Microscopic Scattering Calculations and Data Analysis in Light Nuclei

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Abstract. We report on a consistent, practically converged microscopic calculation of the scattering states in the 4He system employing modern realistic two-nucleon and three-nucleon potentials in the framework of the refined resonating group model (RRGM). The extension to the 5He system is briefly discussed. Comparisons are made for selected examples of phase shifts and data. The extension to heavier nuclei, using semi-realistic NN-forces allows only for qualitative comparisons.

INTRODUCTION

The Refined Resonating Group Model [1] was modified to allow for realistic NN- and NNN-forces. The 4He atomic nucleus is one of the best studied few-body systems, both experimentally and theoretically, as summarized in the recent A/BP 4 compilation [2]. Besides the many textbook examples of gross structure, there are subtle points yielding large effects that are only qualitatively understood. Except for [3] none of the existing calculations aims at a complete understanding of the many features of 4He, which is not surprising in view of the number of different phenomena studied so far [2].

We calculated for the 4He compound-system the S-matrix elements using the realistic two-nucleon potential Argonne AV18 [4] and the three-nucleon Urbana IX (UIX) [5] potential. All two-fragment channels p–3H, n–3He, and d–2H were taken into account together with approximations for the three- and four-body break-up channels. Using V18 and UIX the experimental binding energies of the fragments and the thresholds were reproduced within 20 keV or better.

For the scattering calculation we included the S-, P-, D-, and F-wave contributions to the \( J^\pi = 0^+, 1^+, 2^+, 3^+, 4^+, 0^-, 1^-, 2^-, 3^-, 4^- \) channels. From the R-matrix analysis these channels are known to suffice to describe the data. The results are compared either directly to data or, on a partial-wave-by-partial-wave basis, to a new comprehensive R-matrix analysis [6] of a large amount of data in the 4He system. These calculations need large computer resources. A first extension to the 5He system is on the present limit.

For heavier nuclei we discuss briefly the use of semi-realistic forces, and what can be learned for the data analysis from such kinds of calculations.

RESULTS FOR THE 4HE SCATTERING SYSTEM

Physical observables are complicated bi-linear combinations of S-matrix elements. It is enough that one of these elements is incorrect in the calculation to destroy a direct comparison with experiment. Therefore a comparison of individual partial waves is much more favourable. The procedure of fitting complex, energy-dependent S-matrix elements to data at one energy is in most cases not feasible due to lack of enough data. The way out of this dilemma is the use of an energy-independent R-matrix description, where the energy dependence is given by penetrabilities to connect data at different energies and then calculate the S-matrix from the R-matrix. However, one has to keep in mind that any wrong dataset entering the R-matrix analysis may yield an incorrect parameterisation. Despite this caveat, we consider the results of the R-matrix analysis the closest we can come to a comparison of microscopic calculation and data. Let us now discuss a few typical examples.

In Fig. 1 the calculated 0+ phase shifts for the NN-interaction AV18 are compared to the recent R-matrix analysis. Below the 3He-n threshold, the agreement for the triton-proton \( 1S_0 \) phase shift is perfect; above the agreement is qualitatively good. The energy dependence has to be wrong, due to the missing binding energies. The kink in the proton-triton and 3He-n phase shifts occurs at the calculated deuteron-deuteron threshold. For a better comparison, the d-d phase shifts have been shifted to the experimental threshold.

In Fig. 2 the same R-matrix phase shifts are compared to the full calculation, using the AV18 NN-potential together with the Urbana IX three-nucleon force. Now the thresholds are well reproduced by the calculation and the
FIGURE 1. Elastic $0^+$ phase shifts for all physical two-fragment channels calculated for the AV18 potential alone. The RRGM $1S_0$ phase shifts are displayed as full lines for the t-p ones, as dashed lines for the $^3$He-n ones, and as dotted lines for the d-d ones. The corresponding $R$-matrix results are given by $+$ for t-p, $x$ for $^3$He-n, and $+$ for d-d. The $^5D_0$ d-d phase shifts are especially marked. The RRGM calculated d-d phase shifts are shifted by 0.9 MeV to the experimental threshold.

FIGURE 2. Same as Fig. 1, but now calculated for AV18 and the three-nucleon force UIX.

phase shifts of the analysis agree almost perfectly with the microscopic calculation. Considering that the diagonal $S$-matrix elements near 5 MeV are tiny due to strong isospin coupling, such a degree of agreement is surprising. Only below the $^3$He-n threshold are the calculated triton-proton phase shifts slightly more attractive than those from the analysis.

In Fig. 3 we display the calculated $^3P_2$ phase shifts together with those from the $R$-matrix analysis. In the calculation NN- and NNN-forces are taken into account, as is the coupling to the corresponding $^3F_2$ channels. The triton-proton and $^3$He-n phases do not reach the experimental values. Since this phase shift is dominat- ing the respective vector analyzing powers, these are not too well reproduced in the calculation. The deuteron-deuteron phase shifts are much more repulsive in the cal-

FIGURE 3. Same as Fig. 1, but for the $^3P_2$ phase shifts, calculated for AV18 and the three-nucleon force UIX. The coding is the same as in Fig. 1.

FIGURE 4. $^3$He(n,p)$^3$H total cross section calculated for AV18 (NN) and AV18 together with UIX (NN+NNN) compared to the ENDF standard.

culation than in the analysis. The qualitative agreement is altogether quite good.

