Low-Energy Tests of Lorentz and CPT Symmetry

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Abstract. A review is presented of some recent Lorentz and CPT tests conducted in low-energy atomic and particle systems. These systems are particularly well suited to search for unique low-energy signatures of new physics, including effects that could originate from the Planck scale. Experimental signatures of possible Lorentz and CPT violation are investigated, and bounds are discussed.

INTRODUCTION

There has been a growing interest in testing Lorentz and CPT symmetry in recent years [1]. This is due to both theoretical developments as well as improved experimental capabilities. For example, it has been shown that string theory can lead to violations of CPT and Lorentz symmetry [2]. This is because strings are nonpointlike and have nonlocal interactions. They can therefore evade the CPT theorem. In particular, there are mechanisms in string theory that can induce spontaneous breaking of Lorentz and CPT symmetry. It has also been shown that geometries with noncommutative coordinates can arise naturally in string theory [3] and that Lorentz violation is intrinsic to noncommutative field theories [4].

Experimental signals due to effects in these kinds of theories are expected at the Planck scale, $M_{\text{Pl}} = \sqrt{\hbar c/G} \simeq 10^{19} \text{ GeV}$, where particle physics meets up with gravity. This energy scale is inaccessible in accelerator experiments. However, a promising approach has been to adopt Lorentz and CPT violation as a candidate signal of new physics originating from the Planck scale. The idea is to search for effects that are heavily suppressed at ordinary energies, e.g., with suppression factors proportional to the ratio of a low-energy scale to the Planck scale. Normally, such signals would be unobservable. However, with a unique signal such as Lorentz or CPT violation (which cannot be mimicked in conventional physics) the opportunity arises to search for effects originating from the Planck scale. This approach to testing Planck-scale physics has been aided by the development of a consistent theoretical framework incorporating Lorentz and CPT violation in an extension of the standard model of particle physics [5]. In the context of this framework, it is possible to look for new signatures of Lorentz and CPT violation in atomic and particle systems that might otherwise be overlooked.

Experiments in atomic systems are particularly well suited to this approach since they are often sensitive to extremely low energies. These experiments are routinely sensitive to small frequency shifts at the level of 1 mHz or less. Expressing this as an energy shift in GeV corresponds to a sensitivity of approximately
$4 \times 10^{-27}$ GeV. Such a sensitivity is well within the range of energy one might associate with suppression factors originating from the Planck scale. For example, the fraction $m_p/M_{Pl}$ multiplying the proton mass yields an energy of approximately $10^{-19}$ GeV, while for the electron the fraction $m_e/M_{Pl}$ times the electron mass is about $2.5 \times 10^{-26}$ GeV.

Some examples of atomic and particle systems that are highly sensitive to Lorentz and CPT violation include experiments with photons [6, 7, 8], electrons [9, 10, 11, 12, 13], muons [14, 15, 16], protons [17, 18], and neutrons [19]. These examples include some of the classic tests of Lorentz and CPT symmetry, such as $g - 2$ experiments in Penning traps [20] and atomic-clock comparisons – the so-called Hughes-Drever experiments [21, 22, 23]. In addition to these examples involving leptons and baryons, there are other experiments that provide bounds on mesons [24, 25].

LORENTZ AND CPT SYMMETRY

It appears that nature is invariant under Lorentz symmetry and CPT [26]. The CPT theorem links these symmetries [27]. It states that (under mild technical assumptions) all local relativistic field theories of point particles are symmetric under CPT. A consequence of the CPT theorem is that particles and antiparticles should have exactly equal lifetimes, masses, and magnetic moments.

