Proton Interactions with Nuclei to Probe the Neutron Matter Distribution of the Nucleus

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Abstract. Analyses of intermediate energy nucleon-nucleus scattering made using nonlocal optical potentials obtained by full folding medium modified effective NN interactions are sensitive to the target nucleon densities used. As the effective interaction and the structure details are all preset and no a posteriori adjustment or simplifying approximation is made to the (g-folded) optical potentials, the observables obtained are predictions and they match data well. The procedure, under inverse kinematics, also explains radio-active ion beam data from scattering of exotic, halo nuclei, from Hydrogen.

1. INTRODUCTION

Usually nucleon-Nucleus (NA) elastic scattering data, total as well as angular dependent observables, have been analyzed using NA optical potentials; potentials most commonly taken to be local in form and often to have Woods-Saxon character. However, it has long been known that the optical potential must be nonlocal and markedly so, although it has been assumed also that the energy dependence of the customary (phenomenological) model accounts for that. Of more concern is that the phenomenological approach is not truly predictive. The parameter values chosen, while they may be set from a global survey of data analyses, are subject to uncertainties and ambiguities. While differential cross sections from a global survey of data analyses, are subject to uncertainties and ambiguities. While differential cross sections may be described by such local potentials, different parameter sets that are known to give similar results [1] may lead to different predictions for the total reaction cross section. There is no requirement that those diverse potentials be phase equivalent only that they lead to equivalent differential cross sections.

In contrast to analyses made using phenomenological local form interactions, we report on predictions found on using, complex, nonlocal, optical potentials formed in coordinate space. They are formed by folding realistic, effective, nucleon-nucleon (NN) interactions with density matrices (hereafter simply termed densities) of the nucleus; densities defined from credible nucleonic models of nuclear structure. As the effective NN interactions are determined from a mapping of NN g matrices, i.e. solutions of the Brueckner-Bethe-Goldstone (BBG) equations in nuclear matter, the resultant effective NN interaction is complex, energy, and density dependent. Those properties lead to complex, energy dependent and strongly non-local optical potentials with which, however, good predictions of angular and integral observables, including all spin observables, for nucleon scattering from all nuclei [2, 3] have been obtained. The strong medium dependence varies with energy and also is evident in predictions of total reaction cross sections [4].

2. THE G-FOLDING POTENTIALS

To define the nonlocal interaction for NA scattering in a full folding model, antisymmetry of the wave functions for the projectile and each and every bound nucleon wave function need be taken in the evaluation of multi–particle matrix elements of the (symbolic) form

\[ U_{pa} = \left( \Psi(1\ldots A) \right| \sum_{n=1}^{A} V_{no} \left| \Psi(1\ldots A) \right) \]

with \( \left| \Psi(1\ldots A) \right) \) being the many-body wave function for the ground state of the target and ‘0’ denoting the projectile coordinates. As all nucleons in the target are equivalent, it is useful to choose a specific entry (‘1”) and write \( \sum_{n=1}^{A} V_{no} = A V_{01} \). With the many-body state expanded in cofactors,

\[ \left| \Psi(1\ldots A) \right) = \frac{1}{\sqrt{A}} \sum_{\alpha m} \varphi_{\alpha m}(1) a_{\alpha m} \left| \Psi(1\ldots A) \right) \]

where \( \alpha \) specifies the set of single particle (SP) quantum numbers \( \{n, l, j, \zeta\} \) in which \( \zeta \) is the isospin projection, the optical potential takes the form

\[ U_{pa}(0, 1) = \sum_{\alpha m \alpha' m'} \left( \Psi \left| a_{\alpha' m'}^\dagger a_{\alpha m} \right| \Psi \right) \times \left( \varphi_{\alpha m'}(1) \right| V_{10} \left( \left| \varphi_{\alpha m}(1) \right) - \left| \varphi_{\alpha m}(0) \right) \right) . \]

when pair antisymmetry between the projectile and struck nucleon is taken into account. The nuclear structure information required to evaluate the optical poten-
tials are the many-body matrix elements of the particle-hole operators. They are defined by

\[
P^{\alpha \alpha'J_J J_f \mp M_f}_I = \left\langle \Psi \left| \alpha'_m \alpha_m \right| \Psi \right\rangle = \sum_{I,N} \frac{(-1)^{j-m}}{\sqrt{2j_f+1}} S_{\alpha \alpha'}
\]

\[
\times \left\langle j \, m' | I - N \right\rangle \left\langle J_I M_I | I \, N \right| J_f M_f \right\rangle,
\]

where the \( S_{\alpha \alpha'} \) are one body density matrix elements (OBDME). With \( \alpha_m = (-1)^{j-m} \alpha_{\alpha-m} \)

\[
S_{\alpha \alpha'} = \left\langle \Psi_{J_I} \left| \left[a^\dagger_{\alpha'} \times a_{\alpha}\right]^{I_T} \Psi_{J_f} \right\rangle 
\]

\[
\rightarrow \left\langle \Psi_{J_f} \left| \left[a^\dagger_{\alpha'} \times a_{\alpha}\right]^{I_T} \Psi_{J_f} \right\rangle.
\]

