Development and Assessment of Non-Isotropic Spatial Resolution in PIV

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Abstract

The present study discusses the development of a novel method to interrogate PIV recordings. The technique is introduced as an adaptive resolution in that it is based on the adaptation of the interrogation volume to the local flow field properties. In the first part the evaluation of the PIV spatial resolution based on cross-correlation is discussed. A comparison is proposed between the behaviour of moving average filters with the result of the cross-correlation operator. In both cases the operators act as spatial low-pass filters of the velocity distribution, while the difference is given by the additional error due to the noise level intrinsic of particle image correlation. The error due to the finite extent of the interrogation window is derived with an analytical expression. A local Taylor expansion truncated at the second order models the velocity two-dimensional distribution. The error analysis shows that the error is proportional to the velocity second derivative and to the square of the window (linear) size.

The second part discusses the concept of non-isotropic resolution only possible in multi-dimensional signals. The method is based on the analysis of the velocity second derivatives. The Hessian tensor eigen-values/vectors describe the spatial curvature radius of the velocity distribution. This information is used to modify the aspect ratio of the interrogation window and to orient it in order to minimize the effects of the largest fluctuations. The proposed method is based on interrogation windows of elliptical shape, with a constant area and elongated in the direction of the largest radius of curvature. The implementation of the non-isotropic windowing method within a recursive interrogation scheme which also applies window deformation is described. The non-isotropic interrogation method performance is then assessed by means of Monte-Carlo simulation of particle images motion. The analysis of one-dimensional sinusoidal displacement yields the comparison with moving average filters. The results show that for the one-dimensional case the spatial resolution can be improved by a factor two. A final qualitative comparison is presented on a turbulent separated flow assessing the method robustness in case of real (noisy) PIV images.
1 Introduction

In PIV, the flow velocity at a given location is associated to the motion of an ensemble of tracers in the neighbourhood. Since an ensemble of particle images is required to perform a robust correlation analysis, the spatial resolution is often considered as the limiting factor of the PIV technique. The size of the interrogation windows is often expressed in terms of pixels and depending on the seeding density and on the optical settings of the experiment, the interrogation areas linear size $l$ may vary from about 10 to 50 pixels. The ratio between the size of the illuminated area $L$ and the light sheet thickness $W$ does not vary largely. According to the review of PIV experiments collected by Raffel et. al (1998), the $L/W$ ranges from 50 to $2 \times 10^2$. On the other hand, the CCD sensors used nowadays for particle imaging are typically equipped with $10^3$ pixels along each direction. The spatial resolution associated to the measurement volume dimensions can be expressed by

$$r = \sqrt{l^2 + h^2 + w^2}$$

where $l$, $h$ and $w$ are the measurement volume length, height and width respectively. In most cases the in-plane dimensions $l$ and $h$ are prevailing and the measurement volume covers a thin region of space; in this case $r = \sqrt{l^2 + h^2}$ as shown in Fig. 1a. In some cases, they become as small as $w$ returning a roughly cubic or spherical measurement region (Fig. 1b). In the third case the measurement volume has its largest dimension along the light sheet thickness. In this case, the light sheet thickness is the limiting factor for the spatial resolution ($r = w$), which can be improved only by optical means either reducing $W$ or limiting the depth of focus of the imaging system. However such a situation will not be considered since it is only seldom obtained and for specific applications (e.g. micro-PIV). The first case occurs most frequently in PIV experiments, which justifies the numerous studies devoted to the improvement of the in-plane spatial resolution (Fincham et al., 1997-2000; Gui and Merzkirch, 2000; Nogueira et al., 2001; Scarano et al., 1999; Di Florio et al., 2002 among others). One should however keep in mind that as soon as the case $b$ is approached (cubic measurement volume), further improvements of the in-plane spatial resolution do not necessarily improve the overall measurement accuracy due to the averaging effect in the direction normal to the plane.