Many of the sharpest tests of CPT and Lorentz symmetry are made in particle and atomic systems where the predominant interactions are described by QED. For example, the Hughes-Drever type experiments typically compare two clocks or high-precision magnetometers consisting of different atomic species. The best CPT tests for leptons and baryons cited by the Particle Data Group [28] are made by atomic physicists working with Penning traps. These experiments have obtained bounds of order $2 \times 10^{-12}$ on the relative difference in the $g$-factors of electrons and positrons and of order $9 \times 10^{-11}$ on the relative difference in the charge-to-mass ratios of protons and antiprotons. In addition to these, two experiments at CERN intend to make high-precision spectroscopic comparisons of trapped hydrogen and antihydrogen [29]. One possibility is to compare $1S-2S$ transitions in hydrogen and antihydrogen. These are forbidden transitions and can only occur as two-photon transitions. They have a small relative linewidth of approximately $10^{-15}$. High precision comparisons of these and other transitions in hydrogen and antihydrogen will yield sharp new CPT bounds.

It is interesting to note that of all the experiments testing Lorentz and CPT in matter it is the atomic experiments which have the highest experimental precisions (as opposed to sensitivity). For example, in neutral meson experiments quantities are measured with precisions of approximately $10^{-3}$, while in atomic experiments frequencies are typically measured with precisions of $10^{-9}$ or better. Nonetheless, the CPT bound from the neutral meson experiments is many orders of magnitude better than those from the atomic experiments. It would therefore be desirable to understand the atomic experiments better and to gain greater insight into their sensitivity. Part of the difficulty in doing this stems from the fact
that these experiments all compare different physical quantities, such as masses, $g$ factors, charge-to-mass ratios, and frequencies. One way to find a more meaningful approach to making cross comparisons would be to work within a common theoretical framework.

A number of different ideas for violation of Lorentz or CPT symmetry have been put forward over the years. In order to evade the CPT theorem one or more of the assumptions in the proof of the theorem must be disobeyed. A sampling of some of the theoretical ideas that have been put forward include the following: nonlocal interactions [30], infinite component fields [31], a breakdown of quantum mechanics in gravity [32], and spontaneous Lorentz and CPT violation occurring in the context of string theory [2]. It has also recently been shown that Lorentz violation is intrinsic to noncommutative field theories [4].

To investigate some of the experimental consequences of possible Lorentz or CPT violation, a common approach has been to introduce phenomenological parameters. Examples include the anisotropic inertial mass parameters in the model of Cocconi and Salpeter [33], the $\delta$ parameter used in kaon physics [34], and the TH$\epsilon$ model which couples gravity and electromagnetism [35]. Another approach is to introduce specific types of lagrangian terms that violate Lorentz or CPT symmetry [6]. These approaches are straightforward and are largely model independent. However, they also have the disadvantage that their predictive ability across different experiments is limited. To make further progress, one would want a consistent fundamental theory with Lorentz and CPT violation. This would permit the calculation of phenomenological parameters and the prediction of signals indicating symmetry violation. No such realistic fundamental theory is known at this time. However, a candidate extension of the standard model incorporating CPT and Lorentz violation does exist.

**STANDARD-MODEL EXTENSION**

A useful theoretical tool for studying CPT and Lorentz violation is the standard-model extension [5, 36]. It provides a consistent theoretical framework that includes the standard model (and SU(3)×SU(2)×U(1) gauge invariance) and which allows for small violations of Lorentz and CPT symmetry. It has been shown that any realistic noncommutative field theory is equivalent to a subset of the standard-model extension [4]. To consider experiments in atomic physics it suffices to restrict the standard-model extension to its QED sector and to include only terms that are power-counting renormalizable. The resulting QED extension has energy-momentum conservation, the usual spin-statistics connection, and observer Lorentz covariance. The renormalizability of the QED extension has recently been shown to hold to one-loop [37]. The theory has also been used to study scattering cross sections of electrons and positrons in the presence of CPT and Lorentz violation [38].

