Resonance Analysis in the Region of Unresolved Resonances

A.A. Lukyanov\textsuperscript{1}, N.T. Koyumdjieva\textsuperscript{1,2}, N.B. Janeva\textsuperscript{1}, K.N. Volev\textsuperscript{1,2}, and P. Schillebeeckx\textsuperscript{2}

\textsuperscript{1}Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria  
\textsuperscript{2}EC-JRC Institute for Reference Materials and Measurements, 2440 Geel, Belgium

Abstract. The independent analysis of new experimental data for \(^{232}\text{Th}\) cross sections in the unresolved region performed on the basis of the existing scheme and method of evaluation confirm the previously obtained average resonance parameters. The method of statistical modeling of the resonant cross-section structure in the unresolved resonance region, proposed and developed earlier by introducing the characteristic function of \(R\)-matrix elements distribution and the presentation of this by a ladder of fixed "resonances," is used for calculation of the self-shielding factors of \(^{232}\text{Th}\). The results are given in comparison with those of the code NJOY and experimental data.

INTRODUCTION

The new experimental data for \(^{232}\text{Th}\) cross sections in the resolved and unresolved resonance region \cite{1} and the practical interest to the precision of these, require detailed analysis of the data on the basis of the existing scheme and method of evaluation. The methodology of \(R\)-matrix formalism accepted in code SAMMY \cite{2} is used in this work with some development for unresolved resonance regions for effective accounting of the resonance parameter fluctuations.

Average Cross Sections

The result of cross-section averaging, of \(^{232}\text{Th}\) in particular, over the large energy interval containing many resonances is presented by Hauser-Feshbach formulae \cite{3}:

\begin{equation}
<\sigma_\gamma(E)> = 4\pi k^2 \sum_{l} (2l+1) \left[ \sin^2 \phi_l + \cos 2\phi_l \frac{T_{\text{re}}(E)}{1+T_{\text{re}}(E)} \right]
\end{equation}

\begin{equation}
<\sigma_\gamma(E)> = 4\pi k^2 \sum_{l} (2l+1) \frac{T_{\text{re}}T_{\text{at}}}{T_{\text{at}} + T_{\text{at}}} F_l
\end{equation}

where

\(T_{\text{at}}(E) = \frac{\pi}{2} s_1 \sqrt{E P_1(E)}\), \(T_{\text{at}} = \frac{\pi}{2} T_{\text{at}}/\sqrt{D_i}\)

\(s_1\) is the neutron strength function, and \(\phi_l, P_l\) are the phase and penetrability, defined by the parameter \(kR\), \(R\) is the channel radius (Table 1).

TABLE 1. \(\phi_l, P_l (\rho = kR)\).

<table>
<thead>
<tr>
<th>(l)</th>
<th>(\phi_l)</th>
<th>(P_l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(\rho)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(\rho - \arctan \frac{3\rho}{3 - \rho^2})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\rho^4)</td>
<td></td>
</tr>
</tbody>
</table>

The function \(F\) in \(<\sigma_\gamma>\) accounts for the resonance fluctuation effects and can be chosen roughly the same as in the resolved region by the parameterization of \(^{232}\text{Th}\) radiation capture cross section \cite{1} (Fig. 1).

The combined analysis of the total and radiation cross sections in the energy region 4-100 keV confirms the following parameter set, which is same as the set obtained in the resonance analysis of the recent GELINA \(^{232}\text{Th}\) capture experimental data \cite{1}. The analysis of \(^{232}\text{Th}\) resonance neutron total cross section is illustrated in Fig. 2; the ENDF/B-6(8) data file is used.

The average resonance parameters of \(^{232}\text{Th}\) for the energy interval 4-100 keV confirmed by the present analysis:
Here the energy dependence of radius and strength functions is not taken into account, but it is necessary at $E \geq 100$ keV [4].

\[ R = 9.43 \text{ fm, } s_0 = 0.94 \times 10^{-4}, \]
\[ s_1 = 1.96 \times 10^{-4}, \text{ and } s_2 = 1.24 \times 10^{-4}. \]

\[ f(\sigma_0) = \frac{1}{\sigma_0} \text{ and } \quad f(\sigma_0) = \frac{1}{\sigma_0}, \]

where $\sigma_0$ is the cross section of non-resonance additional isotopes in the media – dilution cross section.

