Nuclear Ground-State Observables from Relativistic Mean-Field Models: Masses, Densities, Radii, Single-Particle Levels

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Abstract. We report on the current status of relativistic mean-field models for the calculation and prediction of nuclear ground-state observables. These models are quite powerful and can be applied to light (A ≳ 16), medium, and heavy nuclei (spherical and deformed) and allow realistic extrapolations to the drip lines and to superheavy nuclei. From a single calculation one obtains a plethora of microscopic information about the chosen nucleus. We discuss several of the corresponding observables that are then simultaneously calculated as well as the accuracy with which they can be determined within the current models. Finally, we discuss recent model enhancements, connections to more fundamental physics, and future work.

INTRODUCTION

As of today, we are facing a plethora of nuclear data including many data on nuclear ground states. These include masses, form factors, life times, etc. Additional information can be deduced from these quantities, such as separation energies, shell gaps, and radii. With new RIA facilities ramping up in the near future, even more data, especially for exotic nuclei, will become available. Nuclear ground-state properties still constitute a great challenge to our theoretical understanding of the nuclear many-body system. Our understanding of its structure progresses with our possibility to describe all ground-state observables simultaneously in one approach. Self-consistent mean-field models constitute such an attempt. The calculation of single-particle wave-functions allows, in principle, the calculation of all ground-state observables.

THEORETICAL FRAMEWORK

Relativistic mean-field (RMF) models [1, 2] have reached a high degree of accuracy in the description of nuclear ground-state observables. Such models, which typically have 6-9 parameters (plus two more for the proton and neutron pairing strengths), deliver all the single-particle wave functions for protons and neutrons self-consistently. All ground-state observables can, in principle, be obtained from them. The various terms in the RMF model Lagrangian are obtained by experience and phenomenology. Most predominantly, strong scalar and vector fields are needed, delivering both the relativistic saturation mechanism of nuclear matter as well as the strong spin-orbit force in nuclei with correct magnitude and sign. Relativistic effects in nuclei reveal themselves not through their kinetic motion (roughly 1/3 of the speed of light), but rather through the strong spin-orbit force. This force is generated in relativistic mean-field models through the large scalar and vector fields of the order of 300 MeV, which add with the same sign for the spin-orbit force, but almost cancel for the binding energy, producing the -16 MeV binding energy in symmetric nuclear matter. Nonlinear isoscalar-scalar terms need to be introduced for a quantitative description of nuclei and nuclear matter. Each term introduces a coupling constant which needs to be adjusted to nuclear ground-state observables. The adjustment process is not unique, and the choices of observables and their chosen uncertainties in the χ² adjustment yield the predictive power of the model for the various kinds of observables. Recently we found that modern RMF models show a trade-off between the binding energy and form-factor-related observables [7].

RMF models can be formulated using ‘mesons’ [5], i.e., boson fields (with certain quantum numbers) that only have a very loose correspondence with the physical meson spectrum occurring in nature, or by employing contact interactions (point couplings) between the nucleons together with gradient terms [3]. Both ways lead to a covariant framework. The Lagrangian of our most successful model to date is given by Eq. (1). The corresponding force, PC-F1 [3], has 9 parameters adjusted simultaneously to nuclear ground-state observables.
Since the ansatz of these models is a covariant Lagrangian from which everything else follows, no assumptions on the nuclear shape or potential have to be made. The actual deformation and density distributions of the nucleus are true predictions of the theory. 

\[
\mathcal{L} = \mathcal{L}^{\text{free}} + \mathcal{L}^{\text{4f}} + \mathcal{L}^{\text{hot}} + \mathcal{L}^{\text{der}} + \mathcal{L}^{\text{em}}
\]

\[
\mathcal{L}^{\text{free}} = - \frac{1}{2} \alpha_S \langle \bar{\psi} \gamma_H \psi \rangle^2 - \frac{1}{2} \alpha_S \langle \bar{\psi} \gamma_H \psi \rangle \langle \bar{\psi} \gamma_H \psi \rangle
\]

\[
\mathcal{L}^{\text{4f}} = - \frac{1}{2} \alpha_S \langle \bar{\psi} \gamma_H \psi \rangle^2 - \frac{1}{2} \alpha_S \langle \bar{\psi} \gamma_H \psi \rangle \langle \bar{\psi} \gamma_H \psi \rangle
\]

\[
\mathcal{L}^{\text{hot}} = - \frac{1}{3} \beta_S \langle \bar{\psi} \gamma_H \psi \rangle^3 - \frac{1}{2} \gamma_S \langle \bar{\psi} \gamma_H \psi \rangle^4
\]

\[
\mathcal{L}^{\text{der}} = - \frac{1}{2} \delta_S \langle \bar{\psi} \gamma_H \psi \rangle \langle \bar{\psi} \gamma_H \psi \rangle - \frac{1}{2} \delta_S \langle \bar{\psi} \gamma_H \psi \rangle \langle \bar{\psi} \gamma_H \psi \rangle
\]

\[
\mathcal{L}^{\text{em}} = - eA \mu \bar{\psi} \left( 1 - \tau_3 \right) / 2 \gamma_H \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}
\]