The modified Dirac equation in the QED extension describing a four component spinor field $\psi$ of mass $m$ and charge $q = -|e|$ in an electric potential $A^\mu$
is

\[(i\Gamma^\mu D_\mu - M)\psi = 0\]  \hspace{1cm} (1)

where

\[\Gamma_\nu = \gamma_\nu + c_{\mu\nu}\gamma^\mu + d_{\mu\nu}\gamma^5\gamma^\mu\]  \hspace{1cm} (2)

and

\[M = m + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}\]  \hspace{1cm} (3)

Here, natural units with \(\hbar = c = 1\) are used, and \(iD_\mu \equiv i\partial_\mu - qA_\mu\). The two terms involving the effective coupling constants \(a_\mu\) and \(b_\mu\) violate CPT, while the three terms involving \(H_{\mu\nu}\), \(c_{\mu\nu}\), and \(d_{\mu\nu}\) preserve CPT. All five terms break Lorentz symmetry.

The recent atomic experiments that test CPT and Lorentz symmetry express the bounds they obtain in terms of the parameters \(a_\mu\), \(b_\mu\), \(c_{\mu\nu}\), \(d_{\mu\nu}\), and \(H_{\mu\nu}\). This provides a straightforward way of making comparisons across different types of experiments and avoids problems that can arise when different physical quantities (\(g\) factors, charge-to-mass ratios, masses, frequencies, etc.) are used in different experiments. It is important to keep in mind as well that each different particle sector in the QED extension has a set of Lorentz-violating parameters that are independent. The parameters of the different sectors are distinguished using superscript labels. A thorough investigation of possible CPT and Lorentz violation must look at as many different particle sectors as possible.

**ATOMIC EXPERIMENTS**

Before discussing these recent experiments individually, it is useful to examine some of the more general results that have emerged from these investigations. First, the sharp distinction between what are considered Lorentz tests and CPT tests has been greatly diminished. Experiments traditionally viewed as CPT tests are also sensitive to Lorentz symmetry and vice versa. In particular, it has been demonstrated that it is possible to test for CPT violation in experiments with particles alone. This has opened up a whole new arena of CPT tests. A second general feature is that the sensitivity to CPT and Lorentz violation in these experiments stems primarily from their ability to detect very small anomalous energy shifts. While many of the experiments were originally designed to measure specific quantities, such as differences in \(g\) factors or charge-to-mass ratios of particles and antiparticles, it is now recognized that they are most effective as CPT and Lorentz tests when all of the energy levels in the system are investigated for possible anomalous shifts. Because of this, several new signatures of CPT and Lorentz violation have been investigated in recent years that were previously overlooked. Examples are given in the following sections. Finally, another common feature of these experiments is that they all have sensitivity to the Planck scale.

**Penning-Trap Experiments**
The aim of the original experiments with Penning traps was to make high-precision comparisons of the $g$ factors and charge-to-mass ratios of particles and antiparticles confined within the trap [20]. This was obtained through measurements of the anomaly frequency $\omega_a$ and the cyclotron frequency $\omega_c$. For example, $g-2 = 2\omega_a/\omega_c$. The frequencies were typically measured to $\sim 10^{-9}$ for the electron, which determines $g$ to $\sim 10^{-12}$. In computing these ratios it was not necessary to keep track of the times when $\omega_a$ and $\omega_c$ were measured. More recently, however, additional signals of possible CPT and Lorentz violation in this system have been found, which has led to two new tests being performed.

The first was a reanalysis performed by Dehmelt’s group of existing data for electrons and positrons in a Penning trap [9]. The goal was to search for an instantaneous difference in the anomaly frequencies of electrons and positrons, which can be nonzero when CPT and Lorentz symmetry are broken. (In contrast the leading-order instantaneous cyclotron frequencies remain equal). The original measurements of $g-2$ did not involve looking for possible instantaneous variations in $\omega_a$. Instead, the ratio $\omega_a/\omega_c$ was obtained using averaged values. The new analysis is especially relevant because it can be shown that the CPT-violating corrections to the anomaly frequency $\omega_a$ can occur even though the $g$ factor remains unchanged. The new bound found by Dehmelt’s group can be expressed in terms of the parameter $b_3$, which is the component of $b^c_\mu$ along the quantization axis in the laboratory frame. They obtained $|b_3| \lesssim 3 \times 10^{-25}$ GeV.

