Laser Ponderomotive Electron-Positron Collider

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Abstract. Relativistic ultrahigh laser fields can produce plasmas through quantum mechanical tunneling ionization mechanism, and accelerate produced electrons and ions to generate a relativistic electron beam and energetic ions in plasmas. This process will be followed by creation of electron-positron pairs through interaction of relativistic electrons with a Coulomb field of a nucleus in plasma ions or a strong laser field. In a relativistic strong laser field, the longitudinal accelerating force exerted on an electron is proportional to the square of the electric field, whereas the transverse quivering force is just linearly proportional to it. This is essence of the relativistic ponderomotive acceleration that dominantly produces energetic particles in interaction of ultraintense laser fields with particle beams and plasma. Therefore a tightly focused laser field can accelerate an electron-positron bunch longitudinally up to a remarkable energy and at the same time confines it transversely in the superposed ponderomotive potential of an intense ultrashort laser pulse. Here we propose acceleration and focusing of the electron-positron pair beam by the ponderomotive acceleration scheme to compose a high energy electron-positron collider with very high luminosity.

INTRODUCTION

The recent progress of ultraintense lasers makes it possible to conceive a novel concept on production and acceleration of an intense electron-positron pair beam, and its application to an electron-positron collider. The strong laser field can produce plasmas through quantum mechanical tunneling ionization mechanism, and accelerate produced electrons and ions to generate a relativistic electron beam and energetic ions in plasmas. This process will be followed by creation of electron-positron pairs through interaction of relativistic electrons with a Coulomb field of a nucleus in plasma ions or a strong laser field. In the ultraintense laser intensities more than $10^{21}$ W/cm$^2$, the pair-production rate rises quickly to enormous values. Since the pair-production occurs in the presence of the laser field and the electrostatic field generated by an ultraintense laser pulse, the produced pairs will be accelerated by the coherent action of those fields to form a relativistic beam. This pair-beam will be useful for applications to high energy collider physics as an electron-positron beam source if it can be accelerated to a very high energy and focused to a very small spot size.

In this paper, the possible pair-production processes in strong laser-plasma interactions are investigated to estimate the number of electron-positron pairs in terms of the laser intensity and the plasma density. We propose acceleration and focusing of the pair beam by the ponderomotive acceleration scheme to compose a high energy electron-positron collider with very high luminosity.
PAIR PRODUCTION PROCESSES IN LASER FIELDS

Pair creation by relativistic electrons in a nuclear field

The creation of an electron-positron pair in the vicinity of a nucleus with charge $Z$ is a process of extremely large multiphoton order, given by $n \omega L + Z \rightarrow e^+ e^-$, where a very large number of photons of the order of $n \sim 2m_e c^2 / \hbar \omega_L$ must be absorbed for any laser frequency $\omega_L$ to create a pair. The pair creation rate per nucleus is expected to be of order $W \sim \exp[-m_e c^2 / \hbar \omega_L] \sim \exp[-10^6]$ sec$^{-1}$. The cross section for this process is so small at optical frequencies as to make it completely negligible in laser-plasma interactions.

Focused laser pulses produce plasmas in matter. Electrons can be accelerated to relativistic energy by electrostatic wakefields collectively generated by intense short laser pulses in plasmas or by direct laser fields as the quiver motion of electrons becomes relativistic. When the incident electron kinetic energy exceeds the pair-production threshold $2m_e c^2$, the high energy electron can produce an electron-positron pair by scattering in the Coulomb potential of a nucleus in the process, often called "trident process".

\[ e + Z \rightarrow e' e^+ e^- \]  

The cross section for the trident pair-production process is first calculated by Bhabha\[2\].

The more exact calculation can be approximated over most of the energy range near the trident production threshold as\[3\]

\[ \sigma_T \approx 9.6 \times 10^{-4} (\alpha r_e Z)^2 (\gamma - 3)^{3.6}, \]

where $\alpha$ is the fine structure constant, $r_e = e^2 / m_e c^2$ is the classical electron radius, $Z$ is the nuclear charge, and $\gamma$ is the Lorentz factor of the electron.

Pair creation by relativistic electrons in a strong laser field

The multiphoton Breit-Wheeler process\[4\] is considered as the trident process

\[ e + n \omega_L \rightarrow e' e^+ e^- \]  

In this process an incoming high energy electron emits a virtual photon that decays into an electron-positron pair in the presence of the absorption of $n$ laser photons. From the energy-momentum conservation for the trident process, the threshold electron Lorentz factor for a head-on collision between laser and electron is given by

\[ \gamma > \frac{2m_e c^2}{n \hbar \omega_L} \left( 1 + \frac{(e^2 A^2)}{m_e^2 c^4} \right), \]

where $A$ is the vector potential of a laser field\[5\].

