Nonlinear Pump Depletion and Electron Dephasing in Laser Wakefield Accelerators

E. Esarey*, B. A. Shadwick*†, C. B. Schroeder* and W. P. Leemans*

*Center for Beam Physics, Lawrence Berkeley National Laboratory, Berkeley, CA 94720
†Institute for Advanced Physics, Suite 199, 10875 US Hwy 285, Conifer, CO 80433

Abstract. The nonlinear evolution of sub-ps laser pulses in underdense plasmas is analyzed for arbitrary laser intensity. Expressions for the nonlinear pump depletion, pulse steepening, and frequency shift of the laser pulse are derived. Numerical calculations based on fluid models that show the interplay between electron dephasing and pump depletion are presented. Implications for an optimized design of a 1 GeV laser-plasma-based accelerator stage are discussed.

INTRODUCTION

There are several mechanisms that can limit the effective accelerator length, and, therefore, the energy gain in a laser wakefield accelerator (LWFA) [1]. One of the most severe is laser diffraction, but this can be overcome through the use of a plasma channel to guide the laser pulse [2–5]. The two main limitations for a channel-guided LWFA are electron dephasing and pump depletion. Electron dephasing is a result of a highly relativistic electron outrunning the wakefield, which typically has a phase velocity $< c$. In the low intensity (or linear) limit, $a_0^2 \ll 1$, the dephasing length is $L_{\text{deph}} \sim \lambda_p^3/\lambda_0^2$, assuming an axially uniform, wide plasma channel, where $\lambda_p$ is the plasma wavelength, $\lambda_0$ is the laser wavelength, and $a_0$ is the normalized peak vector potential of the laser field ($a_0^2$ is the normalized laser intensity). In the high intensity (or nonlinear) limit, $a_0^2 \gg 1$, the dephasing length is $L_{\text{deph}} \sim (\lambda_p^3/\lambda_0^2)a_0$, where a constant of order unity has been neglected. The dephasing limitation could in principle be overcome by staging the laser-plasma accelerator such that, after outrunning the plasma wave, the electron is injected into a new plasma wave at the appropriate phase.

The laser pulse excites a wake as it propagates. Wake excitation leads to loss of laser pulse energy. The pump depletion length (the distance over which the pump loses a significant fraction of its energy) can be estimated by equating the laser pulse energy to the energy left behind in the wake [6, 7]. In the linear limit ($a_0^2 \ll 1$), the pump depletion length is $L_{\text{pd}} \sim (\lambda_p^3/\lambda_0^2)a_0^{-2}$, whereas in the nonlinear limit ($a_0^2 \gg 1$), $L_{\text{pd}} \sim (\lambda_p^3/\lambda_0^2)a_0$. Once pump depletion occurs, staging with a fresh pump pulse is necessary.

Of particular interest is the ideal single-stage energy gain for a channel-guided LWFA assuming an axially uniform, wide plasma channel. For an optimized flat-top pump pulse, the accelerating field is given by $E_{\text{max}}/E_0 = (a_0^2/2)^{-1/2}$, where $E_{\text{max}}$ is the maximum electric field amplitude of the wake behind the laser pulse and $eE_0 = 2\pi mc^2/\lambda_p$. The ideal energy gain $W$ is given by multiplying $E_{\text{max}}$ by the acceleration
length, i.e., either $L_{\text{deph}}$ or $L_{\text{pd}}$. In the linear limit ($a_0^2 \ll 1$), $L_{\text{deph}} \ll L_{\text{pd}}$ and $W \sim mc^2(\lambda_p^2/\lambda_0^2)a_0^2$. In the nonlinear limit ($a_0^2 \gg 1$), $L_{\text{deph}} \sim L_{\text{pd}}$ and $W \sim mc^2(\lambda_p^2/\lambda_0^2)a_0^2$. Note that the nonlinear (or mildly nonlinear regime) is advantageous, since $L_{\text{deph}} \sim L_{\text{pd}}$ which implies efficient use of the pump laser pulse energy in a single stage. Furthermore, the energy gain and accelerating gradients are higher compared to the linear regime. Also note that in the linear regime, even if dephasing is overcome (by staging or density tapering [8–10]) such that the acceleration length is limited by depletion, the energy gain would then scale as $W \sim mc^2(\lambda_p^2/\lambda_0^2)$ for $a_0^2 \ll 1$, i.e., a smaller energy gain, a smaller acceleration gradient, and a longer acceleration length compared to the nonlinear regime.

The remainder of this paper discusses the nonlinear evolution of the laser pulse and presents scalings for the pump depletion lengths valid for relativistic intensities ($a_0 \gtrsim 1$). Numerical calculations based on fluid models are presented, showing the interplay between dephasing and depletion, and the implications for the design of a 1 GeV single-stage, channel-guided LWFA are discussed.