A second new test of CPT and Lorentz violation in the electron sector has been made using only data for the electron [10]. Here, the idea is that the Lorentz and CPT-violating interactions depend on the orientation of the quantization axis in the laboratory frame, which changes as the Earth turns on its axis. As a result, both the cyclotron and anomaly frequencies have small corrections which cause them to exhibit sidereal time variations. Such a signal can be measured using just electrons, which eliminates the need for comparison with positrons. The bounds in this case are given with respect to a nonrotating coordinate system such as celestial equatorial coordinates. The interactions involve a combination of laboratory-frame components that couple to the electron spin. The combination is denoted as $\tilde{b}_3 \equiv b_3 \equiv \tilde{b}_3 - m d_{30} - H_{12}$. The bound can be expressed in terms of components $X, Y, Z$ in the nonrotating frame. It is given as $|\tilde{b}_3| \lesssim 5 \times 10^{-25}$ GeV for $J = X, Y$.

Clock-Comparison Experiments

The Hughes-Drever experiments are classic tests of Lorentz invariance [21]. These experiments look for relative changes between two atomic “clock” frequencies as the Earth rotates. The “clock” frequencies are typically atomic hyperfine or Zeeman transitions. In a 1995 experiment, very sharp bounds at leading-order for the proton, neutron and electron were obtained in the experiment of Berglund et al. These were $\tilde{b}_J \simeq 10^{-27}$ GeV, $\tilde{b}_J \simeq 10^{-30}$ GeV, and $\tilde{b}_J \simeq 10^{-27}$ GeV for $J = X, Y$.

More recently, several new clock-comparison tests have been performed or are in the planning stages. For example, Bear et al. have used a two-species noble-
gas maser to test for Lorentz and CPT violation in the neutron sector [19]. They obtained a bound $|\tilde{b}_j| \lesssim 10^{-31}$ GeV for $J = X, Y$. This is currently the best bound for the neutron sector. As sharp as these bounds are, however, it should be kept in mind that certain assumptions about the nuclear configurations must be made to obtain them. For this reason, these bounds should be viewed as accurate to within one or two orders of magnitude. To obtain cleaner bounds it is necessary to consider simpler atoms or to perform more precise nuclear modeling.

**Hydrogen-Antihydrogen Experiments**

Hydrogen atoms have the simplest nuclear structure. Two experiments at CERN aim to make high-precision spectroscopic measurements of the 1S-2S transitions in hydrogen and antihydrogen. These are forbidden two-photon transitions with a relative linewidth of approximately $10^{-15}$. The magnetic field plays an important role in the sensitivity of these transition to Lorentz and CPT breaking. For example, in free hydrogen in the absence of a magnetic field, the 1S and 2S levels shift by the same amount at leading order, and there are no leading-order corrections to the 1S-2S transition. However, in a magnetic trap there are fields that mix the different spin states in the four hyperfine levels. Since the Lorentz-violating couplings are spin-dependent, there will be sensitivity at leading order to Lorentz and CPT violation in comparisons of 1S-2S transitions in trapped hydrogen and antihydrogen.

An alternative to 1S-2S transitions is to consider measurements of ground-state Zeeman hyperfine transitions in hydrogen alone. It has been shown that these transitions in a hydrogen maser are sensitive to leading-order Lorentz-violating effects. Measurements of these transitions have recently been made using a double-resonance technique [17]. They yield new bounds for the electron and proton. The bound for the proton is $|\tilde{b}_p| \lesssim 10^{-27}$ GeV. Due to the simplicity of the hydrogen nucleus, this is an extremely clean bound. It is currently the best Lorentz and CPT symmetry for the proton.