The measurement volume in case $a$ is clearly non-isotropic. Although uniform all over the measurement plane the in-plane dimensions are different from $W$. The present study introduces the concept of adaptive non-isotropic resolution removing the unnecessary constraint that the in-plane aspect ratio of the interrogation windows $l/h$ is fixed as well as the orientation of the interrogation window. In this case two length scales (principal axes) must be chosen for the interrogation window. In order to determine a criterion able to guide the adaptive resolution method, the response of the cross-correlation (CC) interrogation is investigated using the two-dimensional spatial moving average (MA) filter as a model. In the second part of the paper the mathematical basis of non-isotropic image processing is given. In the last section, the performance of the method is assessed with synthetic and real PIV images.
2 Time and Space Resolution in PIV

The particle image displacement \( D(X; t_1, t_2) \) is defined as the distance travelled by a particle imaged at location \( X \) during the time interval \( \Delta t = t_2 - t_1 \). Given the particle image velocity \( V(X; t) \) the particle image displacement is given by:

\[
D(X; t_1, t_2) = \int_{t_1}^{t_2} V\left[X(t), t\right]dt
\]  

Therefore, the displacement (viz. velocity) obtained from two particle image records is a time-filtered (low-pass) representation of the instantaneous particle velocity. Any velocity fluctuation with a time scale shorter than the time separation between the two records is averaged out. Accordingly, velocity spatial fluctuations will be integrated along the particle trajectory, limiting the measurement spatial resolution. The time integration is neither the only source of error nor the most important. In fact the hardware nowadays available (nano-second pulsed lasers, micro-second interline transfer time CCDs) allows choosing the time separation between records sufficiently short to minimize such form of error. On the other hand the spatial integration process often introduces the largest errors. In fact, in the hypothesis of ideal tracers dynamical behaviour, the displacement of tracers particles can be regarded as a random sampling of the displacement field (Westerweel, 1993) and the local velocity returned by CC is between the mean and the median value of the velocity of the tracers particles in the interrogation window. Therefore, the CC result cannot represent the spatial fluctuations at length scales smaller than that of the interrogation windows.

Considering the entire recording where \( N \) particles are imaged, two major parameters give a basis for the definition of spatial resolution, namely the source density \( N_s \) and the image density \( N_i \) (Adrian and Yao, 1984). \( N_s \) indicates whether the image consists of individual particle images (\( N_s << 1 \)) or particle images overlap and eventually interfere (\( N_s >> 1 \)). \( N_i \) can be also interpreted as the ratio...
between the particle image mean diameter $d_\tau$ and the mean distance between neighbouring particles images $\lambda_p$. The image density returns the number of particle images falling within an interrogation area on average. $N_i$ can be seen as the ratio between the interrogation window area $W_i$ and the square of $\lambda_p$. The source density number has a fundamental role in establishing the limits of the measurement spatial resolution. According to Nyquist criterion, given $\lambda_p$, the highest spatial resolution that can be achieved is $2\lambda_p$.

### 3 Particle Images Cross-Correlation

The particle motion is analysed by cross-correlating the particle image pattern recorded at subsequent time instants. In the hypothesis that the image density is spatially uniform and relatively large ($N_i > 10$) and for velocity differences $\Delta U$ smaller than the particle image diameter $d_\tau$ across the interrogation area, the result of $CC$ can be regarded as a low-pass spatially filtered version of the displacement field. Consequently, velocity fluctuations of wavelength smaller than the window are increasingly suppressed (Willert and Gharib, 1991). This is due to the properties of the convolution operator, which is the common denominator of the cross-correlation as well as $MA$ filters. Fig. 2 shows schematically the similarity between the $CC$ and $MA$ operators. One should retain in mind that the $MA$ filter is directly applied to the velocity spatial distribution (sinusoidal in the example), while the $CC$ operator is applied to the particle images. Since the most common choice for the interrogation window is a square array of pixels, a square top-hat window $MA$ filter of the same size as the correlation window will be considered in the remainder. It will be shown that the $CC$ has a behaviour similar to $MA$ except for the additional noise.

