Abstract

In this paper, we show that the careful study of the two-phase front can improve up-scaling techniques for two phase immiscible flows in heterogeneous reservoirs. A detailed numerical and analytical study of the dynamics of the front shows that stochastic approaches cannot neglect the viscous coupling between the pressure and saturation. A very strong interaction exists between the heterogeneity and the stability or instability of the fluid flow displacement. This coupling is responsible for a qualitative and quantitative change of the form of the large scale equations, that must be accounted for by appropriate up-scaling procedures. Once this problem is solved, it is easier to find optimal discretisation procedures based on physical considerations, in order to accelerate Monte Carlo studies. Examples dealing with stratified and isotropic stochastic media will also be presented.

1 Introduction

From early numerical reservoir models appeared in the sixties, involving several hundreds of cells, to current million-cells stochastic models, grids used to describe reservoir heterogeneities have always been by far too large and expensive in terms of computational requirements for flow simulations. Upscaling from a detailed reservoir model to a coarse grid model is hence still a challenging issue for reservoir engineers, in order to speed up flow simulations. However, the meaning of upscaling has partially been affected by the increasing use of stochastic reservoir modelling techniques, and previous upscaling methods can be revisited within a stochastic framework. For example, when dealing with an infinite number of equally likely reservoir models, should the up-scaling process be able to lead to the same simulation results for each realisation of the permeability field, or should it only allow to quickly derive our uncertainty on the recovery curve with respect to the uncertainty on the field and on the production scenario? This consideration motivates ongoing efforts to derive large-scale two-phase flow equations, with average large-scale parameters related to the stochastic properties of the underlying permeability variations.

We propose a stochastic model allowing to describe the dynamics of the front in heterogeneous porous media. We show that the upscaling process should first involve a physical step to relate local flow properties to large-scale flow properties. This step is independent from the size of gridblocks and from the numerical scheme used for flow simulations, as we focus on the underlying continuous equations. The numerical problem of gridding the large-scale model should be investigated in a subsequent step. A study of the physical problem shows that the viscous coupling is a crucial phenomenon that controls the emergence of different large-scale flow regimes. This leads us to propose a new stochastic model to describe the dynamics of the front in heterogeneous media. This promising model gives answers to the physical problem and should be helpful to optimise up-scaling. This model predicts the emergence of different flow regimes, depending on the frontal mobility ratio. In stratified media, the front may move at the same uniform velocity in some layers, that we called “hydrodynamic layers”. This phenomenon allows an optimal grouping of the original geological layers, taking into account their ordering. When the permeability field is isotropic, we are able to relate the statistical structure of the water oil front (mean, covariance function) to the statistical structure of the underlying permeability map, and to the properties

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of the flowing fluids. The main property of the fluids then appears to be the total mobility ratio evaluated at the front.
In the stable case, this process leads to the formation of a quasi stationary front. Coming back to a local Eulerian description shows that this stationary solution can correspond to the result of a competition between the relaxation process corresponding to the stabilisation of the front, and the velocity perturbation induced by the heterogeneities. This indicates that the average process could be described with a large scale equation having the form of a classical two phase flow saturation transport equation, using a suitable renormalized fractional flow term together with a two phase dispersion term.

2 Dynamics of two-phase immiscible flow fronts

Let us consider a two-phase immiscible displacement in a 2-dimensional porous medium. The injected fluid (fluid 1) is injected on face \( x = 0 \) while the displaced fluid is produced on face \( x = L_x \). We suppose that the porosity and the relative permeability curves are constant and that the absolute permeability field \( K(x,y) \) is heterogeneous with known statistical properties. In particular, we assume the mean permeability and the correlation structure of the permeability field \( C_K(r) \) to be known. We assume capillary and gravity forces to be negligible.
In homogeneous media, two-phase immiscible displacements without capillary forces are characterized by the existence of a saturation shock \( S = S_f \) leading a rarefaction zone where each saturation plane travels at constant speed \( v(S) = U \phi'(S) \), where \( U \) is the filtration velocity and \( \phi \) is the fractional flow term. The fractional flow function depends on the relative permeability curves. An example of such a fractional flow function and saturation profiles for quadratic relative permeability curves is given in Figure 1. The stability of the front with respect to small perturbations depends on the mobility ratio \( M_f \) evaluated across the front (Saffman-Taylor, 1958, Hagoort, 1974, King, 1989, Blunt and Christie, 1993), with standard notations:

\[
M_f = \frac{\lambda(0)}{\lambda(S_f)} = \frac{\mu_1 k_{r1}(0)}{\mu_2 k_{r2}(S_f)} \left[ 1 + \frac{\mu_1 k_{r2}(S_f)}{\mu_2 k_{r1}(S_f)} \right]^{-1}
\]

![Figure 1](image1.png)

**FIG. 1** – (a) Relative permeability curves, (b) fractional flow \( \phi(S) \), (c) local Buckley-Leverett saturation profiles.