Schwinger\[6\] predicted that the spontaneous breakdown of vacuum occurs in a strong static electric field at the critical value, $E_c = m_e^2 c^3 / e \hbar = 1.32 \times 10^{16}$ V/cm. The pair
production in vacuum in the presence of a strong laser field has been discussed by Brezin and Itzykson[7] using the normalized vector potential of the laser field, \( a_0 = eE_{\text{rms}}/\omega_0 m_e c \), where \( E_{\text{rms}} \) is the root mean-square electric field and \( \omega_0 \) is the laser angular frequency. The pair production probabilities per unit time-unit volume are derived for the static (zero frequency) regime, \( a_0 \gg 1 \),

\[
w \simeq \frac{\alpha E^2}{\pi \hbar} \exp \left( -\frac{\pi m_e^2 c^3}{e\hbar E} \right). \tag{5}\]

In the static regime the behavior of the rate can be understood as a quantum-mechanical tunneling. This is analogous to ionization, where a pair is bound in vacuum with binding energy \( V_0 \sim 2m_e c^2 \).

**PRODUCTION OF A HIGH ENERGY ULTRASHORT INTENSE ELECTRON BEAM IN PLASMAS**

In order to produce electron-positron pairs in plasma via the trident process in the presence of either nuclear charge fields or laser fields, initially plasma electrons must be accelerated up to relativistic energy for both cases. The recent laser-plasma interaction experiments have demonstrated relativistic electron acceleration exceeding \( > 200 \text{ MeV} \) for the laser strength parameter \( a_0 \sim 1 \)[8, 9], where \( a_0 \equiv 0.855 \times 10^{-9} \lambda_0 [\mu \text{m}] 1^{1/2} [\text{W/cm}^2] \), \( \lambda_0 \) the laser wavelength and \( I \) the laser intensity.

Here we estimate the energy of electrons accelerated by ultraintense laser pulses in plasmas. The final accelerated energy for initially stationary plasma electrons is given by \( \gamma = (2\gamma^2_g - 1) \gamma_L \), where \( \gamma_g = (1 - \beta^2_g)^{-1/2}, \beta_g = v_g/c, v_g = c(1 - \omega^2_p/\omega_0^2)^{1/2}, \omega_p = (4\pi n_e e^2/m_e)^{1/2} \) the electron plasma frequency in the ambient electron plasma density \( n_e \), and \( \gamma_L = (1 + a_0^2/2)^{1/2} \). The dispersion relation of relativistically strong electromagnetic waves is \( \omega^2 = k^2 c^2 + \omega^2_p/\gamma_L \). This gives a group velocity of the intense laser pulses: \( \beta_g = (1 - \omega^2_p/\gamma_L \omega_0^2)^{1/2} = (1 - n_e/\gamma_L n_c)^{1/2} \), where \( n_c = \pi/(e\lambda^2 \gamma_L) \) is the critical plasma density. It implies that the laser pulses can propagate overdense plasmas for \( n_e < \gamma_L n_c \). This corresponds to the relativistic transparency of overdense plasmas. As \( \gamma_g = \omega_0 \sqrt{\gamma_L}/\omega_p = (\gamma_L n_c/n_e)^{1/2}, \) the final energy is given by

\[
\gamma = 2\gamma^2 L n_c/n_e - \gamma_L \approx a_0^2 n_c/n_e. \tag{6}\]

The density \( n_1 \) of accelerated electrons can be calculated as

\[
n_b = n_e \beta_g/|\beta - \beta_g| = (2\gamma^2 - 1)n_e = 2\gamma n_c - n_e \approx \sqrt{2} a_0 n_c \tag{7}\]

Finally the intense laser pulse propagating the plasma with thickness \( \Delta \) produces an electron beam with bunch length,

\[
l_b = \Delta |\beta - \beta_g|/\beta_g = \Delta (2\gamma^2 - 1) \approx n_e \Delta/(2\gamma_L n_c) \approx n_e \Delta/(\sqrt{2} a_0 n_c). \tag{8}\]

As an example, the intense laser pulse of the wavelength \( \lambda_0 = 0.8 \mu \text{m} \) with the intensity \( I = 2.1 \times 10^{20} \text{ W/cm}^2 (a_0 = 10) \) can accelerate electrons up to the energy of 1.6 GeV in
a plasma of the electron density \( n_e = 5.3 \times 10^{19} \text{ cm}^{-3} \). An electron beam produced from a plasma with thickness \( \Delta = 100 \mu\text{m} \) is compressed to the bunch length \( l_b = 0.2 \mu\text{m} \) (700 attoseconds) with density of \( n_b = 2.4 \times 10^{22} \). The results of the particle-in-cell simulation can show such high energy high intensity electron beam production in plasmas[10].

