Angular Anisotropy of Intermediate Energy Nucleon-Induced Fission of Pb Isotopes and Bi

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Abstract. New results of measured anisotropy coefficients for proton-induced fission of $^{204,206,207,208}$Pb and $^{209}$Bi are presented at proton energies of 48, 98, and 177 MeV. These results, together with earlier ones, are used for an estimation of the dependence of the anisotropy coefficients on proton energy in the framework of the standard statistical model, taking into account the characteristics of the intermediate compound nuclei formed in the process of the interaction of protons with lead and bismuth nuclei.

INTRODUCTION

Measurement of fission-fragment angular anisotropy is a way to determine the state of a fissioning nucleus at the saddle point. This is necessary for the understanding of the key characteristics and dynamics of the fission process. A semi-empirical description of experimental results on the anisotropy of fission induced by protons and neutrons in the energy range above 20 MeV has been presented recently [1], in the framework of the standard statistical (transition state) model [2,3]. Fission of heavy nuclei: $^{232}$Th, $^{235}$U, $^{237}$U, and $^{238}$U was considered. The correlation between experimental data on the angular anisotropy for (n,f) and (p,f) reactions has been analysed, and it has been concluded that the anisotropy for neutron-induced fission may be calculated using the anisotropy for proton-induced fission at the same energies for composite systems having the same fissility parameter, $Z^2/A$, taking into account the difference of the introduced momenta.

In the present work, results of measurements of angular anisotropy in proton-induced fission of $^{204,206,207,208}$Pb and $^{209}$Bi are presented. Data for $^{204}$Pb are obtained for the first time. The other results supplement essentially those of other work in this energy region. As in previous work, the dependence of the anisotropy on proton energy is estimated in the framework of the standard statistical model [2,3], taking into account the characteristics of the intermediate compound nuclei formed in the process of the interaction of protons with lead and bismuth nuclei. The latter are calculated using the TALYS code [4], for which a comparison with experimental values of anisotropy is a new form of testing and verification.

MEASUREMENTS AND EXPERIMENTAL RESULTS

Measurements of fission-fragment angular distributions for intermediate energy proton-induced fission of $^{204,206,207,208}$Pb and $^{209}$Bi were carried out at the proton beam of the Svedberg Laboratory of the Uppsala University, Sweden, at the proton energies 49, 98, and 177 MeV. The experimental setup, measurement conditions, and the procedure of the processing of experimental results are described in detail in our report [5]. The anisotropy factor:

$$C = W'(0°)/W'(90°) - 1,$$

where $W'(0^\circ)$ and $W'(90^\circ)$ are the probabilities of fragment emission at 0° and 90° with respect to the direction of the projectile beam, was obtained by means of fitting of the experimental angular distribution using well known expressions (see, e.g., [6]). The results for $^{209}\text{Bi}$ and $^{204,206,207,208}\text{Pb}$ are presented in Table 1 and shown in Fig. 1, together with earlier results of other authors [7]. As can be seen from the figure, the results of the present work are in good agreement with the earlier data within the stated errors.

### TABLE 1. The anisotropy factor for proton-induced fission fragments.

<table>
<thead>
<tr>
<th>$E_p$ [MeV]</th>
<th>$^{209}\text{Bi}$</th>
<th>$^{208}\text{Pb}$</th>
<th>$^{208}\text{Pb}$</th>
<th>$^{208}\text{Pb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>48±1</td>
<td>0.35±0.05</td>
<td>0.42±0.07</td>
<td>0.35±0.07</td>
<td>0.4±0.1</td>
</tr>
<tr>
<td>98±2</td>
<td>0.22±0.05</td>
<td>0.25±0.1</td>
<td>0.4±0.1</td>
<td>—</td>
</tr>
<tr>
<td>177.3±1.2</td>
<td>0.11±0.05</td>
<td>0.15±0.04</td>
<td>0.17±0.03</td>
<td>0.12±0.03</td>
</tr>
</tbody>
</table>

