New Developments on PBG RF Cavities*

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Abstract. Performance and design features of metal PBG and rod-loaded cavities for single-beam and multi-beam acceleration and rf power generation devices are considered. Fundamental differences of the performance between single-defect and multi-defect structures are identified. Rod-loaded cavity designs are considered for a 6-beam klystron. Preliminary design of the X-band MBK demonstrates feasibility of generating high power with high efficiency in a very compact construction.

INTRODUCTION

We consider generalized rod-loaded configurations in a new class of microwave structures having a 2D periodic boundary (lattice) in the transverse dimensions. For a single-beam accelerator application, Photonic Band Gap (PBG) structures have been studied by several research groups at UCSD, MIT and SLAC [1,2,3,4,5]. N. Kroll proposed damping unwanted HOMs in a single-defect PBG structure loaded by dielectric rods [1]. Theoretical studies of an infinite 2D lattice are based on Floquet theorem and qualitative characterization of the PBG effect with Brillouin diagrams, Bloch waves and zones. Using this approach, the remarkable band gap effect was demonstrated for infinite square lattices ([1]-for dielectric rods, [2,6]-for metal rods) and triangular ones ([1]-for dielectric rods, [7]-for metal rods). Multi-beam and flat-field (for ribbon beam) rod-loaded structures have been proposed by DULY Research Inc. [8,9] (patents are pending).

SINGLE-DEFECT BAND GAP EFFECT

Practical applications require quantitative characterization of the PBG effect in a finite (terminated) structure. Experimental results are available for scattering parameters vs. frequency (e.g. [1]-for dielectric rods, [6]-for metal rods). Earlier numerical studies of the PBG effect were performed in the time-domain calculations of the long-range wakefield in a 7x7 square lattice with metal rods [10]. The key parameter for band gap effect characterization and structure design is Q-factor for HOMs and fundamental mode. To simplify the modeling of absorbing periphery boundary we assumed low conductivity (eight orders less than copper) of the periphery wall and neglected mode structure change caused by such a low conductivity. It is valid for small field perturbation with at least two or more of rod layers around the defect. Otherwise, a realistic absorber with finite thickness still can be used in this model. Frequency domain GdfidL code [11] was used to calculate the

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modal Q-factors influenced by this reduction of conductivity of the cylindrical wall. We consider triangular lattice with fixed number of rods (36), rod radius \( a = 1.6 \) mm and variable lattice spacing \( b = \text{var} \). Up to 250 eigenmodes were computed. In Figure 1 we plotted the ratio of Q-factors for damped and all-copper cavity for TM\(_{010}\) mode \( Q/Q_{\text{Cu}} \), for dipole mode \( Q_{110}/Q_{110\text{Cu}} \), and frequency ratio between the two modes \( f_{110}/f_{010} \).

One can see the Q-factor of the dipole mode is reduced by a factor of \(~30\) for \( 0.139 < a/b < 0.184 \), whereas the fundamental mode Q-factor remains high and changes insignificantly (\(<10\%\)). The dipole mode undergoes bifurcation at \( a/b = 0.21 \), and the fundamental mode has strong bifurcations at \( a/b = 0.186 \) and \( a/b = 0.296 \). In the range \( a/b > 0.213 \) the dipole mode becomes a trapped (defect) mode. The fundamental mode is no longer a defect mode at \( 0.188 < a/b < 0.291 \) and its Q-factor sharply drops by two orders in this sharp forbidden gap. It is in a good qualitative correlation with the dip observed experimentally for the transmission coefficient [1]. In other words, in the bandgap we have propagating waves from the defect to the periphery, whereas outside the gap we deal with evanescent modes leading to high reflections and forming trapped (defect) modes.

