is addressed by Wen et al. [8]. The upscaling used here is a power law method, which allows choosing arithmetic, geometric or harmonic average.

Fluid Flow Simulation
The fluid flow simulation is required to constrain the geological model to production data measured at wells (bottom hole flowing pressure, gas-oil ratio, water cut, etc...). For seismic reservoir monitoring, the saturation / pressure patterns are also required, as output from the fluid flow simulation, in order to compute compressional and shear impedances by using Gassmann’s equation.

Downscaling
The objective of the downscaling is the mapping of physical properties such as pressures and saturations from a coarse grid (fluid flow simulation scale) to a fine grid (geological model scale). The proposed downscaling (Figure 3) method is based on 3D interpolation tools. More complex methods for downscaling can be used but their use leads to an increased CPU time.

Petro-Elastic Modelling
Converting fluid changes to seismic changes always requires the estimation of fluid changes at scales smaller than the scale of the fluid flow simulation. In our approach we propose to perform the petro-elastic simulation at the geological model scale. This requires downscaling of the pressure and saturation distributions computed by the fluid flow simulator. The saturation / pressure patterns obtained from the downscaling module are mapped onto compressional and shear impedances at each fine geostatistical cell. There are a number of quantitative relationships published in the open technical literature to link elastic properties of rocks with their pore space, pore fluid, fluid saturation, pore pressure, and rock composition. Many of these relationships are based on empirical correlation. Other relationships derive from effective medium theory and hence are subject to different types of operating assumptions (Gassmann's equations).
Compressional Impedance: The compressional impedance is defined as the product of the bulk density $\rho_B$ and the compressional velocity $V_p$:

$$I_p = \rho_B V_p$$

First, we need to compute the bulk density using the following formula:

$$\rho_B = (1 - \phi) \rho_{Gr} + \rho_F$$

- $\rho_{Gr}$: the grain density, which is defined for each facies;
- $\rho_F$: the fluid density;
- $\phi$: the porosity: one constant per facies or one value per pixel (3D cube).

The second step is the computation of the compressional velocity $V_p$:

$$V_p = \sqrt{\frac{3K + 4\mu}{3\rho_B}}$$

- $K$: the bulk modulus;
- $\mu$: the shear modulus.

Gassmann's equations allows computing the bulk modulus $K$:

$$K = K_M + \frac{\left[1 - \frac{K_M}{K_{Gr}}\right]}{\frac{\phi}{K_F} + \frac{1 - \phi}{K_{Gr}} - \frac{K_M}{K_{Gr}}} \left(\frac{K_{Gr}}{K_F} + \frac{K_{Gr}}{K_F} + \frac{K_M}{K_{Gr}}\right)$$

- $K_{Gr}$: the bulk modulus of grain;
- $K_F$: the bulk modulus of fluid;
- $K_M$: the bulk modulus of the porous frame.

The Hertz model allows modeling pressure variation effects on $V_p$:

$$V_p(P_e - P_p) = V_p(\Delta P_a)^{h_p}$$

- $P_e$: the confining pressure;
- $P_p$: the pore pressure;
- $\Delta P_a$: the asymptotic differential pressure;
- $h_p$: the Hertz coefficient for compressional waves.

Shear Impedance

$$I_s = \rho_B V_s$$

with $V_s = \sqrt{\frac{\mu}{\rho_B}}$

where $\mu$ corresponds to the shear modulus of porous frame. Furthermore, the impact of pressure variation on shear velocity can be modeled using Hertz model:

$$V_s(P_e - P_p) = V_s(\Delta P_a)^{h_s}$$

where $h_s$ is the Hertz coefficient for shear waves.
Filtering
In order to get the synthetic impedances in the bandwidth of the seismic data, we need to filter compressional and shear impedances obtained by the seismic module.

Upscaling / Downscaling
In order to be able to compare the computed impedances with the synthetic data, we must perform an upscaling or a downscaling to map the obtained data from the geological scale to the seismic data scale. Generally, we use a vertical or horizontal upscaling.

Objective Function: Least-Squares Criterion
The update of a geological model by the dynamic data depends on the minimization of an objective function that measures the difference between the observed data (production and 4D seismic related data) and the obtained simulation results for a fixed value of the parameters $\theta$.

Several formulations are possible to define an objective function. The least-squares formulation is most often used in the oil industry. In this case the objective function is written as follows:

$$J(\theta) = \frac{1}{2} \left( P^{obs} - P(\theta) \right)^T C_{P}^{-1} \left( P^{obs} - P(\theta) \right) + \frac{1}{2} \left( S^{obs} - S(\theta) \right)^T C_{S}^{-1} \left( S^{obs} - S(\theta) \right)$$

- $P^{obs}$: the production data,
- $P(\theta)$: the simulated production result for the value $\theta$,
- $C_{P}$: the covariance matrix on the production data,
- $S^{obs}$: the 4D seismic related data,
- $S(\theta)$: the simulated seismic result for the value $\theta$,
- $C_{S}$: the covariance matrix on the seismic data.

The minimization of the objective function requires the calculation of the derivatives of the simulation results with respect to the parameters to estimate:

$$\frac{\partial P}{\partial \theta}(\theta) \text{ and } \frac{\partial S}{\partial \theta}(\theta)$$

In our work, the finite difference approximation is used for the calculation of the derivative of the simulation results with respect to the parameterization of the fine scale geostatistical model. This choice allows a quick application of our 4D workflow without requiring any additional programming in the fluid flow simulator or the petro-elastic modeling.