### 4 Moving Average Filters

In this section the spatial response of a 2-D square top-hat $MA$ filter is by analysed deriving an expression for the error due to the finite spatial resolution. The derivation is made for the one-dimensional case and the result is then extended to 2-D. The displacement spatial distribution $U(x)$ is locally expressed by a second order polynomial (simple parabola):

$$
U(x) = ax^2 + bx + c
$$

(4.1)

The moving averaged version $U_{ma}$ of the displacement $U$ with a filter of length $l$ is given by:

$$
U_{ma}(x) = \frac{1}{l} \int_{-\frac{l}{2}}^{\frac{l}{2}} U(x-\xi) d\xi
$$

(4.2)
\[ U_{MA}(x) = a \cdot x^2 + b \cdot x + c + \frac{1}{12} a \cdot l^2 = U(x) + \frac{1}{12} a \cdot l^2 \] (4.3)

In the two-dimensional case we can write:
\[ U(x, y) = ax^2 + bx + c + dy^2 + ey + fxy \] (4.4)

\[ U_{MA}(x, y) = U(x, y) + \frac{1}{12} \left( a \cdot l_x^2 + b \cdot l_y^2 \right) \] (4.5)

Therefore the expression of the error \( \varepsilon_{MA} = U_{MA} - U \) reads as:
\[ \varepsilon_{MA}(x, y) = \frac{1}{12} \left( a \cdot l_x^2 + b \cdot l_y^2 \right) \] (4.6)

Fig. 2 Top: a sinusoidal signal processed with a top-hat moving average filter. Bottom: particle image records with a sinusoidal displacement distribution are processed with the cross-correlation operator.

Following the above expression, the error associated to a MA filter depends on the curvature (2nd spatial derivative) of the displacement (or velocity) spatial distribution and it is proportional to the square of the window size in each direction respectively. It is clear that the error decreases when the window size is made smaller. In particular, halving \( l_x \) and \( l_y \) will result in an error reduction of a factor four. However it should be kept in mind that the above results are to be transferred to cross-correlation, in which case a reduction in size of the correlation window is accompanied by a reduction of the image density \( N_I \) with dramatic effects on the correlation signal-to-noise ratio (Scarano, 2002).
Considering a one-dimensional displacement distribution the evaluation of the error can be related directly to the modulation transfer function for a given filter shape. A top-hat function (rectangular window) returns only a fraction of the maximum displacement attained at the crests of the sinusoids. A previous study (Scarano and Riethmuller, 2000), confirmed the similar behaviour of MA and CC. It can be seen that if $W_s$ is smaller than a quarter of wavelength $\Lambda$ then $U_{ma}/U_0 > 0.9$, corresponding to less than 10% error (Fig. 3). The CC results however, show a slightly lower response with respect to the MA data.

Fig. 3 MA and CC analysis of a sinusoidal displacement distribution. CC+D indicates cross-correlation with window deformation.

5 Non-Isotropic Resolution

In several flow problems, the velocity spatial variation exhibits a preferential direction, in which case the radius of curvature of the velocity fluctuations has not the same value in different directions. We can recall for instance the case of a laminar boundary layer flow, where the velocity spatial derivatives in the streamwise direction can be neglected with respect to those in the wall-normal direction. In such a case arranging the measurement volume with a preferential direction orthogonal to the wall-normal direction (Fig. 4) would bring a direct benefit to the measurement resolution. For more complex flows the choice of the preferred direction is not as straightforward and is in general varying along the measured field.
Several measurement techniques probe the flow non-isotropically and are arranged so that the largest dimension is aligned with the direction where the flow is most uniform (e.g. flat Pitot pressure probe in boundary layers, hot wire anemometer, laser Doppler velocimeter). In PIV, the idea of shaping the interrogation volume (viz. window) in such a way to improve the measurement of the in-plane particle motion is not new in itself. Lecordier et al. (1999) proposed the re-orientation of square windows in the flow direction in order to minimize the effect of the velocity gradient along the window diagonal. Di Florio et al. (2002) proposed a windowing, re-shaping and re-orientation method, which stretches the windows in the direction of the flow and proportionally to the measured displacement. The first method aims at improving the correlation signal-to-noise ratio minimizing the velocity difference across the interrogation window, but it relies on the restrictive hypothesis that the velocity gradient is perpendicular to the local trajectory. In conclusion such a method should be listed in the group of methods enhancing the correlation signal but no improvement is expected in terms of resolution. The second method is somewhat similar in that it adopts a window re-shaping and re-orientation. The method aims at improving the spatial resolution in the regions where large velocity fluctuations are found, however the re-shaping and re-orientation criterion is based on the velocity magnitude and direction, which according to linear filter theory is not expected to improve the spatial resolution in case of velocity fluctuations.

