Abstract

This paper discusses two practical strategies to quantify the uncertainty in production and injection forecasts for a field with a long and complex production history with poor quality measurements. These methods are applied to a large offshore field in Africa that has been on production for more than 30 years. The first method follows an advanced Experimental Design framework and requires the use of non-linear Response Surfaces such as kriging. The second method uses sensitivity coefficients; it can be considered as a first pass to evaluate uncertainty before embarking in a more comprehensive analysis. Both methods lead to multiple acceptable representations of the history of the reservoir. The range of outcomes obtained with production forecasts allows for uncertainty quantification. In this case, both methods delivered similar prediction results.

Introduction

Rigorous methods for estimation of uncertainty for reservoirs with a substantial production history are not standard in the oil industry. Monte Carlo simulations coupled with probabilistic inverse theory would constitute the best alternative. The parameter space is sampled randomly with a large number of simulations. Each realization (sample) is assigned a probability that is a function of the fitness of the model to the observations (data to match). This probability is carried over to the predictions. It is in general not practical for real field cases.

Efficient alternatives are being investigated [1,2,3] to offset the large number of simulations required by the method aforementioned. One key step in the search of efficient procedure involves the building of surrogates of reasonable quality with a manageable number of simulations [3]. The fitness of the model to the observed data (history match error function) and the forecasts are modeled using proxies (response surfaces). Multiple solutions to the history match are first identified by sampling the proxy for the fitness to the model function. These estimated acceptable solutions are used to sample the proxy for the forecasts. Statistics such as P10, P90 are extracted from the last sampling process. Both history match and uncertainty estimation processes are driven by a complex use of response surfaces and optimization algorithms that aim to obtain the maximum information from a given number of simulations. In the field case described hereafter, one single appropriate sampling of the parameter space by a large number of simulations followed by a proxy modeling resulted in the identification of a reasonable set of solutions.

The sensitivity coefficient technique puts more emphasize on the use of analytical mathematics. The sensitivity coefficients are the first order derivatives of the simulator output
with respect to the history match and uncertainty analysis parameters [4,5]. In this approach it is assumed that a good match has already been obtained. Derivatives corresponding to both the history match and forecast periods are computed at the “good match” location. It is then possible to calculate the a posteriori covariance matrix of the estimated history match parameters and the covariance matrix of forecasts. Multiple solutions to the history match problem can also be derived. The main limitation is that the analysis is only valid in the neighborhood of the “good match” location.

Field Case Problem Description

The field of interest has over one billion barrels of original oil in place and is overlain by a significant gas cap (Fig. 1). It has been on production for more than 30 years and it is now considered for gas storage. The current simulation model which has a fair history-match (Fig. 3) is used as the basis for evaluating this field for gas storage with continued production. The most likely development plan driving the predictions of this study involves 4 gas injector wells located on the crest of the gas cap. The primary variables of interest for the assessment of uncertainty are gas injection rates and cumulative gas injected.

The main goal of the two methodologies discussed hereafter is to deliver multiple models (combinations of pre-selected parameters at different settings) that match the historical data within the error band of the measurements. The parameters used in the analysis are summarized in Table 1. They include not only the ones that affect the history like fluid contacts and fault transmissibility (depicted in Fig. 1), permeability and vertical communication in different areas of the reservoir, but also parameters associated with the introduction of new wells in the reservoir such as skin effect. The associated uncertainty ranges shown in the table are the results of significant pre-work and sensitivity analysis.

Model Misfit Evaluation

Both methods require the computation of an error function that characterizes the misfit between simulated and actual data. The usual expression in the literature shown in Eq. 1 involves the covariance matrix of the production data and the covariance matrix of the parameters.

$$E_{HM}(\bar{\alpha}_{HM}) = \frac{1}{2}(\bar{d}_{obs} - \bar{d}_{calc})^T C^{-1}_d (\bar{d}_{obs} - \bar{d}_{calc}) + \frac{1}{2}(\bar{\alpha}_{HM} - \bar{\alpha}_{prior})^T C^{-1}_{a\_prior} (\bar{\alpha}_{HM} - \bar{\alpha}_{prior})$$  \hspace{1cm} (1)

The second term of the equation relates to potential prior information available about the parameters. For this study, there was no such reliable information on the cross-correlation between parameters. We consequently focus our effort on the expression of the covariance matrix of the historical data that is assumed to be diagonal. The quality of the match is hence evaluated using a simplified version of Eq. 1:

$$E_{HM}(\bar{\alpha}_{HM}) = \sum_{i=1}^{i_{obs}} w_i (d_{obs}^i - d_{calc}^i)^2$$  \hspace{1cm} (2)