Validation on Synthetic Test Case

PUNQ-MONITOR case description
For our validation test, we are working with the PUNQ-MONITOR case, widely inspired from PUNQ-S3, a simplified case with a small-size reservoir model. The PUNQ-S3 model is a standard test case defined in the PUNQ project (Production forecasting with UNcertainty Quantification) for comparative studies of inversion method [9].

Geological modeling
The geological model of the PUNQ-MONITOR case is made by five geological units:

- Units 1, 3 and 5 are of high reservoir quality;
- Unit 4 is of intermediate reservoir quality;
- Unit 2 is of low reservoir quality.

A geostatistical model based on Gaussian random realizations is used to generate the porosity and permeability fields. The structural parameters of the model are defined in Table 1.
# Geostatistical simulation

The geostatistical simulation is made unit by unit:
- Simulation grid size per unit: $171 \times 252 \times 6$;
- Simulation block size: $20 \text{ m} \times 20 \text{ m} \times 1 \text{ m}$.

The truth porosity and permeability fields for the five units are shown in the Figure 4.

![Reference porosity, log(horizontal permeability) and log(vertical permeability) fields.](image)

## Fluid flow simulation

The geostatistical simulation model is composed of $171 \times 252 \times 30$ grids corresponding to $1.292.760$ grid blocks with a block size of $20 \text{ m} \times 20 \text{ m} \times 1 \text{ m}$. The grid properties for the fluid flow simulation are:
- Simulation grid size: $19 \times 28 \times 5$;
- Simulation block size: $180 \text{ m} \times 180 \text{ m} \times 6 \text{ m}$.

The aggregation, whose rate is $9 \times 9 \times 6$, is performed by using the arithmetic average. The obtained results are shown on Figure 5.

![Porosity and permeability upscaling.](image)

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During the production phase (1967-1978), six production wells have been perforated on units 1-3-5; during the injection phase (1967-1978), six water injection wells have been perforated on units 3-4-5 and one gas injection well has been perforated on unit 5 (Figure 6).

![Figure 6: The Punq-Monitor reservoir - Production data.](image)

During the production history (12 years), we dispose of the surface oil rate, the gas-oil ratio and the water cut for the six production wells (Figure 6).

**Petro-elastic simulation**

A realistic petro-elastic model is used to generate compressional and shear impedances. For the PUNQ-MONITOR case, an unique PEM is defined for the 5 units:

- Grain density: 2.65 g/cm³;
- Bulk modulus of grain ($K_G$): 37 GPa;
- Shear modulus ($\mu$): 45 GPa;
- Bulk modulus of porous frame $K_M$: experimental;

So we need to downscale the flow simulation results (pressure and saturation) from the fluid flow simulation grid (180 m x 180 m x 6 m) to the geostatistical simulation grid (20 m x 20 m x 1 m). Three surveys (base at T0 and monitor at T1 and T2) of compressional and shear impedances for the PUNQ-MONITOR data set are computed using the fine grid data of pressures and saturations after downscaling. Furthermore, we have chosen to filter the obtained data set using the frequencies: 0 Hz - 0 Hz - 100 Hz - 120 Hz corresponding to a low pass filter. To mimic real data set, a third scale is introduced in addition to the geostatistical and fluid flow scales, the seismic scale:

- Seismic grid size: 57 x 84 x 5;
- Seismic grid block size: 60 m x 60 m x 6 m.

Then we must upscale the compressional and shear data set to the seismic data scale: from (20 m x 20 m x 1 m) to (60 m x 60 m x 6 m).
The resulting 4D seismic data can be observed in Figure 7. The main variations in compressional and shear impedances are related to the secondary gas cap expansion due to reservoir depletion.

![Figure 7: 4D seismic data.](image)

**Data Integration**

The proposed 4D history-matching loop is applied to update an initial realization of the geostatistical model with both production and 4D seismic data.

The gradual deformation parameterization was used to update the initial porosity field (Figure 8). A porosity / permeability correlation law (deduced from well data) is used to compute the permeability fields. The correlation laws (one law per unit) are also optimized within the 4D loop. The updated porosity field is represented in the Figure 8. During the 4D history matching process, the objective function was reduced on both data type (production and 4D seismic).

The initial and final (after history matching) objective function terms corresponding to the base compressional impedance for the Unit 1 of the reservoir are illustrated on Figure 9. This objective function behavior (illustrated only for the compressional impedance of Unit 1) is also obtained for the other units and for both compressional and shear impedances.

![Figure 8: Initial vs. updated porosity field.](image)
The match of production data is also achieved at the end of the history matching process. A typical production data matching is given on Figure 10. Same quality matches have been obtained for the other wells (PRO-1, PRO-2, PRO-3, PRO-4, PRO-5, PRO-6) and for the different production data type (surface oil rate, gas oil ratio, water cut).

The match of 4D seismic data is illustrated on Figure 11-Figure 13.
Conclusions
The 4D workflow used in this work takes advantage of a powerful data integration scheme that involves both dynamic and static information, and delivers estimates that:

- Preserve the coherency of the geological model properties;
- Guarantee the match of production and 4D seismic related measurements;
- Benefit from models supported by the physics of fluid flow and acoustic wave propagation.
The proposed approach highlights the applicability of 4D seismic data in geological modeling and reservoir characterization. Interdisciplinary integration of seismic measurements and rock physics with multiphase fluid flow helps to reduce uncertainties on petrophysical property distributions, and as a result, reduces also uncertainties on interpreting seismic attributes for an optimal reservoir management.

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