Abstract

The representation of joint (opening –mode fractures) and fracture swarm development is classically presented by a first order phenomenon involving effective traction orthogonal to fracture plane (regional extension). Although commonly used, this hypothesis seems to us less realistic in many circumstances and may conflict with geological observations. Therefore, we describe fracture growth as a second order phenomena resulting from crack parallel compression.

One example of systematic joint characterisation

Several characteristics of natural fracture networks play a key role in reservoir performance: Trends about major fracture sets orientation, typical joint spacing when periodicity appears, length of fractures when discontinuous, scaling factors when size and spacing are of different magnitudes from one net to the other, persistence of fractures over stratigraphic interfaces is also a major concern. This paper presents physical models that can help to understand or predict these characteristics.
Stress criterion, energy criterion

One could only but be struck by the regularity of parallel systematic jointing and the first question to answer is the rule determining spacing. Rock properties, ability to fracture (toughness) and to deform and flow together with bedding and other geometrical influences play their role too.

It is commonly assumed that failure of a mechanical system is controlled by two principal conflicting modes; one is often assigned to perfect plasticity with flow limit \( \sigma_y \); the other is linked to creation of surfaces with surface energy \( GIC \) (or \( R \)).

Balance between these two modes leads to the definition of a scaling length \( l_0 \) (Irwin) which represents the size for which a transition operates from one mechanism to the other.

\[
l_0 = \frac{RE}{\sigma_y^2} = \left( \frac{K_{IC}}{\sigma_y} \right)^2 \tag{1}
\]

This size requires an additional factor \( \alpha \) in order to represent the precise ability for the system to fail by plasticity or fracture; \( \alpha \) is weighing both geometry and loading \( \lambda \).

Applying to systematic jointing:

According to Kemeny et Cook [8] a model has been developed in the context presented above in order to describe observable parameters such as mean spacing \( w \), crack length \( a \) and bed thickness \( h \).

Geometrical ratios such as \( a/w \) and their variation with overburden and non dimensional toughness \( \kappa \) (see definition below) are also of interest. Wider joint spacing is predicted for joints formed at deeper depths (hydrostatic pressure dominant over bending, loading parameter \( \lambda \)). Non dimensional toughness is linking internal length parameter \( l_0 \) (rock properties) to geometrical scaling parameter \( h \) according to:

\[
l_0 = \left( \frac{K_{IC}}{\sigma_y} \right)^2 = h \kappa^2 \tag{2}
\]

Results have been presented using aspect ratios:

\[
\frac{a}{h} = \frac{4}{3} g_{\kappa}(\lambda) \tag{3a}
\]

\[
\frac{w}{a} = \left( \frac{9}{2} \right)^2 g_{\kappa}(\lambda) \tag{3b}
\]

\[
\frac{w}{h} = (3\kappa)^3 g_{\kappa}(\lambda) \tag{3c}
\]

A fracture swarm model

A brief presentation of the model developed in [3] and [19] is made in the following. It allows some features of the complex phenomenon of failure in compression (Kendall, [9]) to be illustrated. One typical set of results is presented fig. 3.

The first one is a figurative view of the idealised fracture swarm; the axis of symmetry is on the left side; a continuously evolving (along horizontal axis) confinement parameter \( s \) allows to describe the main features of fractures. The important facts are:
• when considering fractures close to the assumed free surface (confining parameter $s$ equals zero) spacing $w$ reaches a well defined limit depending on fracture properties (150 mm for the presented case) and aspect ratio $a/w$ remains unspecified: any ratio is possible and long fractures are statistically the most likely.

• the dual proposition concerns the fracture swarm's margin, where $a/w$ ratio is reaching a specified limit whereas spacing $w$ remains unspecified (no bedding in this model as a scale parameter).

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**Figure 3: Fracture swarm model after [3] and [19]**
Fracture pattern from stratigraphic information: joint propagation across layer interfaces

One example of the powerful theory of singularities matched to asymptotic expansions is presented.

The initial geometry of the geological unit (fig. 4 and 5) involves 'sandstone' bedding bounded by stratigraphic interfaces of a generally weaker material called "shale layer". We imagine a possible sequential fracturing process of propagation across the layer interface: we suggest a communicative mechanism from one jointed sandstone layer to the adjacent one in spite of inhibition by a thin interposed shale layer. Corresponding model consists of one cracked sandstone layer possibly initiating a new joint across the shale interface.