Fig. 5 summarises the different possible approaches to the concept of adaptive and non-isotropic resolution. Case a is representative of the approach followed by Di Florio et al. In this case some benefit could be obtained because less loss-of-pairs would occur than with a fixed window the interrogation method is used. However a window-shift method (Westerweel et al., 1997) would equivalently solve the problem.

In case b the non-isotropic criterion is based on the velocity gradient tensor. In this case, the stretched windows would be such that the loss-of-pairs due to in-plane velocity gradient is minimized in the cross-correlation. No literature studies are found that adopt such a criterion. For this case the deformation of the correlation window is expected to bring an equivalent result.
Case c defines the criterion proposed in the present study. The window size is made smaller in the direction of the minimum curvature radius and relaxed in the orthogonal direction. An increase in directional spatial resolution is expected in this case and the results relative to this approach will be discussed in the following sections.

Finally case d proposes an adaptive resolution approach (Scarano, 2002) based on the largest eigen-value of the velocity Hessian. The interrogation window is reduced when small length scale velocity fluctuations are present. The method was demonstrated to improve the spatial resolution in some cases. As a drawback, \( N_i \) is not constant and also the uncertainty and the signal-to-noise ratio.

The mathematical relation between the velocity fluctuations and the shape and orientation of the interrogation volume, keeping \( N_i \) fixed, is described in the next section. Considering, for instance, the above case of wall boundary layer, the interrogation volume should be arranged so as to decrease the error due to the nonlinear term in the velocity profile.

**5.1 Velocity fluctuations**

In this section a mathematical procedure is proposed to evaluate the spatial distribution of the velocity fluctuation curvature. The procedure is applied separately to the two velocity components. The following derivations are made for the velocity component along the \( x \)-direction \( U \), and the discussion will be completed further with the \( y \)-component \( V \). Considering the velocity distribution as known (or measured) the second spatial derivatives of \( U(x,y) \) can be estimated for instance by means of finite differences or by a least squares regression. This allows to evaluate the Hessian matrix:
When $H$ is non-singular, its eigen-values $\lambda_1$ and $\lambda_2$, and linearly independent eigenvectors, $\overline{I}_1$ and $\overline{I}_2$ can be obtained. The maximum and minimum radius of curvature of the velocity spatial distribution $r_{\text{max}}$ and $r_{\text{min}}$ are perpendicular. The orientation of $r_{\text{min}}$ is given by $\theta$:

$$r_{\text{min}} = \frac{1}{|\lambda_1|}$$ \quad and \quad $$r_{\text{max}} = \frac{1}{|\lambda_2|}$$ \quad (5.2)

$$\theta = \tan^{-1}\left(\frac{\lambda_1(\overline{I}_1)}{\lambda_2(\overline{I}_2)}\right)$$ \quad (5.3)

Repeating the procedure for $V(x, y)$ we will finally obtain the following field parameters:

$$\theta_u(x, y); \quad r_{u_{\text{min}}}(x, y); \quad r_{u_{\text{max}}}(x, y); \quad \theta_v(x, y); \quad r_{v_{\text{min}}}(x, y); \quad r_{v_{\text{max}}}(x, y);$$ \quad (5.4)

Which are necessary to establish the directions of the minimum and maximum curvature radii and their ratios.