In Figure 2 we plotted normalized frequencies \( \omega b/c \), Q-factor reduction caused by damping of the retained modes, and number of retained modes as a function of the geometrical parameter \( a/b \). As a criterion for the modes to be retained we used the constraint \( Q/Q_{\text{Cu}} > 1/25 \). Figure 2 may be compared to the TM mode eigenfrequency diagram obtained for an infinite lattice [6]. Using \( Q/Q_{\text{Cu}} > 1/10 \) as a criterion for trapped modes, we obtained the condition \( a/b < 0.2 \) for providing TM\(_{010}\) single mode performance. This condition is the same as that found earlier for infinite lattice [6]. For \( Q/Q_{\text{Cu}} > 1/25 \) we have the condition \( a/b \leq 0.15 \). Along with the requirement of unchanged Q-factor of the fundamental mode it gives \( a/b = 0.15 \) which actually was adopted in the design [6].
Time-Domain Simulations of a Single-Defect Structure

To model a practical case with realistic absorbing material, an example of an X-band structure from [6] was simulated in the time domain with both Gd1 [12] and CST Microwave Studio® codes [13]. From S-curves shown in Figure 3 we found that insertion of the Eccosorb® material reduces the loaded Q-factor of the quadrupole mode by a factor of ~7.8 (from 631 to 73). This model was applied also for dipole mode transmission analysis. Enlarged waveguides (twice as wide as WR90) were used to allow propagation of the asymmetric mode. The same absorber resulted in a factor of ~10 reduction of the loaded Q-factor (from 590 to 58) calculated from the S-curves given in Figure 4. Since the excitation ports are placed on the periphery of the lattice (close to the absorber), it is difficult to determine for this setup how the absorber would change the Q-factor of the fundamental mode.

![Figure 3](image1.png)

**FIGURE 3.** S-parameter simulation of a 1-defect, 11-GHz structure coupled with two WR90 waveguides. On the left: no absorber, on the right: Eccosorb® material is placed with 3.2mm thickness, \(\varepsilon=25, \mu=1.1, \text{tg}\sigma_\varepsilon=0.2, \text{and tg}\sigma_\mu=4\). The insets show the field pattern at ~16GHz.

Another CST MWS configuration considered had no waveguide ports but a discrete excitation was placed in the defect. For the quadrupole mode, a factor of ~39-44 reduction of the Q-factor (from 5131-5618 to 131-127) was obtained using idealized absorber having \(\varepsilon=1=\mu, \text{tg}\sigma_\varepsilon=8, \text{tg}\sigma_\mu=4\) and direct calculation of Q-factor. Further increase of loss-tangents does not reduce the Q-factor. The fundamental mode Q-factor remains nearly unchanged.

![Figure 4](image2.png)

**FIGURE 4.** S-parameter simulation of a 1-defect, 11-GHz structure coupled with two waveguides of 3.16 cm width. On the left: no absorber; on the right: Eccosorb® material is inserted (the same as in Figure 3 simulation). The insets show the dipole mode field pattern (~16.9 GHz).
These results indicate that the material and geometry have to be optimized carefully to match and damp specific HOMs. Thus a dense modal spectrum in a bandgap structure [9] can be essentially rarified with a periphery absorber. The degree of this rarefaction may depend on the criterion of what is the maximum Q-factor allowed for damped modes and how it is implemented. There is a somewhat similar situation in classic open structures in which strong modal rarefaction occurs due to radiation losses into an open space [14]. In this way PBG structures may be considered as an intermediate class between closed and open ones. It can be very effective for damping long-range HOM wakefields (e.g., multi-bunch BBU), but ineffective for short-range wake suppression [9] (e.g., single-bunch BBU). Nevertheless a nearly single mode, with suppression of long-range HOM wakefields, seems still feasible for a single-defect, finite-lattice PBG structure.

MULTI-DEFECT STRUCTURES

We found earlier [9] that the 6 trapped modes in a 6-defect cavity have relatively small frequency separation (a few percentages only). Additionally, 2-defect, 3-defect (triangular lattice), and 4-defect (square lattice) structures have been analyzed. In the vicinity of $a/b \approx 0.12$ we found correspondingly 2, 3 and 4 defect modes having also small frequency separation. Such a behavior can be explained as an analogy with the system of weakly coupled mechanic oscillators [8,15].