**LASER PULSE EVOLUTION**

The nonlinear evolution of a short laser pulse in an underdense plasma in the 1D, quasi-static limit is described by the coupled equations

$$2 \left(ik_0 + \partial_\zeta\right) \partial_\zeta \hat{a} = k_p^2 \hat{a}/(1 + \phi),$$

(1)

$$\partial_\zeta^2 \phi = \left(k_p^2/2\right)\left[\gamma_\perp^2(1 + \phi)^{-2} - 1\right],$$

(2)

where $a = eA_\perp/mc^2$ is normalized vector potential, $a = (\hat{a}/2)\exp(ik_0 \zeta) + \text{c.c.}$, $\zeta = z - ct$ is the co-moving variable, $k_0$ is the wavenumber of the laser, $k_p = \alpha_p/c$ is the plasma wavenumber, $\phi = e\Phi/mc^2$ is the normalized scalar potential, and $\gamma_\perp^2 = 1 + |\hat{a}|^2/2$. In deriving Eqs. (1) and (2), the small term $\partial_\zeta^2 \hat{a}$ has been neglected on the left-side of the wave operator and an averaging over the fast laser frequency has been performed.

From the above model two exact global conservation relations can be obtained. The first is conservation of laser action $\partial_\zeta \mathcal{A} = 0$, where the wave action is defined as $\mathcal{A} = \int d\zeta |\hat{a}|^2$ and the wavenumber of the laser field is given by $k = k_0 + \partial_\zeta \theta$ with $\hat{a} = |\hat{a}| \exp(i\theta)$. The second is conservation of energy,

$$\partial_\zeta \mathcal{W} = -(k_p/k_0)^2(E_{\text{max}}/E_0)^2.$$  

(3)

where the normalized laser pulse energy is defined as

$$\mathcal{W} = \int d\zeta \left|(i + k_0^{-1} \partial_\zeta) \hat{a}\right|^2.$$  

(4)

Equation (3) describes laser energy depletion into a plasma wave. For a flat-top laser pulse of optimal length, $E_{\text{max}}/E_0 = (\gamma_\perp^2 - 1)/\gamma_\perp$.

These two global conservation relations can be linearized to determine the initial evolution of the laser energy, $\delta \mathcal{W}$, the wavenumber shift $\delta k(z, \zeta) = \partial_\zeta \theta(z, \zeta)$ averaged
over the pulse profile, and the normalized laser intensity perturbation \( \delta a^2(z, \zeta) = |\dot{a}|^2 - a_0^2(\zeta) \) averaged over the pulse profile, i.e.,

\[
\delta \mathcal{W} = -\frac{z}{L_{pd}}, \quad (5)
\]

\[
\delta k = \int d\zeta a_0^2(\delta k / k_0) = -\frac{z}{L_{pd}}, \quad (6)
\]

\[
\delta a^2 = \int d\zeta a_0^2(\delta a^2 / a_0^2) = \frac{z}{L_{pd}}, \quad (7)
\]

where \( L_{pd} = (k_0/k_p)^2(E_0/E_{max})^2 \) is the characteristic pump depletion length. Equation (5) describes the laser pump depletion, Eq. (6) describes the initial red-shifting of the laser frequency, and Eq. (7) describes the initial steepening of the laser pulse intensity. For a flat-top laser pulse of optimal length, the pump depletion length is

\[
L_{pd} = \frac{\lambda_p^3}{\lambda_0^2} \begin{cases} \frac{2}{a_0^2}, & \text{for } a_0^2 \ll 1, \\ \sqrt{\frac{2}{\pi}} a_0, & \text{for } a_0^2 \gg 1. \end{cases} \quad (8)
\]

The dephasing length can be estimated by deriving approximate analytical solutions to Eqs. (1) and (2). Instead of presenting this rather lengthy analytical calculation, results on electron dephasing and the interplay with pump depletion, studied via numerical solutions to the nonlinear 1D cold fluid equations [11], will be presented.

**NUMERICAL SOLUTIONS TO THE COLD FLUID EQUATIONS**

The self-consistent evolution of the laser pulse and plasma wake over long propagation distances \( z \sim L_{pd} \) is studied via numerical solutions to the nonlinear, 1D cold fluid equations [11]. Figure 1 shows the evolution of the wake electric field and Fig. 2 shows the evolution of the laser driver. The sharp increase in the wake amplitude near \( \omega_p t = 5000 \) is due to the compression, and accompanying amplitude increase (an increase in laser intensity and frequency red-shifting), of the driver. Subsequently the laser pulse lengthens dramatically and the wake amplitude falls sharply. These phenomena limit the overall energy efficiency of the acceleration. Once the pulse begins to lengthen, the energy remaining in the pulse is only weakly coupled to the plasma (compared with the original resonant coupling), and excessively long propagation distances would be required to obtain additional acceleration in this regime. The spectrum of the laser pulse is shown in Fig. 3. In addition to the formation of sidebands (related to the envelope distortions), red-shifting due to depletion is clearly evident.

