Abstract

Enhanced Oil Recovery (EOR) methods include injection of different fluids into reservoirs to improve oil displacement. Displacement of oil by any of these fluids involves complex physico-chemical processes of interphase mass transfer, phase transitions and transport properties changes. These processes can be divided into two main categories: thermodynamical and hydrodynamical ones. They occur simultaneously during the displacement, and are coupled in the modern mathematical models of EOR. The model for one-dimensional displacement of oil by gas is analyzed in this paper. The main result is the splitting of thermodynamical and hydrodynamical parts in the EOR mathematical model. The introduction of a potential associated with one of the conservation laws and its use as an independent variable reduces the number of equations. The algorithm to solve the problem includes the solution of the derived lifting hyperbolic equation and inversion of the coordinate transformation. The reduced auxiliary system contains just thermodynamical (equilibrium fractions of each phase) variables and the lifting equation contains just hydrodynamical (phases relative permeabilities and viscosities) parameters while the initial EOR model contains both thermodynamical and hydrodynamical functions. So, the problem of EOR displacement was divided into two independent problems: one thermodynamical and one hydrodynamical. Therefore, phase transitions occurring during displacement are determined by the auxiliary system, i.e., they are independent of hydrodynamic properties of fluids and rock. For example, the minimum miscibility pressure (MMP) is independent of relative permeabilities and phases viscosities. In this paper, the splitting technique is used for the development of an analytical model for the non-self-similar problem of displacement of oil by rich gas solvent slug with lean gas drive.

Introduction

Gas based methods of enhanced oil recovery include injection of different gases (methane, rich hydrocarbon gases, carbon dioxide and various combinations) in order to improve displacement of oil by mass exchange between oleic and gas phases [1]. One dimensional displacement of oil by gas in large scale approximation is described by an (n-1)x(n-1) hyperbolic system of conservation laws, where n is the number of components. Continuous gas injection results in a Riemann problem for this hyperbolic system. Displacement of oil by a gas slug with another gas drive is described by the initial and boundary value problem with piece-wise initial data.

A hyperbolic system for gas flooding is similar to the one of polymer flooding. The solution of the Riemann problem for displacement of oil by a polymer solution was obtained for the system...
The case \( n=2 \) where the \( i \)-th adsorbed concentration depends only on the concentration of \( i \)-th component in aqueous phase was solved and several Riemann problems solutions have been found [3]. A semi-analytical Riemann problem solver for two-phase \( n \)-component polymer flooding was developed more than ten years ago [4,5]. The solution method is based on the observation that the concentration waves in 2-phase environment can be lifted from one phase multi component flow. Non-self-similar problems with wave interactions have been solved for \( n=1 \) and 2 [6].

Semi-analytical solutions for \( n \)-component gas flooding were obtained by numerical combination of shocks and rarefactions [7-9]. The reduction of the gas flood system dimension was developed through the lifting of the concentration waves from the system with lower dimension and exact solutions were obtained for the displacement of \( n \)-component ideal mixtures [10,11]. These solutions were used for different initial data corresponding to injection of different gases in different reservoir oils [12,13].

In the current paper, a potential function associated with the conservation of the \( n \)-th component is introduced. The potential is used instead of time as an independent variable and another potential is used instead of linear co-ordinate. The change of independent variables reduces the order of the system. The reduced (auxiliary) system contains only thermodynamic functions while the original system contains both thermodynamic functions and transport properties. It is shown that the “concentration part” of the solution of the initial-boundary value problem for the \((n-1)\times(n-1)\) system satisfies the auxiliary \((n-2)\times(n-2)\) system. The equations for projection and lifting of elementary waves have been derived. These equations allow the construction of the solution from the auxiliary system solution. We also present the solution of the non-self-similar problem of oil displacement by a rich gas slug with lean gas drive.

**Mathematical Model for Gas Flooding**

We consider 1D two-phase multicomponent gas flooding under the following conditions:

- neglected capillary pressure and diffusion
- instantaneous thermodynamic equilibrium
- constant pressure and temperature
- equal component individual densities in both phases.