5.2 Adapated non-isotropic window

From the two radii of curvature a two-dimensional interrogation window of elliptical shape can be obtained. The eccentricity of the ellipse $e$ is defined as $e = 1 - \alpha/\beta$ where $\alpha$ and $\beta$ are the ellipse semi-axes. The semi-axes ratio is directly obtained considering the Hessian matrix eigen-values ratio $\lambda_1/\lambda_2$. However, in order to exclude the case of a very eccentric or a degenerated ellipse occurring when $e$ tends to 1 the range of values for the eccentricity is restricted to the interval $[0, e_{\text{max}}]$. Moreover we propose an exponential weighting of the eccentricity with a function of the ratio between the minimum radius of curvature $r$ and the equivalent circular window linear size $l$. This weighting takes into account the fact that the higher is the ratio $r_{\text{min}}/l$, the less important is the non-isotropic weighting (the modulation error becomes smaller) and the interrogation window tends to become isotropic. The proposed expression for the eccentricity reads as:
With a fixed value of $\sigma = 10^2$. The equivalence of the weighted window and the rectangular one (of dimensions $l_x$ and $l_y$) is based on the relation
\[ 2\alpha \cdot 2\beta = l_x l_y \]  
(5.6)

From the definition of $e$, the expression of $\alpha$ and $\beta$ as function of $l_x$, $l_y$, and $e$ is obtained.
\[ \alpha = \frac{l_x l_y}{2} (1-e)^\frac{1}{2} \]  
(5.7)
\[ \beta = \frac{l_x l_y}{2} (1-e)^{-\frac{1}{2}} \]  
(5.8)

The ellipse is expressed in canonical form in the $\xi-\eta$ system of co-ordinates given by the eigen-vectors:
\[ \frac{\xi^2}{\alpha^2} + \frac{\eta^2}{\beta^2} = 1 \]  
(5.9)

In order to obtain the equation of the ellipse in the $x$-$y$ frame of reference the following transformation is applied (Fig. 6).

\[
\begin{bmatrix}
\xi \\
\eta
\end{bmatrix} = \begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]  
(5.10)
Finally, the expression of the ellipse in the x-y frame of reference is obtained as:

\[
\begin{align*}
&x^2 \left( \frac{\cos^2(\theta)}{a^2} + \frac{\sin^2(\theta)}{b^2} \right) + y^2 \left( \frac{\sin^2(\theta)}{a^2} + \frac{\cos^2(\theta)}{b^2} \right) + \ldots \\
&\ldots + xy \left( 2\sin(\theta)\cos(\theta) \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \right) = 1
\end{align*}
\]

(5.11)

Once the equation of the ellipse is obtained in the x-y frame of reference, different weighting schemes can be applied in order to shape the interrogation windows accordingly. In the present study, the elliptical shape of the interrogation windows is attained by means of a Gaussian weighting function applied to the square interrogation windows. It should be mentioned that this is not the only way to proceed and the alternative is to select a top-hat like elliptical interrogation areas using eq. 3-11 to select the image pixels falling inside the ellipse. However the discussion of alternative methods goes beyond the scope of the present study. The Gaussian weighting function obtained from the above expressions is:

\[
G(x, y) = \exp \left[ - \left( ax^2 + by^2 + cxy \right) \right]
\]

(5.12)

The weighting function is applied to the interrogation windows prior to performing the correlation and the weighted intensity distribution \( I_w \) in the interrogation window is obtained from the original grey level \( I \) as follows:

\[
\begin{align*}
I_w^x &= I^x \cdot G \\
I_w^y &= I^y \cdot G
\end{align*}
\]

(5.13)

The weighting operation is followed by a pixel intensity normalization. Finally the normalized correlation function is computed.