In contrast to these findings the $D$- and $F$-waves show in the $R$-matrix analysis quite some $J$-splitting for the deuteron-deuteron channels, as well as for the nucleon three-nucleon channels, whereas the microscopic calculation yields essentially no splitting. The magnitude of the phase shifts is, however, quite small in the energy range considered. Let us now compare observables directly.

Due to recent experiments, see [7] and [8], the complex spin-dependent scattering lengths of neutrons off $^3$He gained much interest. These observables are closely related to the total neutron cross section. This cross section is one of the neutron standard cross sections. In Fig. 4 the low-energy cross section for various nucleon interactions is compared to the standard data. They are a bit overpredicted by the AV18 alone and a bit on the lower side for AV18 + UIX. These results are published in detail in [9]. Around 250 keV the cross section starts
to deviate from the $1/v$ behaviour. The results from AV18 alone stay quite close to the $R$-matrix analysis, which is the basis of the ENDF standard, whereas the results for AV18 + UIX start to oscillate around the $R$-matrix ones and then fall well below them.

In Fig. 5 we compare the calculated differential cross section for the reaction $^2$H(d,n)$^3$He at a rather low energy with the results from the $R$-matrix analysis and data [10]. In all calculations, $F$-waves are absolutely necessary to reproduce the steep rise at forward and backward angles. The NN-forces alone overshoot the data appreciably, whereas the full calculation reproduces the data nicely. The $R$-matrix analysis misses the forward/backward rise, due to a lack of data in this angle range. These missing data and similar cases might be the reason for the unwanted large J-splitting of the higher partial-waves, discussed above. These examples are an excerpt of a forthcoming detailed paper [11].

PRELIMINARY RESULTS ON $^5$HE

To go from four nucleons to five nucleons requires many more computer resources, because now we have to deal with the wave function of the $\alpha$-particle in one kind of channel and a product of triton and deuteron wave functions in the other channels. Taking the triton and deuteron wave functions from the previous chapter and the there-calculated ground-state wave function for $^4$He is beyond the available computer resources. Therefore we reduced the complexity of the wave functions, so that we lost two MeV on the binding energy of the $^4$He ground state for the full calculation using AV18 and UIX. The triton and deuteron wave function were reduced accordingly, in order that the full calculation yield almost the correct threshold energy. We estimate the computer time necessary for calculating $S$- and $P$-wave $\alpha$-n and triton-deuteron $S$-matrix elements to more than an order of magnitude more than the $^4$He calculation altogether.

In Fig. 6 the $^2S_{1/2}\alpha$-n and triton-deuteron phase shifts are compared to an old analysis [12] and a recent one [13]. The NN-results alone already nicely reproduce the $\alpha$-neutron results of the analysis, those of the full calculation even better. The calculated t-d results are more repulsive than the analysis. The results for the AV18 alone had to be shifted by 2.5 MeV to the experimental threshold. The effects of the NNN-force are tiny for these partial waves. Note, the full calculation for the t-d channel is not yet fully converged. The missing attraction beyond the t-d threshold might be due to the yet-missing $^4D_{1/2}$ t-d channel.

EXTENSION TO HEAVIER NUCLEI

For heavier nuclei the situation becomes much more complicated because of (many) low-lying narrow resonances and of (many) excited states of the fragments. Due to computer-time limitations, mainly caused by the NNN-forces, a calculation from first principles is not yet feasible within the refined resonating group framework. Therefore one has to reduce the complexity of the problem by relying on much simpler nuclear forces. These semi-realistic or effective forces allow for a binding of the lightest nuclei already with pure $S$-wave components, by reducing the repulsive core in the central part of the nuclear interaction. For these simplified forces calculations up to mass number 10 [14] have been done and even heavier ones are feasible. In multi-channel calculations these forces tend to overbind the bound states of the compound nucleus, see e.g., [15].
We tried to study the neutron standard reactions $^6\text{Li}(n,t)^4\text{He}$ and $^{10}\text{B}(n,\alpha)^7\text{Li}$ using forces similar to those of [15]. The main question that now arises is the binding energies, and the related question of obtaining all the relative thresholds from one to-be-determined force. To have a stable scattering calculation, the various fragment-wave functions have to be the ones most deeply bound in the chosen model space for the corresponding Hamiltonian. This makes the search for a potential very tedious, especially as the calculated radii should be close to the experimental values, since otherwise the low-energy reaction cross sections might be completely wrong. A typical result is at low energy attraction between the fragments, due to the low density in the outer regions, but when the fragments come closer to each other a strong repulsion due to the Pauli principle arises. Such a situation is quite common and results in too-large reaction cross sections.

Since the analysis of nuclear data in the light nuclear systems is done using relatively few $R$-matrix poles per partial wave, a direct calculation of these poles would be very helpful. Studies in this direction found in scattering calculations that many poles are necessary to describe the scattering-wave functions [16]. Only the low-lying ones, which are well known anyhow, are relatively stable against small parameter variations, whereas close above thresholds many poles accumulate, which fluctuate with small parameter changes. We found no reliable procedure to combine these calculated poles to a few that correspond to resonances.

To compare to an $R$-matrix analysis, the best way we found is to calculate the individual partial-wave cross section and compare these. For the $^6\text{Li}(n,t)^4\text{He}$ reaction qualitative agreement could be found; see a contribution to this conference [17]. For the $^{10}\text{B}(n,\alpha)^7\text{Li}$ reaction, which is much more complicated due to the many resonances in the energy range considered, work is still in progress.

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