### FIGURE 1. Energy dependence of the anisotropy factor for proton-induced fission.

**SEMI-EMPIRICAL ANALYSIS OF ANGULAR ANISOTROPY**

In the transition state model [2,3] the anisotropy factor $C$ is merely connected with the mean square of the angular momentum of a nucleus at the saddle point $\langle I^2 \rangle$ and the dispersion of the momentum projection on the axis of the fissioning nucleus, $K^2_0$:

$$C = \langle I^2 \rangle / K^2_0,$$

where $K^2_0$ is determined by the effective moment of inertia, $J_{\text{eff}}$, and temperature, $T$, of a nucleus in this state,

$$K^2_0 = (1/\hbar^2) J_{\text{eff}} T.$$

To describe the dependence of the anisotropy factor on the projectile energy $E$, it is necessary to take into account the change with $E$ of $\langle I^2 \rangle$ and $K^2_0$. For calculation of the maximum angular momentum introduced by protons, the following expression was used [8]:

$$I_{\text{max}}^2 = 2\langle I^2 \rangle = 4.17 E - 30.$$

A change of $K^2_0$ can be connected, in principle, to a change of both $J_{\text{eff}}$ and $T$. However, in this work, the energy dependence of $J_{\text{eff}}$ is not taken into account, because on one hand the minimum proton energy, for which experimental data exist, is about 25 MeV, and therefore the excitation energy at the saddle point is almost equal to a critical value, $\sim 6$ MeV, above which a transition begins from superfluid to the normal Fermi-gas state [9]. On the other hand, at higher energies, where shell effects could be revealed, the most striking feature of the experimental behaviour of $J_{\text{eff}}$ is found to be a sudden change of its dependence on the parameter $Z^2/A$ at $Z^2/A = 30$ predicted by the liquid-drop model, which indicates a liquid-drop state of the nucleus at the saddle point [9]. Unfortunately, theoretical calculations of the influence of possible shell effects on the moment of inertia for nuclei in the lead region were not carried out. $J_{\text{eff}}$ can also depend on the angular momentum, but the dependence is found to be strong only for $I_0 \geq 20 \, \hbar$. However, for such light particles such as protons and neutrons, the value of $I$ does not exceed $20 \, \hbar$ even at the maximum energy 200 MeV considered here. A small value of introduced angular momentum reduces, of course, the value of the anisotropy and makes a measurement of it difficult, but allows for avoiding the necessity of taking into
account the dependence on $I$ of $J_{\text{eff}}$ and other parameters of the model: the fission barrier $B_f$ and the level density parameter $\alpha$. This is a stimulating factor for carrying out measurements of the angular anisotropy for light particles.

The temperature of a nucleus at the saddle point is calculated by the following relationship:

$$T = \left( \frac{E^*}{\alpha_J} \right)^{1/2}$$

$$E^* = E + B_p - B_f - E_v - E_d,$$

where $E$ is the proton energy in the center of mass system, $B_p$ is the proton binding energy, $E_v$ is the energy taken away by neutrons evaporated before the saddle point, and $E_d$ is the energy taken away by the cascade and pre-equilibrium particles. Calculations show that even at $E=200$ MeV, the temperature $T \leq 2$ MeV, and therefore the change of $B_f$ with temperature was not taken into account. The dependence $B_f(I)$ was also not taken into account, because, like the dependence $J_{\text{eff}}(I)$, it is found to be weak at $L \geq 20$ h. [10]. The values of $B_p$ and $B_f$ used are from [9] and [11], respectively. For calculation of $E_v$ it was supposed that one neutron is evaporated for every 6 MeV of excitation energy, which is close to the value from the statistical model. Cascade and pre-equilibrium particles arise in faster processes preceding fission in reactions with intermediate energy nucleons. As a result of these processes, a broad set of residual nuclei arises, which differ in nucleon composition, excitation energy, and magnitude and spatial orientation of the angular momentum. At sufficient excitation energy, such residual nuclei become “ancestors” for chains of emission (multi-chance) fission, i.e., for fissioning nuclei having new masses, excitation energies, and angular momenta. Each one of such nuclei introduces its own contribution to the anisotropy in accordance with $\langle f^2 \rangle$ and $K_0^2$. Therefore a quantitative analysis of the anisotropy requires a “differential” approach. Such an approach was applied in our previous work [12]. Here an “integral” description is given, where $\langle f^2 \rangle$ and $K_0^2$ are averaged over the states of the fissioning nuclei. The use of this approach is caused by the complexity of a quantitative consideration of the mentioned processes and is adequate for the tasks of the present work.