6 Performance Assessment

Synthetic PIV images with a known displacement spatial distribution are analysed. The non-isotropic resolution scheme is compared with an isotropic scheme with equivalent window size and following the same interrogation method. The cross-correlation results are also compared with the output of a two-dimensional equivalent MA filter.
6.1 Synthetic PIV images

Particle images are obtained with Monte Carlo simulation. The Gaussian particle light pattern is numerically integrated on a 500×500 pixels array. The particle image diameter is \( d_p = 2 \) pixels and the particle image density is 0.1 particles per pixel (ppp), yielding about 100 particle images on a 32×32 pixels window. The interrogation method follows a multi-grid scheme with progressive reduction of the interrogation window. After that interrogation is performed iteratively until the results converge within a threshold set at 0.03 pixels. The interrogation method also includes the relative deformation of correlation windows, which compensates for the in-plane particle motion up to the first order derivatives of the displacement spatial distribution (Huang et al. 1993, Scarano and Riethmüller, 2000).

First a one-dimensional sinusoidal particle images displacement is chosen to evaluate the method’s accuracy. The assessment with synthetic images is completed with the application to the case of a normal shock wave type of flow.

6.2 One-dimensional sinusoid

The analysis of a one-dimensional sinusoidal displacement distribution is performed to evaluate the spectral response of the interrogation method. Different values of the window linear size are chosen with \( l = \{11, 21, 31\} \). A large window overlap factor is applied to obtain the velocity distribution over a pixel grid, which allows not to introduce uncertainties associated to the interpolation schemes. The velocity fluctuations are along the \( y \)-coordinate; therefore \( U_y \) is the only nonzero term in the Hessian. The sinusoid amplitude is 2 pixels. Changing the window size and the sinusoid wavelength \( \Lambda \), the amplitude response diagram is obtained as a function of the normalized window size \( l^* = l/\Lambda \).

![Fig. 7](image)

**Fig. 7** One-dimensional sinusoidal displacement distribution: vector field and interrogation windows. Left: U-component; right: V-component. Window size \( l = 16 \) pixels.

Fig. 7 shows the displacement vector field and the pattern of the interrogation windows. The ellipses show a maximum eccentricity at the sinusoid crests where
also the second derivative reaches its maximum. At the velocity inflection points (crossing the zero) the eccentricity is practically zero and circular (isotropic) interrogation windows are returned. The method also returns circular windows when the V-component is to be measured, which is uniformly zero.

The result of the interrogation is summarized in Fig. 8. The top-hat window and the Gaussian weighted (circular) window yield the same result when \( l^* < 0.5 \). For larger values of \( l^* \) the values of the Gaussian weighted window analysis are higher than the top-hat. Moreover for \( l^* > 1 \) the Gaussian weighted interrogation does not show any sign reversal, which occurs for the top-hat window. A detailed discussion on the effects of weighting functions on correlation windows is given by Nogueira et al. (1999, 2002). Comparing the results of the non-isotropic and isotropic methods, for \( l^* = 0.5 \) (the window size is half a wavelength) the error of the isotropic method is about 40% and it reduces to 19% for the non-isotropic case. One can conclude that the non-isotropic interrogation method reduces the error to about half in the range \( 0 < l^* < 0.5 \). For larger values of \( l^* \) the improvement is less significant and the different data series merge while approaching zero. It may be concluded that when \( l^* > 0.5 \) the adaptive resolution method becomes ineffective since the error due to the lack of resolution goes beyond the possibility to correct for it. At this point only super-resolution interrogation methods may offer a viable
solution (Keane and Adrian, 1995 and Nogueira et al. 2001). However, the most important part of the diagram is that with relatively small values of $l^*$, representing the situation in which the velocity fluctuations length-scale are actually resolved within the measurement spatial resolution.