PAIR-BEAM PRODUCTION YIELD IN PLASMAS

A trident process in the nuclear field

The pair creation rate by means of the trident process in a volume of which the characteristic length \( l \lambda_0 \) is

\[
\frac{dN_p}{dt} = (l \lambda_0)^3 n_i n_e \sigma_T v_e, \tag{9}
\]

where \( n_i \) is the ion (nucleus) density, \( n_e \) is the electron density and \( v_e = \beta c = (c/\gamma)(\gamma^2 - 1)^{1/2} \) is the velocity of the electron. In a plasma containing of charge \( Z_i = n_e/n_i \), substituting Eq. (6) into the electron energy \( \gamma \gg 3 \), the pair production yield is given by

\[
N_{pair} \approx 0.48 \pi^3 \frac{\alpha^2 \omega_0^2 Z^3}{10^3} \left( \frac{n_e}{n_i} \right)^2 \left( \frac{r_0}{\lambda_0} \right)^2 \left( \frac{\Delta}{\lambda_0} \right)^2 \tag{10}
\]

\[
\approx 2.8 \times 10^{-45} Z^3 I^4 \text{[W/cm}^2\text{]} n_e^{-2} \text{[cm}^{-3}\text{]} r_0^2 \text{[\mu m]} \Delta^2 \text{[\mu m]}, \tag{11}
\]

where \( r_0 \) is the laser spot radius and \( \Delta \) is the plasma thickness. As an example, when a laser pulse with \( I = 1 \times 10^{22} \text{ W/cm}^2 \) focused on the spot size of \( r_0 = 10 \mu\text{m} \) is propagating a thickness of \( \Delta = 100 \mu\text{m} \) in the Xe (\( Z = 54 \)) plasma with density of \( n_e = 1 \times 10^{20} \text{ W/cm}^3 \), the number of pairs produced is \( N_{pair} \approx 4.4 \times 10^{14} \).

A trident process in the counter-propagating laser field

For a head-on collision between an electron and a laser field, a nonzero Lorentz and gauge invariant parameter can be formed as[5]

\[
\chi = 2 \gamma \frac{e \hbar E_L}{m_e c^3} = \frac{2 \gamma E_L}{E_c} = \frac{\hbar \omega_0}{m_e c^2} = 2 \gamma a_0 \frac{\lambda_c}{\lambda_0}, \tag{12}
\]

where \( \lambda_c/2\pi = \hbar/m_e c \approx 3.86 \times 10^{-11} \text{ cm} \) is the Compton wavelength of the electron. Let us consider the \( N_e \) electrons with energy \( \gamma m_e c^2 \) crossing the laser field with \( E_L \). Integrating the probability over volume \( \Delta V \) and time \( \Delta t \) for each electron crossing, i.e. \( \Delta V = (\lambda_c/2\pi)^3 \) and \( \Delta t = \tau_L/2\gamma \) for the time of interaction with laser pulse of duration \( \tau_L \) in the electron rest frame, assuming that a pulse length is smaller than the Rayleigh
length. The number of pairs produced per laser shot is given by
\[
N_{\text{pair}} \approx \frac{8\pi^3 a_0^4}{\alpha} \left( \frac{r_0^2 \Delta}{\lambda_0^3} \right) \left( \frac{c \tau_L}{\lambda_0} \right) \exp \left[ -\frac{\pi}{\chi} \right],
\]
\[
\approx 5 \times 10^{-33} I^2 [\text{W/cm}^2] r_0^2 [\mu\text{m}] \Delta [\mu\text{m}] \tau_L [\text{fs}] \exp[-\pi/\chi]. \tag{13}
\]
where the invariant parameter is
\[
\chi \approx \frac{4\pi^2}{\alpha} a_0^3 n_e \lambda_0 \approx 3.38 \times 10^{-12} I^{3/2} [\text{W/cm}^2] n_e [\text{cm}^{-3}]. \tag{14}
\]
As an example, when a laser pulse with \( I = 1 \times 10^{22} \text{ W/cm}^2 \) and \( \tau_L = 20 \text{ fs} \) focused on the spot size of \( r_0 = 10 \mu\text{m} \) is propagating a thickness of \( \Delta = 100 \mu\text{m} \) in plasma of density \( n_e = 1 \times 10^{20} \text{ cm}^{-3} \), the number of pairs produced is \( N_{\text{pair}} \approx 9 \times 10^{16} \) with \( \chi \approx 34 \).