*Figure 4: step-over mechanism, after Hegelson et Aydin (1991), [5]*

Observation shows how vertical joint may appear as continuous in sandstone whereas discontinuous across shale. Two possibilities arise:

- in spite of discontinuities, fractures are well aligned in a vertical sense (straight through propagation mode)
- fractures are out of plane with each other, giving the overall vertical joint trace a discontinuous character (step over, fig. 4).

The third possibility is that not enough energy is allowable for initiation of any mechanism. Fracture arrest - "termination", "abutment" - then occurs (fig 5).

The effect of loading and geometry has been discussed and summarised in maps of mechanisms in the plane: horizontal effective stress versus overburden effective stress (fig. 6).

An important result is that straight through propagation is relevant for stiff shale layers and small overburden while step-over is characteristic of soft shale layers and deep configurations.

*Figure 5: The three types of fracture behaviour*
Geometrical effects such as $a/e$ aspect ratios, contrast of Young modulus and failure characteristics have been reviewed. When both fracture mechanisms are candidates, the less energy demanding one is favoured (Lawn principle).

The solution is obtained by matching parameters relative to far field and those relative to the near field. Taking simultaneously into account the relatively thin shale layer (ratio $e/a$) and crack tip singularity at the interface is not easy with finite elements. It is valuable to use mathematical techniques close to homogenization with fine scale for local perturbations and a global one for general trends. The coupling is obtained in the technique of matched asymptotics.

The far field

Away from intercalation, we approximate the displacement field by an "external" elastic description where only the parent crack in an homogeneous material is visible. Close to the crack tip, when considering a two-component loading - one is the horizontal effective stress $\sigma_t$ correlated with the scalar $K_I$, expressing the intensity of the singular contribution, the other is the structural component induced by the vertical part $T$ of the stress tensor. The "outer" expansion is expressed as (known as Williams's expansion, 1956):

$$U_0^0(x_1, x_2) = U_0^0(r, \theta) = U(0,0) + K_I \sqrt{r} u(r, \theta) + TrT(\theta) + ... \quad (4)$$

The near field

The previous expansion gives an accurate approximation far from the perturbation formed by the shale intercalation. Let us consider now the stretched dimensionless variables $y_1$ and $y_2$. The interbedding thickness is now 1, whatever its actual size. The dimensionless relative crack extension length is defined by $\mu = l/e$.

We postulate the existence of functions $V_i(y_1, y_2)$ defined in an unbounded domain, such that:

$$U^e(x_1, x_2) = U^e(ey_1, ey_2) = F_0(e)V_0^0(y_1, y_2) + F_1(e)V_1^1(y_1, y_2) + F_2(e)V_2^2(y_1, y_2) + ... \quad (5a)$$

with:

$$\lim_{e \to 0} \frac{F_{i+1}}{F_i} = 0 \quad \text{when} \quad e \to 0 \quad \text{...(5b)}$$

Real problem

The near field away from the intercalation must match the far field approaching the intercalation. These conditions yield for unperturbed configuration ($\mu=0$).

$$U_0^0(x_1, x_2) = U(0,0) + K_I \sqrt{e} V_1^1(y_1, y_2) + TeV_2^2(y_1, y_2) + ... \quad (6)$$

Stress field writes:

$$\sigma^e(x_1, x_2) = \frac{K_I}{\sqrt{e}} \sigma_{\infty} V_1^1(y_1, y_2) + T \sigma_{\infty} V_2^2(y_1, y_2) + ... \quad (7)$$

Displacement fields $V_1$ and $V_2$ and associated stress fields are calculated once for ever, for a given contrast $E_2/E_1$ between intercalation and bedding. Expression (7) explains why the actual stress field results as a simple weighed combination involving components ($K_I$ and $T$) and geometry $e$.

$V_1$ is the field linked to horizontal "singular" loading (stress component $\sigma_t$).

$V_2$ is the field linked to vertical "structural" loading ($T<0$).

Nucleation mechanisms and energy release rate
Field information \( V_1 \) and \( V_2 \) is extended towards perturbed configuration with a short initiated crack (\( \mu > 0 \)) beyond the intercalation (step over) or through the intercalation (straight through penetration).