### 6.3 One-dimensional compression front (normal shock wave)

In this case the velocity spatial fluctuation is in the same direction as the flow. The only non zero term of the Hessian matrix is therefore $U_{xx}$. The present case is representative of the situation encountered in compressible flows where the particle tracers decelerate abruptly across a shock wave. However due to the particles finite response the tracers cannot follow the flow with fidelity after the shock. The particle relaxation time/length is a crucial issue in high speed flow diagnostics and it requires a careful assessment. In many cases the limited spatial resolution of the measurement may constitute a major constraint to either estimate the position of the shock wave or the particle tracers’ relaxation length. It is therefore crucial to limit the smoothing effect of the PIV measurement across the shock.

![Fig. 9](image-url)

Fig. 9 shows the velocity vector field and the interrogation window distribution relative to the measurement of the U-component. The maximum eccentricity is attained at about the shock location and it decreases downstream.

The results of the CC show that the response of the non-isotropic interrogation window with $l = 41$ pixels can be compared with the isotropic top-hat window with $l = 21$ pixels. This confirms the result obtained from the sinusoidal displacement. In this case adopting the Hessian criterion is crucial, since the velocity difference is normal to the streamlines. A method based on the value and the direction of the velocity to re-shape the windows and re-orient them along the streamlines would further decrease the measurement spatial resolution.
6.4 Turbulent BFS flow

The robustness and applicability of the non-isotropic method is assessed with a sample application to PIV records obtained from real experiments. An instantaneous snapshot of the turbulent flow past a backward facing step is analysed with four different interrogation methods. The result given in terms of instantaneous vorticity distribution is shown in Fig. 11. The picture in the top-left corner shows the result obtained by cross-correlation with window discrete shift at a window size of $23 \times 23$ pixels. Large vorticity peaks in the shear layer and in vortex cores are due to the discontinuous behaviour of the correlation in regions with a large velocity gradient. In comparison, the result obtained with the window deformation method (top-right) shows a more regular vorticity distribution mostly due to the signal recover in the sheared regions. When the adaptive resolution scheme is applied with the isotropic method, the vorticity map shows higher peaks (about 30%) at the cost of an increase of measurement noise estimated at 15%. Finally the adaptive resolution analysis performed with the non-isotropic method (basic window size $23 \times 23$ pixels, with maximum aspect ratio $11 \times 45$) returns almost the same improvement in terms of peak vorticity while the noise is kept at the same level as in the case of uniform window size.
Fig. 11 – Backward facing step flow; vorticity spatial distribution. Top-left: cross correlation with window discrete shift (23×23 pixels); Top-right: cross correlation with window deformation (23×23 pixels); Bottom-left: cross-correlation with adaptive (isotropic) resolution with window deformation (31×15×15 pixels); Bottom right: cross-correlation with adaptive (non-isotropic) resolution with window deformation (l = 23 pixels).

7. Conclusions

The measurement error of the cross-correlation PIV interrogation has been investigated. The effect of the poor spatial resolution has been studied through the analogy between the CC analysis and MA filters. The results for top-hat rectangular filters show that the spatial resolution can be improved only if the effective size of the filter is reduced. The error grows with the square of the filter size and is proportional to the second derivatives of the velocity spatial distribution. It was therefore concluded that the driving criterion to reduce the measurement error due to poor resolution must be based on the spatial curvature of the velocity distribution.

The concept of non-isotropic spatial resolution has been introduced. A mathematical basis has been given to evaluate the essential parameters needed to locally adapt the properties of the interrogation windows, fixed keeping the interrogation area. The Gaussian elliptical windowing has been proposed as a possible choice.

The method has been implemented within the existing PIV image analysis software based on CC and iterative window deformation. The performance of the non-isotropic interrogation technique has been assessed using simulated PIV images of a reference particle motion distribution. The analysis of one-dimensional sinusoids has showed that the modulation error of the isotropic interrogation method can be reduced of about 50% in the range 0 < l* < 0.5. The analysis of the simulated particle motion across a normal shock wave returned a similar result. Finally the assessment performed on real PIV images from a turbulent backward facing step flow has confirmed the method viability on real flow problems returning a visible increase in spatial resolution.
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