**LASER ELECTRON-POSITRON COLLIDER**

**Relativistic ponderomotive acceleration and focusing of a pair beam**

High energy booster acceleration of a pair-beam can be accomplished by the relativistic ponderomotive acceleration with focusing in vacuum or tenuous plasma. In the ponderomotive acceleration\[11\], the final energy is obtained approximately by \( \gamma_f \approx a_0^2 \) for a particle initially at rest. The accelerated final energy is written as
\[
E_f [\text{GeV}] \approx 0.37 \times 10^{-21} I [\text{W/cm}^2] \lambda_0^3 [\mu\text{m}]. \tag{15}
\]
As an example, the laser intensity \( I = 1 \times 10^{24} \text{ W/cm}^2 \) of \( \lambda_0 = 0.8 \mu\text{m} \) can accelerate the electron beam up to 240 GeV.

The focusing of an electron beam will be accomplished by the higher order Hermite-Gaussian modes. The focusing force is obtained from the ponderomotive potential \( U \) as \( F_r/m_e c^2 = \partial U/\partial r \). In the fundamental Hermite-Gaussian mode referred to as a Gaussian mode, the ponderomotive potential propagating in vacuum is given by
\[
U_0(r,z,t) = a_0^2 \sigma_{\perp 0}^2 \exp \left[ -\frac{r^2}{2\sigma_{\perp 0}^2} \right] \exp \left[ \frac{-(z-ct)^2}{2\sigma_z^2} \right], \tag{16}
\]
where \( \sigma_{\perp 0} \) is the rms spot size at \( z = 0 \), \( \sigma_{\perp} = \sigma_{\perp 0} \sqrt{1+z^2/Z_R^2} \) the rms spot size at \( z \), \( Z_R \) the Rayleigh length, and \( \sigma_z \) the rms laser pulse length. Since \( \partial U_0/\partial r < 0 \), the ponderomotive potential of a Gaussian mode exerts defocusing forces on off-axis particles that are quickly expelled from the laser beam in the radial direction. The focusing force can be produced by superposition of a Gaussian mode and higher order modes of which the ponderomotive potential creates a potential well in the radial direction\[12\]:
\[
U_1(r,z,t) = a_1^2 r^2 \sigma_{\perp 0}^2 \exp \left[ -\frac{r^2}{2\sigma_{\perp 0}^2} \right] \exp \left[ \frac{-(z-ct)^2}{2\sigma_z^2} \right], \tag{17}
\]
where \( a_1 \) is the dimension less vector potential of the first order mode. The focusing strength at \( r = 0 \), and \( z - ct = 0 \) is

\[
K_F = \frac{F_r}{\gamma mc^2} = \frac{\partial U}{\gamma r \partial r} = \frac{2a_1^2 - a_0^2}{\gamma \sigma^2 \perp},
\]

(18)

where \( U = U_0 + U_1 \) is the total ponderomotive potential. Then the beam envelope equation on the rms beam radius \( \sigma_{rb} \) is written as

\[
\frac{d^2 \sigma_{rb}}{dz^2} + K_F \sigma_{rb} - \frac{r_e N_b}{\sqrt{2\pi} \beta^2 \gamma^3 \sigma_{rb} \sigma_{zb}} - \frac{\varepsilon_b^2}{\sigma_{rb}^3} = 0,
\]

(19)

where \( N_b \) is the number of electrons in the bunch, \( \sigma_{zb} \) the rms bunch length, \( \varepsilon_b = \varepsilon/\gamma \beta \) the geometrical beam emittance, and \( \varepsilon_n \) is the normalized beam emittance. In this equation the third term and the fourth term are attributed to a space charge force and the thermal emittance, respectively. The equilibrium beam radius is obtained from \( d^2 \sigma_{rb}/dz^2 = 0 \).

First let us consider an equilibrium beam size of the electrons or the positrons focused by the laser ponderomotive potential well in the case of the radial expansion of the beam due to the space charge force. Assuming \( a_1 = a_0 \), \( \sigma_{b\perp} = r_0/2 \), and \( \sigma_{zb} \approx \lambda_0/2\pi \), the equilibrium beam size is given by

\[
\sigma_{rb}[\text{pm}] \approx \frac{(2\pi)^{1/4} r_0}{2a_0^{5/2}} \sqrt{\frac{r_e N}{\lambda_0}} \approx \frac{2 \times 10^{24} \sqrt{N}}{I^{5/4}[\text{W/cm}^2]} \frac{r_0[\mu\text{m}]}{\lambda_0[\mu\text{m}]}.
\]

(20)

As an example, for \( N = 1 \times 10^{10} \), \( \lambda_0 = 0.8 \mu\text{m} \), the laser pulse of the peak intensity of \( I = 1 \times 10^{22} \text{W/cm}^2 \) can focus the spot radius to \( \sigma_{rb} \approx 1.2 \text{nm} \).