Under thermodynamic equilibrium conditions there are \( n-2 \) independent phase fractions. We choose components \( i=2,3,\ldots,n-1 \) in gas phase as the components of the vector of independent phase fractions:

\[
\bar{g} = (c_{2g}, c_{3g}, \ldots, c_{(n-1)g})
\]

In this case, the total two-phase flux is conserved, and \( n \) mass balances for \( n \) components are replaced by \( n-1 \) volume conservation laws for \( n-1 \) components:

\[
\frac{\partial C_i}{\partial t_D} + \frac{\partial F_i}{\partial x_D} = 0, \quad x_D = \frac{x}{l}, f_D = \frac{ut}{\Phi l}
\]

where the overall \( i \)-th component fraction and flux are

\[
C_i = c_{ig} S + c_{ig} (1-S), \quad F_i = c_{ig} f + c_{ig} (1-f)
\]

Here \( f \) is the fractional flow for liquid phase.
At this point we introduce new thermodynamic geometric variables:

\[ \alpha_i(g) = \frac{c_{i1} - c_{ig}}{c_{il} - c_{ig}}, \quad \beta_i(g) = c_{ig} - \alpha_i, \quad i = 2, 3, \ldots, n - 1 \]  

(4)

Figure 1 shows the geometrical meaning of \( \alpha_i \) and \( \beta_i \). Vortices 1, 2, \ldots, n correspond to pure components in phase diagram. Tie line GL connects equilibrium phase compositions, \( G_iL_i \) is the tie line projection on the plane \((C_i, C_n)\). The slope of the straight line \( G_iL_i \) is equal to \( \alpha_i \), and the intersection of \( G_iL_i \) with the axes \( C_i \) is equal to \( \beta_i \).

Applying the new variables, system (2) becomes:

\[ \frac{\partial C}{\partial t_D} + \frac{\partial F}{\partial x_D} \left( C, \beta \right) = 0 \]
\[ \frac{\partial \left( \alpha \beta C + \beta \right)}{\partial t_D} + \frac{\partial \left( \alpha \beta F + \beta \right)}{\partial x_D} = 0 \]  

(5)

In system (5), \( C \) is equal to \( C_n \), the overall volumetric fraction of \( n \)-th component, and \( F \) is equal to \( F_n \), the overall volumetric fractional flow of \( n \)-th component. The unknowns in the new system of \( n-1 \) equations are \( C \) and \( \beta_i, i = 2, 3, \ldots, n-1 \).

Initial and boundary conditions for continuous gas injection correspond to given compositions of injected gas and displaced oil (Riemann problem). In the case of the displacement of oil by a rich gas slug with lean gas drive, the boundary conditions determine the non-self-similar problem:

\[ C(0, t_D); C_n', t_D \leq 1, \quad \beta_i(0, t_D); \beta_i(g'), t_D \leq 1 \]
\[ C(t_D, t_D); C_n', t_D > 1, \quad \beta_i(0, t_D); \beta_i(g'), t_D > 1 \]  

(6)

**Splitting Between Thermodynamics and Hydrodynamics**

The conservation law form of the first equation (5) allows the introduction of the following potential:

\[ C = -\frac{\partial \varphi}{\partial x_D}, \quad F = \frac{\partial \varphi}{\partial t_D} \]  

(7)

The potential \( \varphi(x_D, t_D) \) is equal to the \( n \)-th component volume flowing via a trajectory connecting points \((0,0)\) and \((x_D, t_D)\):

\[ \varphi(x_D, t_D) = \int_{0,0}^{x_D, t_D} F \, dt_D - C \, dx_D \]  

(8)

The integral (8) is a function of \( x_D \) and \( t_D \), and is independent of the trajectory. Now we introduce the variable:

\[ \psi = x_D - t_D \]  

(9)

From the incompressibility of the total flux follows that \( \psi(x_D, t_D) \) is equal to the overall mixture volume flowing via a trajectory connecting points \((0,0)\) and \((x_D, t_D)\). After the following transformation of independent variables

\[ \Theta(\psi, \varphi) \rightarrow (\psi, \varphi) \]  

(10)

system (5) becomes
\[
\frac{\partial}{\partial \phi} \left( \frac{C}{F-C} \right) - \frac{\partial}{\partial \psi} \left( \frac{1}{F-C} \right) = 0 \tag{11}
\]
\[
\frac{\partial \beta}{\partial \phi} + \frac{\partial \alpha(\beta)}{\partial \psi} = 0 \tag{12}
\]