In heterogeneous media, the coupled saturation, pressure and velocity maps are perturbed by the permeability fluctuations, and their evolution depends on the interaction between heterogeneity and viscous coupling effects. This non-linearity is responsible the emergence of different flow regimes (Lenormand, 1996, Furtado and Pereira, 2003).
However, a front line is still defined, corresponding to the position of the saturation shock \( S(x, y, t) = S_f \) (Figure 2b). Choosing to work in a Lagrangian framework, we focus on the perturbations of the front line:
\[ \dot{x}(y, t) = x(y, t) - ct, \]

where \( c = U \varphi'(S_f) \) is the mean velocity of the front. \( U \) is the mean filtration velocity imposed by the constant injection rate. Our goal is to find a relation between \( \dot{x}(y, t) \) and the properties of the underlying heterogeneous permeability field. We work in the spectral domain, with Fourier modes \( \delta_x(y, t) \) of the front defined by:

\[ \delta_x(y, t) = \int \dot{x}(y, t) e^{i\delta y} dy \]

Using analytical methods proposed by King and Dunayevsky, 1989, and series perturbation expansions in powers of \( \sigma_{\ln(K)} \), it is possible to derive the following equation driving \( \delta_x(y, t) \):

\[ \frac{\partial \delta_x(y, t)}{\partial t} = -c\varphi' \left( \frac{M_f - 1}{M_f + 1} \right) \delta_x(y, t) + \eta_y(t), \]

The first term in the right hand side corresponds to the relaxation (or amplification) due to the viscous coupling. The second term, \( \eta_y(t) \), is a random excitation source term describing the effect of heterogeneities. Statistical properties (mean, covariance function) of \( \eta_y(t) \) may be found by in Dagan, 1989. This equation is exact up to first order in \( \sigma_{\ln K} \). This is an Ornstein-Uhlenbeck process, having a well defined complete analytical solution. In the absence of the relaxation term, we directly recover the dynamics of a passive tracer leading to an Eulerian macrodispersion equation (Gelhar and Axness, 1983).

FIG. 2 – (a) One realization of the \( \ln(K) \) field, and (b) the associated saturation map.

In the **stable** case \( (M_f > 1) \), the solution of this equation is the result of a competition between the viscous relaxation and the random excitation. The **marginal** case \( M_f = 1 \) corresponds to the **passive tracer** case. When \( M_f < 1 \), the flow is **instable**, and our developments quickly become irrelevant, as strong nonlinearities must accounted for to describe fully developed viscous-fingering patterns. Koval like descriptions can be applied to describe some special regimes like vertical equilibrium (Yortsos, 2001). As the stability criterion is the value of the mobility ratio across the front, two-phase displacements may be stable even when the fluid in place is more viscous than the injected fluid. Actually, a large number of relative permeability curves are such that the mobility experiences a positive shock across the front even for large end-point mobility ratios.
3  Statistical properties of the front.

The formal solution this model allows to estimate statistical properties of the front as a function of the permeability covariance function $C_{kk}(r)$. Let us define the long-time front covariance as the limit:

$$C_{\delta\delta}(h) = \lim_{t \to +\infty} \langle \delta(x) \delta(y + h, t) \rangle,$$

Using $\langle \delta(x) \rangle = 0$, we can find the relationship between the front covariance and the covariance of the permeability field. Introducing its Fourier transform, defined by:

$$\hat{C}_{\delta\delta}(q) = \int C_{\delta\delta}(h) e^{i\mathbf{h} \cdot \mathbf{x}} \, dh,$$

we have (Artus et al., to appear):

$$\hat{C}_{\delta\delta}(q) = \int dq_x \frac{\hat{C}_{\eta\eta}(q_x, q)}{\langle U \rangle^2 q_x^2 + A^2 q^2},$$

$$A = \left( \frac{u}{\bar{u}} \right) M_f - 1, M_f + 1.$$

The knowledge of the covariance of $\eta_{q}(t)$ provides $C_{\delta\delta}(\mathbf{h})$. The presence of the $1/q^2$ kernel implies divergence in 2D. In particular, the variance $C_{\delta\delta}(0)$ which allows to estimate the front thickness is given by $C_{\delta\delta}(0) = \lim_{t \to +\infty} \langle \delta(x)^2 \rangle$:

$$C_{\delta\delta}(0) = \frac{1}{4\pi^2} \int dq_x dq_{\eta} \frac{\hat{C}_{\eta\eta}(q_x, q)}{\langle U \rangle^2 q_x^2 + A^2 q^2}. $$

This integral is infra-red ($q = 0$) divergent. In order to avoid this singularity, it is more convenient to introduce the variogram of the front, $\gamma_x(h)$, defined by:

$$\gamma_x(h) = \frac{1}{2} \lim_{t \to +\infty} \langle \delta(x) \rangle \langle \delta(y + h, t) - \delta(x, y, t) \rangle.$$

The singularity is recovered studying the behaviour when $h \to +\infty$ of $\gamma_x(h)$. In the stratified case, it can be shown that the variogram of the front behaves linearly at long distances:

$$\gamma_x(h) \propto \frac{\hat{C}_{\delta\delta}(q = 0)}{2A^2} h, h \to +\infty$$