If an electron-positron pair beam is focused, the space charge force will be neglected. The focused beam size can be limited by equilibrium between the ponderomotive focusing and the thermal emittance expansion. Assuming \( a_1 = a_0 \), \( \sigma_{b\perp} = r_0/2 \), \( \varepsilon_b \approx \varepsilon_n/\gamma \approx \lambda_0/(2\pi a_0^3) \), an estimate of the focused beam size is

\[
\sigma_{rb}[\text{pm}] \approx \frac{\sqrt{\varepsilon_n \sigma_{b\perp}}}{a_0} = \frac{1}{2a_0^3} \sqrt{\frac{\lambda_0 r_0}{\pi}} \approx \frac{4 \times 10^{23}}{I[\text{W/cm}^2]} \sqrt{\frac{r_0}{\lambda_0}}.
\]

(21)

As an example, for \( \lambda_0 = 0.8 \mu\text{m} \) and \( r_0 = 10 \mu\text{m} \), the laser pulse of the peak intensity of \( I = 1 \times 10^{22} \text{W/cm}^2 \) can focus the spot radius to \( \sigma_{rb} \approx 0.14 \text{nm} \).

**Luminosity of an electron-positron collider**

It is conceivable that two counter propagating laser-accelerated beams make it possible to produce the \( e^+e^- \), \( e^-e^- \), and \( e^+e^+ \) high energy collisions. The colliding beam energy is given by Eq. (15). We can estimate the collision luminosity, \( L = N^2 f_{rep}/4\pi \sigma_{pb}^2 \) at the repetition frequency of the colliding laser pulses for the new concept collider.
from above discussions on the accelerated energy and the focused beam size due to the ponderomotive acceleration mechanism.

In the space charge limited case, the collider luminosity will be given by

$$L[\text{cm}^{-2}\text{s}^{-1}] \approx \frac{a_0^5 \lambda_0 N f_{rep}}{\sqrt{2 \pi^{3/2}} r_e r_0^2} \approx 2 \times 10^{-30} I^{5/2}[\text{W/cm}^2] \lambda_0^6[\mu\text{m}] r_0^{-2}[\mu\text{m}] N f_{rep}[\text{Hz}]. \quad (22)$$

In the emittance-limited case, where the electron-positron pair beams are collided with no separation, the luminosity results in

$$L[\text{cm}^{-2}\text{s}^{-1}] \approx \frac{a_0^4 \lambda_0^2 N^2 f_{rep}}{r_0 \lambda_0} \approx 5.3 \times 10^{-27} I^2[\text{W/cm}^2] \lambda_0^3[\mu\text{m}] r_0^{-1}[\mu\text{m}] N^2 f_{rep}[\text{Hz}]. \quad (23)$$

As an example, in order to accelerate the pair beams to the center-of-mass collision energy of 10 GeV, the laser intensity of $I = 2.1 \times 10^{22} \text{W/cm}^2$ is required. For $N = 1 \times 10^{10}$, $\lambda_0 = 0.8 \mu\text{m}$, $r_0 = 10 \mu\text{m}$, and $f_{rep} = 10 \text{Hz}$, the space-charge limited luminosity is $3.35 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ and the emittance limited luminosity becomes $1.2 \times 10^{38} \text{cm}^{-2}\text{s}^{-1}$. This is four orders of magnitude higher than the conventional B factories.

**CONCLUSIONS**

The pair-production processes in ultra-strong laser-plasma interactions have been investigated to estimate the number of electron-positron pairs in terms of the laser intensity and the plasma density. Since the pair-production occurs in the presence of the laser field and the electrostatic field generated by an ultraintense laser pulse, the produced pairs will be accelerated by the coherent action of those fields to form a relativistic beam. This pair-beam will be useful for applications to high energy collider physics as an electron-positron beam source if it can be accelerated to a very high energy and focused to a very small spot size by the ponderomotive acceleration mechanism. We propose a new concept of a high energy, high luminosity electron-positron collider driven by the ultra-strong lasers in a micro-scale size.

**REFERENCES**