The most important feature of the system (11), (12) is the independence of the n-2 equations (12) from the first equation (11). The unknowns in the system (12) are \(\beta_i, i=2,3,...,n-1\). The hyperbolic equation (11) contains the unknown \(C(\psi, \phi)\) and the known vector function \(\beta_i(\psi, \phi)\), which is the solution of (12). The system (12) is called the auxiliary system of the large system (5), and equation (11) is denoted as the lifting equation. It is important to mention that system (5) contains thermodynamic functions and transport properties, while the auxiliary system contains only thermodynamic functions.

It is worth mentioning that the elementary wave speeds (shocks, rarefactions) of the auxiliary system are linked with the wave speeds of the large system by
\[
D = \frac{F+V}{C+V} \tag{13}
\]

**Analytical Model for Oil Displacement by Rich Gas Slug with Lean Gas Drive**

We consider one- and two-phase three-component displacement with all tie lines on phase diagram intersecting in one point (Figure 2). Points 1, 2 and 3 correspond to pure components and points I, J and D denote compositions of displaced oil and gases in slug and in drive.

The initial and boundary value problem corresponding to slug injection is given by:
\[
\begin{align*}
t_d & = 0, \phi = -C^I \psi, \quad \beta = \beta^I, \quad x_d = 0; \psi = \begin{cases} -F^I \psi, & -1 < \psi < 0 \\ F^I - F^D (\psi + 1), & -\infty < \psi < -1 \end{cases} \\
C & = \begin{cases} C^I, & -\infty < \psi < -1 \\ C^D, & -F^I - F^D (\psi + 1), & -\infty < \psi < -1 \end{cases} \\
\beta & = \begin{cases} \beta^I, & -1 < \psi < 0 \\ \beta^D, & -F^I - F^D (\psi + 1), & -\infty < \psi < -1 \end{cases}
\end{align*} \tag{14}
\]

The compositional model for \(n=3\) is a 2x2 hyperbolic system. For the considered case of thermodynamic behavior, the auxiliary system is a linear hyperbolic equation, and \(\alpha(\beta) = \Gamma \beta\). Velocity of shocks and characteristics in the auxiliary system are fixed along the straight line with the slope \(-C^I\) (Figure 3). Either gaseous solvent in the slug or drive gas do not contain the third component, so \(F^J = F^D = 0\) in the boundary condition. Therefore, the boundary condition is set along the axes \(\phi = 0\) (Figure 3).

The solution of the linear auxiliary problem is (Figure 3):
\[
\beta(\psi, \phi) = \begin{cases} 
\beta^D, & \psi < \Gamma \phi - 1, \psi > 0 \\
\beta^I, & \phi - 1 < \psi < \Gamma \phi, \phi > 0 \\
\beta^I, & -C^I < \psi < \Gamma^{-1}, \psi > 0
\end{cases} \tag{15}
\]

The solution of the lifting problem in zones J and I coincide with the self-similar solution of the initial-boundary value problem. Zone J is filled by a centered rarefaction wave. It allows the calculation of \(C\) ahead of the shock \(\psi = \Gamma \phi - 1\), which is \(C^I(\Gamma \phi - 1, \psi)\). The Hugoniot-Rankine
condition on this shock allows the calculation of C behind the shock, C'(Γφ-1,φ). The C-values propagate into zone D by characteristics from the rear of the shock ψ=Γφ-1. The solution of the auxiliary and lifting problems β(ψ,φ) and C(ψ,φ) permits the calculation of the inverse mapping:

\[
x_D = \int_{(0,0)}^{(ψ,φ)} \frac{dφ}{F-C} + \frac{F}{F-C} dψ,
\]

\[
t_D = \int_{(0,0)}^{(ψ,φ)} \frac{dφ}{F-C} + \frac{C}{F-C} dψ
\]

\[ (16) \]

It results in the exact analytical solution of the slug injection problem. Figures 4 and 5 show the solution of the slug problem in spaces (C,F) and (xD,tD).