**Spin-Polarized Matter**

A recent experiment at the University of Washington uses a spin-polarized torsion pendulum to achieve high sensitivity to Lorentz violation in the electron sector [13]. Its sensitivity stems from the combined effect of a large number of aligned electron spins. The experiment uses stacked toroidal magnets that have a net electron spin $S \simeq 8 \times 10^{22}$, but which have a negligible magnetic field. The pendulum is suspended on a turntable and a time-varying harmonic signal is sought. An analysis of this system reveals that in addition to a signal with the period of the rotating turntable, the effects of Lorentz and CPT violation induce additional time variations with a sidereal period caused by Earth’s rotation. The group at the University of Washington has analyzed their data and has obtained a bound on the electron parameters equal to $|\tilde{b}_e| \lesssim 10^{-29}$ GeV for $J = X, Y$ and
$|\tilde{b}_e| \lesssim 10^{-28}$ GeV [13]. These are currently the best Lorentz and CPT bounds for the electron.

**Muon Experiments**

Experiments with muons involve second-generation leptons. They provide independent Lorentz and CPT tests. There are several different kinds of experiments with muons that are currently being conducted, including muonium experiments [14] and $g - 2$ experiments with muons at Brookhaven [15]. In muonium, the experiments measuring the frequencies of ground-state Zeeman hyperfine transitions in a strong magnetic field have the best sensitivity to Lorentz and CPT violation. A recent analysis has looked for sidereal time variations in these transitions. A bound at a level of $|\tilde{b}_\mu| \leq 5 \times 10^{-22}$ GeV has been obtained [14]. The $g - 2$ experiments with positive muons are relativistic with “magic” boost parameter $\delta = 29.3$. Bounds on Lorentz-violation parameters should be attainable in these experiments at a level of $10^{-25}$ GeV. These experiments are currently underway at Brookhaven and their results should be forthcoming in the near future.

**SUMMARY AND CONCLUSIONS**

Five new sets of Lorentz and CPT bounds have been obtained in recent years for the electron, proton, neutron, and muon. The leading-order bounds are summarized in Table 1. All of these limits are within the range of sensitivity associated with suppression factors arising from the Planck scale. However, as sharp as these bounds are, there is still room for improvement, and it is likely that the next few years will continue to provide increasingly sharp new tests of Lorentz and CPT symmetry in QED systems. In particular, it should be possible to obtain bounds on many of the parameters that do not appear in Table 1.

One promising approach is to conduct atomic clock-comparison tests in a space satellite [23]. These will have several advantages over traditional ground-based experiments, which are typically insensitive to the direction $Z$ of Earth’s axis and ignore boost effects associated with timelike directions. For example, a clock-comparison experiment conducted aboard the International Space Station (ISS) would be in a laboratory frame that is both rotating and boosted. It would therefore immediately gain sensitivity to both the $Z$ and timelike directions. This would more than triple the number of Lorentz-violation parameters that are accessible in a clock-comparison experiment. Since there are several missions already planned for the ISS which will compare Cs and Rb atomic clocks and H masers, the opportunity to perform these new Lorentz and CPT tests is quite real. Another advantage of an experiment aboard the ISS is that the time needed to acquire data would be greatly reduced (by approximately a factor of 16). In addition, new types of signals would emerge that have no analogue in traditional Earth-based experiments. The combination of these advantages should result in substantially improved limits on Lorentz and CPT violation.
<table>
<thead>
<tr>
<th>Expt</th>
<th>Sector</th>
<th>Params ($J = X, Y$)</th>
<th>Bound (GeV)</th>
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<tr>
<td>Hg-Cs clock comparison</td>
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Table 1: Summary of leading-order bounds.

In summary, by using a general framework we are able to analyze Lorentz and CPT tests in a variety of atomic experiments. We find that experiments that have traditionally been considered Lorentz tests are also sensitive to CPT and vice versa. We find that it is also possible to make very precise tests of CPT in matter alone. Many of the bounds that have been obtained are well within the range of suppression factors associated with the Planck scale. The atomic experiments complement those in particle physics and together they are able to test the robustness of the standard model to increasing levels of precision.

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