Unlike 1-defect structure, the absorbing periphery boundary does not discriminate multipole modes in a multi-defect structure: the Q-factor for all trapped (defect) modes remains always very close to each other. For $a/b=0.12$ and $d/\lambda=0.4$ (where $d$ is the longitudinal gap) the Q-factor reduction for the dominant modes is about the same (~one order for periphery wall conductivity reduced by 8 orders). All the other higher order modes (up to 250 eigenmodes were simulated for different lattices and numbers of defects) have very low Q-factor reduced by three orders by the artificial periphery boundary. Qualitatively such a behavior is quite understandable: instead of a single defect the group of defects acts as a whole cluster of dominant (trapped) modes having therefore close frequencies and Q-factors.

As an example let us consider a 55-rod, 6-defect triangular lattice using the same approach as above. In Figure 5 we plotted the Q-factor reduction caused by an absorbing boundary for the fundamental mode, $Q_{01}/Q_{01Cu}$; and for the closest trapped higher order mode (which is usually dipole mode), $Q_{\text{trap}}/Q_{\text{Cutrap}}$; along with the frequency separation between the two modes. One can see that in the region $a/b<0.13$ the Q-factors of the two modes are very close. In Figure 6 we depicted normalized frequencies $\omega b/c$, Q-factor reduction caused by damping of the retained modes, and number of retained modes as a function of $a/b$ for a 6-defect structure. As a criterion for the modes to be retained we used the same constraint $Q/Q_{Cu}>1/25$. As seen from Figures 5,6 absorbing periphery cannot be used to damp any of the dominant (multipole) HOMs. The dominant (defect) modes are the following: a monopole mode, two dipole modes, two quadrupole modes, and a sextupole mode (see [8,9]). Both sextupole and monopole defect modes are candidates for a rod-loaded multi-beam klystron (MBK): both modes have good field uniformity between the defects, and
have about the same shunt impedance and frequency separation. Below we consider
the monopole mode for the sake of input coupler design simplicity.

<table>
<thead>
<tr>
<th>Lattice spacing b, cm</th>
<th>Q-factor ratio</th>
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<tbody>
<tr>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
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<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>1.6</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Frequency separation, %</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
</tr>
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<tbody>
<tr>
<td>df/f</td>
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**FIGURE 5.** Q-factor for damped 6-defect cavity related to all-copper Q-factor for monopole and closest HOM. Right ordinate axis: minimal frequency separation between the modes. Rod radius is a=0.79mm, longitudinal gap d=6.91mm. The inset shows a 2D view of the cavity at b=6.4mm.

The frequency separation can be improved by enhancement of the coupling between the defects. One of the methods is usage of relatively thin rods a/b≤0.07 (see Figure 3). Additional methods comprise decreasing the radius of periphery boundary and a topology of minimum distance between the defects with Rdef/b=√3 (see Figure 7) instead of Rdef/b=2 (see Figure 5 inset). These means resulted in ~5.8% frequency separation between monopole and nearest dipole defect modes. The corresponding 25-rod structure is shown in Figures 7, 8. The strong inter-defect coupling gave rise to a maximum field displacement with respect to defect center (see Figure 7), which was taken into account in 3D cavity design.

**FIGURE 6.** TM_{mn0} mode eigenfrequencies, number of dominant modes with Q/Q_{Cu}>1/25, and Q-factor reduction Q/Q_{Cu} for a 6-defect cavity loaded by 55 rods; cavity length d=6.91mm.

**FIGURE 7.** Superfish contour plot of the fundamental mode for one quarter of a 6-defect, 25-rod cavity having a topology of minimum distance between defects with a/b=0.066.