The determination of the effective perturbed configuration chosen by the process (\( \mu_c \)) requires two failure criteria [14]. One is linked with bulk properties and is rather material science specific. It involves a strength criterion which writes:

\[
\sigma \geq \sigma_c
\]

The other is an energy criterion which refers to crack propagation and is rather mechanical science relevant. It writes:

\[
G \geq G_c
\]

\( G_c \) is the toughness of the material and relates to a surface energy requirement. In the following, Griffith theory is extended to finite increments \( l = \mu e \). Infinitesimal variation in the classical Griffith theory appears to be too restrictive and sometimes leading to contradictions.

Taking account of the particular shape of the variation of \( \sigma(\mu) \) and \( G(\mu) \) along the potential crack, it is easy to show that these conditions provide lower and upper bound for the crack jump length \( \mu \). Strain energy release rate \( G \) allows to find an undervalue \( \mu_G \) of allowable \( \mu \), whereas \( \sigma \) allows to find overvalues \( \mu_\sigma \).

Looking for initiation conditions is equivalent to find the zero value measure of segment \([\sigma_0, \sigma]\) by adjusting the two-component loading \((T, \sigma)\). This implies at the same time adjustment of functions \( \sigma(\mu) \) and \( G(\mu) \).

For that purpose, the change in potential energy between the two states is expressed as a contour independent integral which can be calculated along the contour between near and far field. This writes as a bilinear form [11]:

\[
-\delta W_p = \psi(u(x_1, x_2, e, l), u(x_1, x_2, e, 0))
\]

\( u(x_1, x_2, e, l) \) is the unperturbed displacement field before initiation

\( u(x_1, x_2, e, 0) \) is the perturbed displacement field after initiation; \( l = \mu e \)

We show that the energy release rate can be expressed as:

\[
G = K^2_l[A(\mu) + mB(\mu) + m^2C(\mu)]
\]

where \( m \) is a non dimensional parameter evaluating the compared intensities of the structural effect \( T \) and the singular effect \( (K_l) \).

\[
m = \frac{T\sqrt{e}}{K_l} = \frac{T}{5\sigma_i} \sqrt{\frac{e}{a}}
\]

The effect of a "biaxial" loading mixed with geometry is apparent in (11). The practical consequence is that the structural effect is maximum when bending of the intercalation is favoured, which is obtained with low \( e/a \) values and large relative overburden \( T \) with respect to \( \sigma_i \).

In (11):

\( A(\mu) \) is an energy contribution relative to \( V_1^2 \)

\( C(\mu) \) is an energy contribution relative to \( V_2^2 \)

\( B(\mu) \) is an energy contribution relative to \( V_1^1 \) and \( V_2^2 \) coupling.
Application maps:

Occurrence of fracture mechanisms as well as abutment events are presented in a \((T, \sigma_t)\) reference as mentioned before. Limits of zones satisfy criteria (equalities) and inequalities are obtained in the failure zones. Examples corresponding to typical values are presented in fig.6.

Conclusion and further work

We think that a better characterisation of fractured reservoirs is obtained by pointing out the salient role of overburden stress. Classical presentation of joints is made through a periodic spatial description under influence of a far field effective horizontal traction. Bending of layers and interbedding behaviour may improve numerical model. Some situations are well described by this class of models but tractions and free surfaces do not seem to be very "natural" and straightforward hypotheses. We think that vertical jointing can also be described by overburden influence where fracture is stimulated and magnified by stress concentrators which transform, as a local effect, vertical into horizontal stress. The shale (weak rock) intercalation is a typical candidate to this role.

The practical form of the model has been obtained thanks to matched asymptotics technique: compatibility is forced between singular intercalation field prone to initiating joint and far field where the influence of intercalation weakens.

Two applications have been performed:

- The first one is the subject of this presentation; a map of pertinent mechanisms is obtained exhibiting three possibilities: abutment, straight through fracture, step-over. Sensitivity to parameters and to failure criteria need to be further investigated.
- The second is in progress. Fracture swarm configuration is analysed through the same technique of matched asymptotics. Near field is expressed as that of a crack of finite width. This distinctive feature implies an increased extension force. Mechanism is considered in the model as a steady state one. Typical scale is far above that of preceding case, so that no significant influence of bedding interfaces can be stated on fracture swarm propagation.

References