RMF models are currently being reinterpreted in the spirit of density functional theory as approximations to the exact (but unknown) density functional [6]. Viewed as a relativistic Kohn-Sham scheme, the single-particle wave functions have no exact physical meaning. At the Fermi surface, however, one can expect good correspondence with the physical single-particle states. Also, differences between them are as in, for example, spin-orbit splittings, are expected to have better accuracy than the energy levels themselves. RMF models are effective field theories for nucleonic degrees of freedom below an energy scale of $\Lambda < 1$ GeV. Short-distance physics including vacuum contributions, nucleon form factors, and short-range correlations are absorbed in the various terms and coupling constants. In this context, the model can be related to low-momentum QCD by applying naive dimensional analysis (NDA) or QCD scaling to the coupling constants [8, 9]. This method should lead to dimensionless coupling constants of order 1 if the Lagrangian is organized in such a way that more complicated processes, i.e., many-body-forces, become weaker with increasing order. As of today, modern high-quality RMF forces fulfill this criterion, indicating that QCD scaling is realized in finite nuclei [3].

**PREDICTIVE POWER**

We first look at the accuracy obtained in the adjustment of modern RMF forces in Fig. 1. Both forces have been adjusted to the nuclei and types of observables shown in the figure. PC-F1 is a point-coupling force, while NL-Z2 employs finite-range boson fields. One sees that the binding energies are described most accurately with an average accuracy of 0.25%. The radii, in most cases, are reproduced within about 0.5%. The surface thicknesses show the largest errors. The low-density part of the nucleus still poses problems in these approaches, see also [7]. Possible cures are the introduction of explicit nucleon form-factors in the iterative procedure (they are used *a posteriori* for the calculation of the charge density after the iterative calculation has been completed) or higher order and/or density dependent gradient terms.

Comparing the average errors between PC-F1 (point-coupling interaction) and NL-Z2 (finite-range interaction), we see slightly different trends. NL-Z2 is superior with respect to binding energies and the surface, while it performs less well concerning radii. The total $\chi^2$ of NL-Z2 is 34% larger than the one of PC-F1, but the differences in performance are not dramatic.

From the calculated single-particle wave-functions, the charge density and hence the form-factor can be extracted. The form-factor of $^{48}$Ca is shown in Fig. 2 for two point-coupling forces (PC-F1 and PC-LA [4]) and two finite-range forces (NL-Z2 and NL3 [11]). The first root and the height of the first minimum are of special importance, as they correspond to the diffraction radius and the surface thickness, respectively. We see that all forces overestimate the first root of the form factor, leading to a slightly too small diffraction radius. The following maximum and minimum are reproduced with great accuracy. Note, however, that this nucleus was part of the adjustment procedure and that for other nuclei these deviations can be greater.

Modern RMF models reproduce correctly the magic numbers for known nuclei. However, similar to nonrelativistic mean-field models, they have the tendency to overestimate shell-structure in terms of separation energies and magic gaps. The inclusion of ground-state correlations in these calculations can lead to a significant improvement [12]. Though in many cases the experimental single-particle energies are reproduced with good accuracy, in some cases their ordering is reversed. From a strict density-functional theory point of view, this observation does not come as a surprise, since the wave-functions are only viewed as auxiliary fields without any strong correspondence to the physical single-particle states. On the other hand, many observables need an accurate reproduction of physical states. Future efforts should include adjustments that are sensitive to the specific shell-structure of nuclei.
FIGURE 1. Percentage errors for the observables binding energy, diffraction radius, surface thickness, and rms charge radius for PC-F1 (filled diamonds) and NL-Z2 (open squares) are seen on the left. The right panels show the absolute mean errors for the corresponding observables. The dashed lines indicate the chosen uncertainties $\Delta O$ in the adjustment procedure.
In the work described above, we have been interested in describing simultaneously the energy and the form factor of the nucleus. Other recent applications of non-relativistic mean-field models, which will most probably be followed by similar work with RMF models, are the design of self-consistent mass models [13].

OUTLOOK

Modern RMF models have reached a high accuracy in describing nuclear ground-state observables. They are relevant for nuclear data applications where a consistent set (stemming from one single calculation) of ground-state observables for a given nucleus is required. Also, we may hope that future-generation relativistic mean-field models will provide reliable extrapolations to unexplored regions of the nuclear chart.

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REFERENCES