Because of limited confidence in production data at the well level, the misfit function evaluation was restricted to field-wide variables: field water production rate (FWPR) and field gas production rate (FGPR). The weights were assigned according to the reliability of the measurements.
Experimental Design and non-linear Response Surfaces

Traditional Experimental Design techniques have been developed to gain maximum information about a system with a limited, carefully selected, set of experiments [6,7]. The response of interest (e.g. recoveries) is influenced by several variables (e.g. permeability), but the form of the relationship is typically unknown. Usually, a low-order polynomial is employed and most designs attempt to minimize the variance of the estimated coefficients, ensuring a reliable proxy is obtained. However, it is very unlikely that a polynomial model will be a reasonable approximation of the true functional relationship over the entire uncertainty space of the independent variables. This is especially true when the response considered is highly non-linear. In history-matching problems, the objective is to find regions of the parameter space where the misfit function has small values. By nature, the misfit function exhibits a complex behavior that cannot be accurately captured by sampling strategies dictated by classical [7] experimental design techniques, where parameters are set at two or three different levels (low, mid and high for instance).

Uniform design techniques provide an efficient way of combining different settings of the parameters [8]. Given a fixed number of runs pre-determined by the experimenter, parameters are organized within each combination so that the coverage of the parameter space is optimized. A simple example in Fig. 2 illustrates the strength of the uniform design as an efficient “space filling” design. Based on our experience, for 9 variables, at least a few hundred combinations are necessary to appropriately span the parameter space. In this case 600 simulation models were built. Fig. 7 shows the distribution of each parameter’s values in all 600 combinations. By comparison, conventional factorial designs such as central composite or Box-Behnken designs would most likely investigate only 3 extreme settings. Although not originally expected, this initial 600 model ensemble combined with proxy modeling proved to be enough in this case to identify an acceptable number of multiple matches to the production history.

The performance of the newly built models is then assessed, both against history and predictions. Each simulation (history + predictions) takes approximately 3 hours. Current distributed computing capabilities make the problem easily tractable: all runs can be done in 3 or 4 days. At this stage two response surfaces are built: one for the measure of the mismatch between observed and simulated data and one for the corresponding prediction variable, cumulative gas injected for instance. Multi-dimensional, non-linear interpolation algorithms are used to better capture the complex behavior of the proxy surfaces. Kriging techniques ensure a perfect fit of the model on the data and have shown robust predictive capabilities in previous sensitivity studies as well as in the literature [8]. The mismatch surface is then sampled with Monte-Carlo techniques within the variability range of parameters and a subset of parameters satisfying a pre-defined threshold is retained. This amounts to discarding all the models that do not preserve the history within the pre-defined measurement error band. The production results for 60 models satisfying the threshold, shown in Fig. 5, demonstrate the efficiency of the selection method. All models are acceptable representations of the past behavior of the reservoir and the spread of responses in prediction allows for uncertainty quantification. The prediction results of the base case, shown in Fig. 3, are also within the brackets defined by the 60 models. The distribution of the possible settings of the 9 parameters featured in the 60 selected models that match the history is revealed in Fig. 7. Except for the water-oil contact (WOC) and for the gas-oil contact (GOC) to a lesser extent, the selected models bear almost every possible setting of the parameters, thus ensuring significant differences among the models. It builds a good level of confidence in the uncertainty ranges estimated in prediction. Fig. 4 shows the spread of responses for both history and predictions for all 600 runs and for the sample 60 runs with smaller error. The spread of
cumulative gas injection responses is reduced by about 50% when we add the historical data constraints.

The same selected combinations of parameters sampled on the second surface provide acceptable estimates of the predicted variables. Statistical analysis on the population of points evaluated on the kriging surface allows for uncertainty quantification. The results for the cumulative gas injection response, presented in Fig. 6, revealed an uncertainty range of 20% at the end of field life. The same analysis repeated at each year of production defines a band of uncertainty as shown in Fig. 8.

The methodology presented not only provides the distribution of acceptable parameters but also guides the experimenter towards a better knowledge of the reservoir. For example, the scatter plot between WOC and GOC in Fig. 9 reveals interaction between parameters. The 60 combinations selected, circled in magenta, show a clear trend between the 2 fluid contacts. It underlines the importance of volume as a key component of a “good match” in this case. If a model has a low WOC, a reasonable match will be obtained if the GOC is low as well, thus preserving the same overall volume. Some of the combinations of parameters might also produce models that have a smaller error than the base case in use. Such models can deliver valuable information regarding the key parameters that might improve the match.