For elastic scattering from a target with zero spin, the OBDME reduce often to be the shell occupancies of the target. Then one obtains a characteristic form,

\[
U(r_0, r_1; E) = \sum_{\alpha \alpha'} \left( 2J + 1 \right) S_{\alpha \alpha'}
\]

\[
\delta(r_0 - r_1) \int \varphi_{\alpha m'}(s) U^{D}(R_{0s}, E) \varphi_{\alpha m}(s) ds 
\]

\[
+ \varphi_{\alpha m'}(r_1) U^{Ex}(R_{01}, E) \varphi_{\alpha m}(r_0),
\]

where \( U^{D} \) and \( U^{Ex} \) are appropriate combinations of the multipoles of the effective interaction for the direct and exchange contributions respectively [2].

The nuclear ground state densities are to be generated from nucleon-based models of structure. For example, for \(^{12}\text{C}\) a complete \((0 + 2)\hbar\omega\) shell model [3] was used. As another example, in some calculations with \(^{208}\text{Pb}\), Skyrme-Hartree-Fock (SHF) wave functions [5] have been considered. But the key features of those structure models for use in forming \( g \)-folding optical potentials are the OBDME and the SP wave functions. They are also the primary features in defining the size of a nucleus, what ‘skin thickness’ there may be, and the variation of matter densities through the surface region.

3. RESULTS AND DISCUSSION

Differential cross sections, and in some cases analyzing powers, from the scattering of nucleons from nuclei (and by inverse kinematics, of nuclei from hydrogen), reflect specific properties of the structure input to the analyses [2]. With elastic scattering it seems that data, typically taken at momentum transfers of 1 fm\(^{-1}\) and greater, with cross section values typically of the order of a mb/sr, are quite sensitive to the surface matter densities of the nucleus involved. Three cases are considered. First is that of \(^{208}\text{Pb}\). Previously analyses [6] of 200 MeV data revealed that by using the \( g \)-folding optical potential approach one could distinguish between diverse SHF models of its structure and suggest that the neutron skin was 0.17 fm. The second example is that of scattering from Sn isotopes to show just how matter distribution changes from proton to neutron drip line may be revealed by RIB scattering from hydrogen; a fuller study has been submitted for publication [7]. Finally we consider scattering of \(^{6}\text{He} \) ions from hydrogen as functions of momentum transfer to show how elastic and inelastic scattering data can be used to identify neutron halo character.

3.1. Probing the matter profiles of \(^{208}\text{Pb}\)

Studies of the matter distributions of \(^{208}\text{Pb}\), and of its neutron density profile particularly, are quite topical. Indeed to specify its neutron rms radius is the aim of a parity violation electron scattering experiment. In contrast to proton rms radii that are known to within an accuracy of \( \sim 0.02 \) fm, neutron rms radii at best have been known to an accuracy of \( \sim 0.3 \) fm. Lately, however, the neutron rms radius in \(^{208}\text{Pb}\) was assessed using the Friedman-Pandharipande neutron equation of state as constraint, to be \( 0.16 \pm 0.02 \) fm larger than the proton value.

Recently [6], proton scattering was studied seeking to select between a commonly used prescription for \(^{208}\text{Pb}\), i.e. of a packed shell nucleus with oscillator functions, the HO model, and more detailed SHF prescriptions [5]. The oscillator energy \( \hbar\omega \) for the neutron states was 7.2 MeV so that both this HO model and a SHF model (built upon the SKM* force) had the same nucleon rms radii. Data at 200 MeV were used in the analyses. Here we show that those results are consistent with both proton and neutron scattering at energies \( \sim 100 \) MeV.