The slope of the plasma wave phase fronts in Fig. 1 is a measure of the wake phase velocity \( v_p \) (a vertical line corresponds to \( v_p = c \)). The decrease in phase velocity is due to the increase in the nonlinear plasma wavelength as the wake grows due to self-steepening of the laser pulse (\( a^2 \) increasing) during the depletion/evolution process. The dramatic increase in phase velocity for late times is due to a drastic reduction in
FIGURE 1. Plasma wave electric field \( (E/E_0) \) excited by a Gaussian laser pulse with \( a_0 = 2, k_p L = 2 \), and \( k_0 = 10 k_p \), versus propagation time \( (\omega_p t) \) and wake phase \( (k_p (ct - z)) \).

FIGURE 2. Evolution of the laser pulse envelope for the parameters of Fig. 1. The steepening of the pulse envelope is responsible for the increase in wake amplitude near \( \omega_p t = 5000 \). The rapid lengthening of the pulse for \( \omega_p t > 5000 \) corresponds to the dramatic drop in wake amplitude seen in Fig. 1.

...wake amplitude (reduction in nonlinear plasma wavelength), since the depleted/distorted pulse no longer effectively couples to the plasma and cannot drive the wake to large amplitudes. Figure 4 shows the energy gain for a \( v = c \) particle at fixed phase \( \psi = k_p (ct - z) \). The qualitative change in the wake structure coincides with termination of acceleration. Note that the single-stage energy gain can be large, well in excess of 1 GeV.
FIGURE 3. Evolution of the spectrum of the laser pulse of Fig. 2.

FIGURE 4. Energy gain (or loss), neglecting beam-loading, for a $v = c$ particle at fixed phase $\psi = k_p(ct - z)$ in the wake field of Fig. 1.

for the parameters of Fig. 4.

Figure 5 shows the laser energy depletion length versus $a_0^2$ for depletion of 50%, 25%, 20%, 10%, 5%, and 1% of the initial laser energy. Figure 6 shows the distance (dephasing length, $L_{\text{deph}}$) at which a $v = c$ particle dephases from the wake field. The dephasing length was calculated for the initial phase that led to greatest energy gain.

The interplay between dephasing and depletion is summarized in Fig. 7, which shows the fraction of laser energy remaining at a propagation distance of $L_{\text{deph}}$. Figure 7 clearly
FIGURE 5. Laser propagation distance to deplete 50%, 25%, 20%, 10%, 5%, and 1% of the laser energy versus initial laser intensity $a_0^2$ for a Gaussian laser pulse with $k_p L = 2$ and $k_0 = 10k_p$.

FIGURE 6. Distance at which a $\nu = c$ particle dephases from the wake versus $a_0$ for several values of $k = k_0/k_p$. This length is shown for the initial phase $\psi = k_p(ct - z)$ that results in largest energy gain.

shows that for $a_0^2 \ll 1$, dephasing occurs before pump depletion (the laser pulse has lost very little energy), whereas for $a_0^2 \geq 1$, the laser pulse has lost a significant portion of its initial energy at the point of dephasing, implying an efficient single-stage configuration.

CONCLUSION

The trade off and interplay between pump depletion and electron dephasing has been examined in the limit of a wide, axially uniform plasma channel. Of particular importance are the implications for the design of an optimized single-stage of a channel-guided
FIGURE 7. Fraction of remaining laser pulse energy after propagating the dephasing length of Fig. 6.

LWFA. In the linear limit \((a_0^2 \ll 1)\), \(L_{\text{deph}} \sim (\lambda_p^3 / \lambda_0^2)\), \(L_{\text{pd}} \sim (\lambda_p^3 / \lambda_0^2) a_0^{-2}\) \((L_{\text{deph}} \ll L_{\text{pd}})\) and \(W \sim mc^2 (\lambda_p^2 / \lambda_0^2) a_0^2\) for a single-stage. Even if the dephasing limit could be overcome, through density tapering or multiple stages, the ideal energy gain as limited by pump depletion would be \(W \sim mc^2 (\lambda_p^2 / \lambda_0^2)\) for \(a_0^2 \ll 1\). In the nonlinear limit \((a_0^2 \gg 1)\), \(L_{\text{deph}} \sim L_{\text{pd}} \sim (\lambda_p^3 / \lambda_0^2) a_0\) and the single-stage energy gain is \(W \sim mc^2 (\lambda_p^2 / \lambda_0^2) a_0^2\). These approximate scaling laws, as well as the detailed numerical calculations presented above, indicate that the energy gain and accelerating gradient are higher for the nonlinear regime, compared to the linear regime for both the untapered (i.e., uniform density) or tapered (or multi-staged) cases.

ACKNOWLEDGMENTS

This work was performed under the auspices the U.S. Department of Energy, Office of High Energy Physics, under contract number DE-AC-03-76SF0098.

REFERENCES