We use the code HARFOR for the calculation of these functionals in the unresolved region and the average cross sections as well [6]. Here the detailed energy dependence of resonance cross sections is modeled by the resonance ladder with fixed parameters. In the expression for the U-function of R-matrix theory (for non-fissile nuclei in Rich-Moore approximation [6]):

\[ U = e^{2\pi iK} - iK^{1/2} \sum_{\lambda} e^{x_{\lambda} + \epsilon_{\lambda}}/\epsilon_{\lambda}, \]

\[ \xi_{\lambda} = \frac{\pi E_{\lambda}}{D} \quad \epsilon_{\lambda} = \frac{\Gamma_{\lambda}}{\Gamma_n}, \]

K is chosen into the form [7]:

\[ K(\epsilon) = i(T_n / N) \sum_{\lambda=1}^{N} \xi_{\lambda} \text{ctg}[(\epsilon_{\lambda} - \epsilon - IT_n) / N] \]

with some parameter set at $\xi_{\lambda}$ and $\epsilon_{\lambda}$ in the interval $ND$. The contribution of the levels below and above this interval is supposed periodically repeated, which leads to our presentation of the function K (3) by Eq. (4).

For the choice of parameters $\xi_{\lambda}$ and $\epsilon_{\lambda}$ the method of correspondence is used of the characteristic function of K for the infinite number of the members in the sum over $\lambda$ with statistically distributed resonance parameters (Porter-Thomas distribution for $\xi_{\lambda}$ and Wigner distribution for $\epsilon_{\lambda}$) and the result obtained with the model function $K(\epsilon)$ (4).

The characteristic function of the joint statistical distribution of the real and imaginary parts of K is determined as follows [8]:

**Self-Shielding Factors**

The evaluation of the factors characterizing the resonance self-shielding in the unresolved region is more complicated. These factors are: transmission with relatively thick targets $<\sigma_{\text{pet}}>$, measured self-indication functions $<\sigma_{\epsilon} e^{i\sigma_{\text{pet}}}>$, and self-shielding coefficients used in reactor calculations [4,5] like:

\[ f(\sigma_0) = \frac{1}{\sigma_0} \text{ and } \quad f(\sigma_0) = \frac{1}{\sigma_0}, \]

FIGURE 1. $^{232}$Th capture cross section experimental data [1].

FIGURE 2. Fitting of $^{232}$Th total cross section in the energy interval 4–100 keV.
where \( \gamma = T_n(U+U'), \beta = T_n(U-U') \). We calculated the multiple integral with Wigner distribution \( Q(\varepsilon_\lambda) \) by the Monte-Carlo method as a three-parameter function.

The equivalent model function \( \hat{X}(\varepsilon) \) (4) with fixed parameters of the resonance ladder is the following integral:

\[
\hat{X}(\gamma, \beta, T_\gamma) = \frac{1}{\pi N} \int_0^{\pi N} \exp \left( -\frac{1}{N} \sum_{\lambda=0}^{\pi N-1} \varepsilon_{\lambda} \left( \frac{2T_\gamma}{N} \sin(2(\varepsilon - \varepsilon_{\lambda})/N) - \cos(2(\varepsilon - \varepsilon_{\lambda})/N) \right) \right)
\]

The optimal number of the resonance ladder terms and their parameters are found by fitting this expression to Eq. (5). The cross sections for our periodical ladder with account of Doppler broadening are presented as periodical functions also, and the resonance averaged functionals of the cross sections are determined as average over the period.

The code HARFOR has been used for calculations of average cross sections and self-shielding factors. The main advantage of this method and computer code is the possibility of calculation in the unresolved resonance region the self-shielding factors, which are used for reactor constant calculations and analysis of transmission and self-indication experimental data.

In Figs. 4 and 5 we present a comparison of the value of self-shielding factors calculated by the NJOY Nuclear Data Processing System used for converting evaluated nuclear data in the ENDF format into libraries useful for application calculations with the same data prepared by HARFOR.

**FIGURE 4.** Calculated self-shielding factors for \(^{232}\text{Th}\) for diluted cross section \(\sigma_0=1,100\) barn with the average resonance parameters from endf 6.8 in the energy range 4-50 keV using NJOY and HARFOR

**FIGURE 5.** Calculated self-indication factors for neutron capture for \(^{232}\text{Th}\) for diluted cross section \(\sigma_0=1,100\) barn with the average resonance parameters from endf 6.8 in the energy range 4-50 keV using NJOY and HARFOR

**CONCLUSION**

The reported results illustrate once more the efficiency of the method and code HARFOR for calculation of the average resonance cross sections and especially their functionals. This approach is unique...
and offers the possibility of calculation of the self-shielding factors in the unresolved region directly from average resonance parameters.

The unresolved problems of the evaluation of neutron resonance data are mainly in the intermediate-energy region between resolved and unresolved resonances and are connected with the detailed description of intermediate structure.

ACKNOWLEDGMENTS

This work is partly supported by IAEA in the frame of the CRP Evaluated Nuclear Data for Thorium-Uranium Fuel Cycle.

REFERENCES