In Fig. 1, \( g \)-folding model predictions are compared with data from the scattering of 98 MeV protons [8] and of 96 MeV neutrons [9]. They are shown on the left and right of this figure respectively. The two predictions shown for each case, were found using the HO (dashed curve) and SHF/SKM* (solid curve) structure model information. Clearly only on using the SHF model structure is the data replicated, and that is so for both data sets. Since the isoscalar force, and in the \(^{3}\text{S}_1\) channel particularly, is the strongest component in the effective interaction [2], the scattering of neutrons (protons) is most influenced by the proton (neutron) matter of the target nuclei. The expectation [6] of \( \sim 0.17 \) fm for the neutron skin thickness of \(^{208}\text{Pb}\) is confirmed by these lower energy results for which the \( g \)-matrices differ markedly from those at 200 MeV.
FIGURE 1. Differential cross sections from 98 MeV protons (left) and 96 MeV neutrons (right) scattering from $^{208}$Pb. Curves are identified in the text.

3.2. Probing the matter profile of the Sn isotopes

Proton and neutron densities have been obtained for the even-even isotopes of Sn from $^{100}$Sn to $^{176}$Sn using a Skyrme-Hartree-Fock-Bogoliubov SHFB model for their structure. The Skyrme force Sly4 has been used. Then the matter densities so defined have been used with the effective $NN$ interactions to specify $g$-folding optical potentials for the elastic scattering of protons with energies in the range 40 to 200 MeV. Those potentials have been used to make predictions of the differential cross sections and spin observables for proton scattering. Specifically we have calculated cross sections for the scattering of all the even mass isotopes $^{100−176}$Sn from hydrogen at an energy of 200A MeV, and for lower energies, from those isotopes for which data exist. The proton and neutron matter distributions that result for a select set of these isotopes are shown in Figs. 2 and 3 respectively. The proton number of course is fixed at 50 and so as the neutron number increases, we see that those 50 protons extend over an increasing volume. That is due to the strong attractive neutron-proton interactions. In concert, the central charge density must, and does, decrease. The effect is a $\sim 33\%$ increase in the charge volume across the mass range. However the proton surface diffuseness, the distance over which the charge density falls from 90% to 10% of its central value, does not vary greatly over the set of nuclei. But the slope of the densities do change.

The neutron densities structure variation with increasing mass is quite different from that of the protons. Such is not unexpected as the proton number is fixed at 50 with the neutron number increasing to give the mass range.

The general trend that the neutron rms radii increase is evident as the half central density is reached at radii ranging from $\sim 5$ fm in $^{100}$Sn to $\sim 6.5$ fm for $^{170}$Sn. Also a strong oscillation develops in the central density, which on average also increases from $\sim 0.08$ neutrons/fm$^3$ in $^{100}$Sn to $\sim 0.1$ neutrons/fm$^3$ for $^{170}$Sn.

Cross sections as ratios to Rutherford for the scattering of 49.35 MeV protons are shown in Fig. 4. The data [10] display a variation with target mass that is not readily mapped by our calculated results. With $^{122,124}$Sn, the peak values of the ratios exceed our predictions by some 30 to 40%. However, the trend of data as well as a number of specific details are well enough matched that the results make the SLy4 structure credible.
3.3. Probing the matter profile of $^6$He

We consider last the scattering of $^6$He ions from hydrogen for which cross section data has been taken now at a number of energies. Also we consider first the 24.5A MeV [11] and 40.9A MeV [12] data for which cross sections have been found for a fair range of momentum transfers as well as for excitation of the first excited ($2^+$) state at 1.8 MeV. In Fig. 5, the data as functions of momentum transfer are compared with predictions. Therein the elastic and inelastic scattering results are shown in the top and bottom panels respectively. The 24.5A MeV and 40.9A MeV results were obtained from calculations made assuming that $^6$He has a neutron halo [12]. The inelastic scattering cross sections were evaluated under a distorted wave approximation with the $g$-folding optical potentials used to get the distorted waves, the same shell model giving the transition OBDME, and the relevant effective $NN$ interactions used as the transition interaction. No core polarization corrections were made.

The 24.5A and 40.9A MeV results are displayed by the solid and dashed curves respectively; predictions that agree well with the data. Note that non–halo model elastic scattering results [11, 12] are more than a factor of two too large for momentum transfers greater than $\sim 1.2$ fm$^{-1}$ while the non–halo halo model predictions for the inelastic scattering are a factor of two too small for momentum transfer values less than 1 fm$^{-1}$. These results are consistent, reflecting greater sensitivity of the inelastic scattering analyses, compared to those of elastic scattering, to matter distributions in the surface region and beyond of the target.

The halo expectation for $^6$He is corroborated by the very good agreement for the total reaction cross section obtained with the halo specification. Using a $4\hbar\omega$ shell model [2] of structure, values of 353 mb and 406 mb for the no–halo and halo cases respectively are to be compared with the experimental value of $426 \pm 21$ mb [12].

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REFERENCES