If unknowns are constant along a straight characteristic line in the plane (ψ,φ), they are also constant along a straight characteristic line in the plane (xD,tD). Therefore, the auxiliary centered wave maps into the centered C-wave 4-5. The simple auxiliary wave behind the shock ψ=Γφ-1 maps into a simple wave 7-8 behind the shock x_D(tD). Points 4 and 7, 5 and 8 are linked by...
Hugoniot-Rankine conditions. The solution of the problem of oil displacement by gaseous solvent slug with lean gas drive is:

\[
\begin{align*}
\beta(x_D, t_D) &= \begin{cases} 
\beta^0, x_D < t_D - 1, x_D < x_0(t_D) \\
\beta^1, t_D - 1 < x_D < D_s t_D, x_0(t_D) < x_D < D_s t_D \\
\beta^1, x_D > D_s t_D 
\end{cases} \\
C(x_D, t_D) &= \begin{cases} 
C^0, x_D < t_D - 1, x_D < \frac{D_s}{1 - D_s} \\
C^1, t_D - 1 < x_D < D_s t_D \\
C^L(x_D, t_D) = \frac{D_s}{1 - D_s} < x_D < x_0(t_D) \\
C^L \left( \frac{x_D}{t_D} \right), x_0(t_D) < x_D < D_s t_D, \\
C^L, D_s t_D < x_D < D_s t_D \\
C^L, D_s t_D < x_D < D_s t_D 
\end{cases}
\end{align*}
\]

where

\[
D_s = \frac{F_s}{C_s} = F'_c \left( C_4, \beta^1 \right), D_s = \frac{F_s + \Gamma^{-1}}{C_4 + \Gamma^{-1}} = F'_c \left( C_5, \beta^1 \right) = \frac{F_s + \Gamma^{-1}}{C_6 + \Gamma^{-1}}, D_s = \frac{F' - F_s}{C' - C_s}
\]

The function \( C^R(x_D/t_D) \) in the slug region is determined by the centered wave

\[
\frac{x_D}{t_D} = F'_c \left( C^R \left( \frac{x_D}{t_D} \right), \beta^1 \right) : \begin{cases} x_0(t_D) < x_D < D_s t_D \\
D_s t_D < x_D < D_s t_D 
\end{cases}
\]

The third component fraction ahead of the shock can be found from the transcendental equation

\[
F' (x_0(t_D)) + \Gamma^{-1} - F'_c \left( C^+ (x_0), \beta^1 \right) = \left( \Gamma t_D \right)^{1}
\]

This equation is obtained by the inverse mapping of the front \( \psi = \Gamma \varphi - 1 \). It allows the calculation of the slug rear front

\[
\frac{x_0(t_D)}{t_D} = F'_c \left( C^+ \left( x_0(t_D) \right), \beta^1 \right)
\]

The Hugoniot-Rankine condition on the front \( x_0(t_D) \) is

\[
\frac{F' \left( C^+ \left( x_0(t_D) \right), \beta^1 \right) + \Gamma^{-1}}{C^+ \left( x_0(t_D) \right) + \Gamma^{-1}} = \frac{F' \left( C^+ \left( x_0(t_D) \right), \beta^1 \right) + \Gamma^{-1}}{C^+ \left( x_0(t_D) \right) + \Gamma^{-1}}
\]

The function \( C^L(x_D, t_D) \) behind the slug is calculated by

\[
C^L(\frac{x_D}{t_D}) = C^- \left( x_0(t_D) \right)
\]

The values of the third component fraction propagate into the zone behind the slug from the rear slug front \( x_0(t_D) \):

\[
\frac{x_D - x_0(t_D)}{t_D - t_0} = F'_c \left( C^- \left( x_0(t_D) \right), \beta^0 \right)
\]

Defining new dimensionless variables

\[
x_D = \frac{\Phi x}{\Delta}, t_D = \frac{u t}{\Lambda}
\]

(25)
where \( \Delta \) is the slug volume, follows that the slug is injected up to the moment \( t_D=1 \).

Up to time \( t_D=1 \), the solution of the slug problem coincides with the self-similar solution of the continuous gas injection. The profile for components distribution along the reservoir corresponds to the path \( J \to 4 \to 5 \to 6 \to I \), where \( \text{A} \to \text{A}^-' \) denotes shock of the variable \( \text{A} \), and \( \text{A} \to \text{B} \) denotes rarefaction waves. Two banks form during the displacement ahead of the front of the injected solvent gas \( J \): the bank with varying composition 4-5 and the bank 6. The velocities of the bank 6, of the bank 4-5 and of the injected gas \( J \) are \( D_6 \), \( D_5 \) and \( D_4 \) respectively.