**Sensitivity Coefficients Method**

As reported in the petroleum engineering literature [4,5], the sensitivity coefficient method allows approximate estimation of uncertainty in flow prediction for reservoir models constrained to production history. The purpose of its use in the present work is to compare its performance with a higher order method when applied to a real field case. The sensitivity coefficients are the first order derivatives of the simulator output with respect to the parameters selected for forecast and for history match. A critical assumption is the existence of at least one good match that could have been obtained with traditional manual history match or with automatic methods. Derivatives corresponding to both the history match and forecast periods are computed at the “good match” location.

In this approach the uncertainty problem is posed in the framework of the inverse problem theory. The method requires the construction of the history match Hessian - the matrix of the second derivatives of the misfit function $E$ in Eqn. 1 - calculated using sensitivity coefficient information. The a posteriori covariance matrix of the history match parameters $C_{\alpha, HM}$ is obtained by inverting the Hessian matrix. The covariance matrix $C_{\alpha, HM}$ can be used to estimate multiple solutions to the history match problem that would result in an error below a predetermined threshold [9]. The covariance matrix of the parameters for the uncertainty analysis in the forecasts $C_{\alpha, pred}$ is obtained by adding terms to the matrix $C_{\alpha, HM}$ that take into account new parameters only impacting the forecast, such as skin at new wells. The matrix $C_{\alpha, pred}$ is manipulated together with the sensitivity coefficient information corresponding to the forecast period to obtain a covariance matrix of the predicted variables, such as field gas production rate ($FGPR$):

$$C_{FGPR} = \text{Covariance}[FGPR] = G \times C_{\alpha, pred} \times G^T \quad (3)$$

The diagonal terms of $C_{FGPR}$, the covariance matrix of $FGPR$, represent the variances of the forecast. The standard deviation $\sigma$, which is the square root of the variance, is a function of time: $\sigma = \sigma(t)$. The range of variability in the forecast is plotted as a 2.56 $\sigma$-wide band around the base forecast (80% of the forecasts are within that band).
The quality of the sensitivity coefficient approach is verified by running actual reservoir simulation for a subset of the realizations generated with this method. For clarity purposes, these results are not presented here.

The results for cumulative gas injection forecasts are shown in Fig. 9. They are compared with the results from the more accurate method using complex response surfaces. The agreement is remarkable considering the simplicity of the sensitivity coefficients method. Besides, it is relatively simple to implement and it does not require a large CPU load. In the course of an uncertainty assessment study, the outcomes of the method can be considered as early indicators of the uncertainty level and of the significance of parameters.

Conclusions

- The methods investigated lead to multiple acceptable representations of the history of the reservoir. The range of outcomes obtained with production forecasts allows for uncertainty quantification.

- “Space filling” Experimental Design techniques coupled with the right Response Surface Methodology provide an efficient way of assessing uncertainties associated with flow predictions.

- Sensitivity Coefficients techniques though in general less flexible and more approximate proved efficient as a first pass method for uncertainty assessment.

- The application of these methods allows the experimenter to gain a better insight in the geological model, which is not possible with other traditional approaches.

- Building accurate response surfaces as efficient surrogates in high dimension problems require a large number of simulation runs that can be CPU intensive.

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References


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Table 1: Parameters and uncertainty ranges

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Figure 1: Geological model: fluid contacts and reservoir compartmentalization.

Figure 2: Space coverage for a typical uniform design.
Figure 3: Field production for history-matched base case

Figure 4: Cumulative water production and cumulative gas injection for (top) the initial 600 combinations (bottom) the selected 60 runs characterized by a small mismatch.

Figure 5: 60 acceptable models provide a good match of the 30 year long historical data and define an uncertainty band for the 30 year long forecast.

Figure 6: Uncertainty ranges at end of field life from Monte-Carlo simulations on the kriged surface.
Figure 7: Distribution of the 9 history-match parameters. In dark blue, all 600 combinations, in light blue, the 60 selected runs that satisfy the matching criterion.

Figure 8: Gas Injection forecasts from advanced Experimental Design techniques and from the computation of Sensitivity Coefficients.

Figure 9: Parameter settings for WOC and GOC. All 600 combinations and the selected set of 60.