In the 2D isotropic case, a logarithmic behaviour arises at long distances:

$$\gamma_x(h) \propto \frac{\hat{C}_{\delta\delta}(q = 0)}{2A^2} \log h, h \to +\infty$$

In the 3D isotropic case, full convergence is ensured even for $h \to +\infty$. However, we did not numerically investigate the 3D case.
4 Monte Carlo tests

We illustrate these results with Monte-Carlo studies, both in the stratified case (strata parallel to mean flow) and in the heterogeneous isotropic case. In both cases, we consider here a very stable situation ($\mu_1/\mu_2 = 10$). Any variogram estimation is obtained through 100 simulations of two phase flows in heterogeneous realizations.

The considered stratified medium was 700m×200m, with a variogram-based stratified permeability field. Fig 3 (a) shows the occurrence of a stationary front. This emergence of stationary fronts was also observed in laboratory experiments using miscible fluids (Loggia, 1996). On the Fig 3, the variogram was gaussian with a correlation length of $L_c = 10$m and $\sigma^2_{\ln(K)} = 0.1$.

![Fig 3](a) Front displacement in a stratified medium, stable case (b) Variogram of the front $\gamma_x (h)$ with adimensional time, emergence of a limiting behaviour

We also considered variogram-based realizations of 200m×200m isotropic media, with $L_c = 10$m and $\sigma^2_{\ln(K)} = 0.1$. The evolution of the front and its associated variogram are shown on Fig 4. The stabilisation and the emergence of a stationary behaviour can be observed.

![Fig 4](a) Front shape at several different time in the isotropic medium, stable case. (b) Variogram $\gamma_x (h)$ with adimensional time, a limit does emerge.
5 Hydrodynamic layers

The stochastic developments presented above are valid at first order only. When permeability contrasts are small, a mobility ratio at front $M_f > 1$ is sufficient to compensate for permeability contrasts and lead to a stationary front. As soon as large permeability contrasts exist in the medium, the mobility ratio should be higher to increase the viscous crossflow between strata and stabilize the front.

For example, for flow under vertical equilibrium (V.E.) in a two-layer stratified medium, it can be shown (Hearn, 1974, Loggia, 1996) that the criterion for a stationary front to exist is $M_f > K_2/K_1$. Under V.E. assumption this pattern is extendible to media with multiple layers: after ordering the layers with respect to their permeability, it is possible to predict the emergence of zones in the medium across which the front is stationary. The width and position of these zones of course depend on the mobility ratio. When the V.E. assumption does not apply, laboratory and numerical experiments confirm the existence of stationary zones in stratified media, depending on the value of the mobility ratio. In these cases, however, the actual ordering of the layers is of crucial importance (Loggia, 1996, Artus, 2003). The existence of these zones is of particular interest for upscaling, as the layers across which the front is stationary (hydrodynamic layers) could be identified and separately averaged with a minimal loss of information concerning the dynamics of the front. This simple idea is illustrated on Figure 5, where a reservoir model formed with several geological layers is upscaled using only the hydrodynamic layers. Note that as the position of the layers depends on the mobility ratio, the optimum coarse grid will also change with the value of $M_f$.

We illustrate this approach on a simple 4-layer stratified medium (Figure 6, Model 1). The computational grid is formed with $40 \times 500$ squared gridblocks. The width of each “geological” layer is 10 blocks. The relative permeability curves are linear, $k_{r1}(S)=S$ and $k_{r2}(S)=1-S$. The injected fluid has a viscosity $\mu_1 = 4cp$ while the viscosity of the fluid in place is $\mu_2 = 1cp$. For this particular configuration, the front is quickly stationary across the two first layers, while the two last layers are flooded at different speeds (figure 7a). Using this information, we averaged the two first layers into a single one using a mean permeability (Figure 6, model 2). The comparison of the evolution of the front between model 1 and model 2 show that this process clearly did not impact the main properties of the front dynamics, leading to a minimal loss of information. The water cut at outlet face is well recovered with model 2 (Figure 7b). Note that as the front is the averaged layers is stationary, it does not change while flooding and the error made by averaging the two layers is hence independent of the travelled length.
The existence and position of the hydrodynamic layers depends on the viscosity ratio. A method has been developed to identify them without relying on the results of a full fluid flow simulation (Nötinger and Artus, 2002).

6 Conclusions

In order to account for the stochastic nature of current reservoir models, up-scaling should be able to relate fine-scale random flow properties to new appropriate average properties at large scale. Sensitivity tests could hence be performed directly on large-scale models, using appropriate coarse simulation grids. This approach requires to investigate the physics of two-phase flow at large scale in heterogeneous media. We have proposed a stochastic model for the dynamics of the front. This model allows to predict the emergence of different flow regimes in the reservoir, depending on viscosity and permeability contrasts. Using this information, an optimal coarse grid can be used for flow simulations, using large scale parameters. This approach is illustrated on a simple stratified medium.

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Références


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