At the moment \( t_D=1 \), there appears the front of the displacement of gaseous solvent \( J \) by the drive gas \( D \) moving with unity speed. At the moment \( t_D=(1-D_4)^{-1} \) the front catches up the solvent front, the profile \( D \to 7 \to 4 \) appears. The shock \( D \to 7 \) is immobile and realizes the complete evaporation of the heavy component residue, after passing the slug, into the drive gas. The position of the shock frontier is \( x_D=D_4(1-D_4)^{-1} \). Behind the frontier \( x_D=D_4(1-D_4)^{-1} \) just the lean gas \( D \) moves via the "dry" rock; ahead of this frontier and behind the slug gas moves at the presence of immobile liquid phase. Composition of the fluid just ahead of the frontier does not change with time and corresponds to point 7. From the moment \( t_D=(1-D_4)^{-1} \), the drive gas invades the slug and dissolves it. The speed of the slug rear front \( x_0(t_D) \) decreases with time and tends to \( D_5 \) when time tends to infinity. The composition inside the slug tends to point 5, composition behind the rear slug front tends to point 8, slug volume stabilizes. During continuous solvent injection \( (t_D<1) \) the composition profile follows the path \( J \to 4 \to 5 \to 6 \to I \). Until the collapse of pure solvent slug \( (t_D<(1-D_4)^{-1}) \) the profile is \( D \to J \to 4 \to 5 \to 6 \to I \). During the dissolved slug motion \( (t_D>>(1-D_4)^{-1}) \) the compositional profile along the reservoir tends to \( D \to 7 \to 8 \to 5 \to 6 \to I \).

Discussions

Phase transitions occurring during gas-based EOR displacements throughout the reservoir are determined just by thermodynamics of the oil-gas system and are independent of transport properties. The solution of the large system \( \beta_i(x_D,t_D) \) realizes the mapping from the plane \((x_D,t_D)\) to the set of tie lines in \( n \)-vortices tetrahedron of an \( n \)-component phase diagram. The image of the domain of the plane \((x_D,t_D); x_D>0, t_D>0, \) defines 2D surfaces in the tetrahedron. The auxiliary solution \( \beta_i(\psi,\phi) \) also maps the domain of the plane \((\psi,\phi)\), where the initial-boundary value problem is defined, into 2D surface in the tetrahedron. From the splitting of the compositional model (5) into auxiliary (12) and lifting (11) equations follow that these surfaces coincide.

The auxiliary solution depends on thermodynamic functions \( \alpha_i \) and \( \beta_i \) and on the composition fractions of the initial and boundary conditions. So, the 2D solution image in the tetrahedron is independent of transport properties, i.e., fractional flow curves, relative phase permeability and phase viscosities. For 3-D displacements, the splitting is valid if and only if the total mobility is constant. It allows applying the obtained 1-D analytical solutions in streamline simulators.

Conclusions

The \((n-1)\times(n-1)\) system of conservation laws for two-phase \( n \) component flow in porous media with interphase mass transfer can be splitted into an \((n-2)\times(n-2)\) auxiliary system and one independent lifting equation. The splitting is achieved from the change of independent variables \((x_D,t_D)\) to flow potentials \((\psi,\phi)\). This change of coordinates also transforms the conservation law for the \( n \)-th component into the lifting equation.
The lifting procedure for the solution of the large system consists of:

- solution of the auxiliary system
- solution of the lifting equation
- inverse transformation of independent variables: \((\psi, \phi)\) to \((x_D, t_D)\).

The auxiliary system contains only equilibrium thermodynamic variables (equilibrium fractions of each phase), while the large system contains both hydrodynamic (phases relative permeabilities and viscosities) functions and equilibrium thermodynamic variables. Therefore, phase transitions occurring during displacement are determined by the auxiliary system, i.e., they are independent of hydrodynamic properties of fluids and rock. For example, the minimum miscibility pressure (MMP) is independent of relative permeabilities and phases viscosities.

References