**FIGURE 8.** Gd1 model of 1/8th of the rod-loaded cavity for a 6-beam X-band klystron. Beamlet axis radius is 10.6mm, cavity radius 18.8mm, 2a=0.89mm, b=6.76mm, beam aperture radius 3mm, interaction gap=5.75mm, cavity gap d=10mm.
Conceptual Design of 6-Beam Rod-Loaded Compact Klystron

Target values of our design are: 100-150 MW power, microperveance $\mu P=0.24-0.6 \mu \text{A/V}^{3/2}$, voltage $U=340-500\text{kV}$, and $P_b=42.5\text{MW}$ power per beam $I_b=85-125\text{A}$. Input and output cavities are assumed rod-loaded. Effective field distribution between the defects [9] enables to have any idle cavity as rod-loaded one (see Figure 8), or a set of six isolated pill-box-like cavities. It does not affect significantly $R$ and $Q$ per beamlet, but enables to eliminate beam-to-beam coupling in the idle cavities.

For input coupling the natural way is a coaxial feeder connected directly to the central rod. It is capable of providing coupling coefficient $\beta_c$ ranging from 1 to 18. The output coupler has to produce $\beta_c$ in excess of $\sim 50$. To this end we increase the outer diameter of the cavity (keeping the same number of rods) and use two rectangular waveguides (see Figure 9). A schematic design in Figure 10 shows eight rod-loaded cavities with coaxial input and two-port output. It is very compact, making it easy to apply conventional solenoidal focusing.

![Figure 9](image-url)  
**FIGURE 9.** CST MWS model for outcoupling cavity excited by 6 beams. Cavity radius 26mm, $Q_e=50$.

![Figure 10](image-url)  
**FIGURE 10.** Schematic layout of 8-cavity 6-beam klystron.

To obtain close-to-maximum values of both $R/Q$ and frequency separation we have chosen $a/b=0.066$ and made preliminary optimization of a reentrant-type cavity design for a 6-beam X-band klystron. Its CAD view and geometrical parameters are given in Figure 8. The parameters computed with the code Gd1 are as follows: $f=11.4\text{ GHz}$, $Q_o=4000$, and $R/Q=22.7\Omega$ per beamlet. Another 6-beam X-band klystron design uses annular $\text{TM}_{12}; 1_0$ cavities [16]. Comparison indicates about 50% higher shunt impedance ($R$) and over $\sim 3$ times higher value $R/Q$ for rod-loaded $\text{TM}_{010}$ variant.

The klystron performance was evaluated using the 1D AJ Disk code from SLAC [17], with $R/Q$ and $Q$ values obtained above. Beamlet aperture filling factor used in simulations is 0.86. 1D dynamics is characterized in Figure 11 for $U=500\text{ kV}$, $I_b=85\text{A}$ ($\mu P=0.24$). A summary on rod-loaded klystron preliminary designs is given in Table 1. One can compare the results with the “annular” design [16] at the same anode voltage $U=340\text{kV}$, beam current $I_b=125\text{A}$, efficiency $\sim 54\%$ and aperture $\varnothing=6\text{mm}$. Our design has the following features: $\sim 4.5$ times smaller transverse dimension of the idle and input cavities (18.8 mm vs. 84 mm); shorter interaction length ($\sim 1.7$ times: $\sim 0.26$ m vs. 0.45 m); higher frequency separation (5.7% vs. 3.7% for idle cavities); no lower order modes; and higher gain (up to 72 dB vs. 56 dB).
TABLE 1. AJ Disk simulation results.

<table>
<thead>
<tr>
<th>Number of cavities N</th>
<th>Interaction length, m</th>
<th>Input cavity external Qe</th>
<th>Microperveance $\mu P$</th>
<th>Voltage U, kV</th>
<th>Efficiency $\eta$, %</th>
<th>RF output P, MW</th>
<th>Input cavity external Qe</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 – 8</td>
<td>0.2-0.3</td>
<td>~ 3100</td>
<td>0.24, 0.6</td>
<td>500, 340</td>
<td>55-63, 45-56</td>
<td>13-150, 114-129</td>
<td>73, 39-42</td>
</tr>
</tbody>
</table>

$\mu P=0.24$, U=500kV, $I_b=85A$, $P=150MW$, $\eta=60\%$

FIGURE 11 An example of AJ Disk simulation.

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REFERENCES