In accordance with this approach, we use the following relation to estimate the energy dependence of the anisotropy:

$$C = \left( \frac{f^2}{J_{\text{eff}}} \right)^{1/2} \left[ \frac{1}{4} k \left( E + B_p - B_f - \Delta E \right) \right],$$

where the coefficient $k$ takes into account the angular momentum carried away and disoriented during the cascade and pre-equilibrium emission of the particles. The energy $\Delta E$ carried away by these particles was calculated using the code TALYS. In the absence at present of similar calculations for disoriented angular momenta, the coefficient $K(E)$ was fitted with the goal of getting a better description of the energy dependence of the anisotropy for the $^{209}$Bi(p,f) reaction, for which a larger amount of experimental data exists. This parameter is equal to 1 below 20 MeV. Starting from this energy, i.e., the energy where direct processes arise, the coefficient falls, leading to a weak dependence of the anisotropy on $E$ at $E \geq 150$ MeV. For all other nuclides, as well as for the (n,f) reactions, $K(E)$ was supposed to be the same (just as for the value of $\Delta E$ which, in accordance with calculations, is found to be almost the same for all nuclides). Of all the considered nuclides, only $^{209}$Bi has a large spin, $I_o=4/2$. However, in this case the correction to the anisotropy,

$$\left[ \frac{1}{2} \right] - 2I_0(I_0 + 1)/18K_0^2,$$

is found to be small due to a large value of $K_0^2$ already at a nucleon energy of 25 MeV.

The dependence of the anisotropy factor calculated in this way is shown in Fig. 2. It is seen that the calculated curves satisfactorily describe the experimental results. The anisotropy drops as the proton energy exceeds the threshold value ($E_a = B_2 - B_2$), shown in the figure by arrows. Then, the anisotropy increases, reflecting the increase of introduced angular momentum, and then drops again as a result of the disorientation of the momentum by fast particles preceding fission (and some increase of the excitation energy of the intermediate compound nuclei). The main difference in the behaviour of the anisotropy for different nuclei takes place at low proton energies and is connected with the difference of the fission barriers (and, correspondingly, $E_a$). At higher energies individual differences disappear, as should be expected. The maximum of the anisotropy factor for all nuclides, with $C$=0.4, is found to be at an energy of about 70 MeV, while at an energy of about 150 MeV and higher, the anisotropy remains constant at a level of $C$=0.1-0.15.

In the same manner, the anisotropy for neutron-induced fission was calculated. Values of $B_n$ and $B_f$ for composite nuclides were taken from [11] and [9]. The introduced angular momentum was adopted to be $\langle f^2 \rangle = 2.5 E$ [9]. $E_v$ did not change, and $E_d$ was
calculated using the code TALYS. Small corrections were introduced to the moment of inertia, $J_{\text{eff}}$, i.e., from 2% to 10%, on the already mentioned dependence $J_{\text{eff}} (Z^2/A)$ [9]. The results are presented in Fig. 2.

![Figure 2](image)

**FIGURE 2.** Energy dependence of the anisotropy factor for neutron-induced fission.

Comparison of Figs. 1 and 2 indicates somewhat higher values of the anisotropy coefficient for neutrons, $C_n$, than for protons, $C_p$, especially on the left side of the maximum, which is connected with the higher value of $\langle I^2 \rangle$. However, the energy dependences of $C_n$ and $C_p$ are essentially the same. Only one experimental result is published for $C_n$ at $E_n = 75$ MeV [12], shown in Fig. 2. It has large errors, but does not contradict the calculated data. Measurements of the anisotropy of neutron-induced fission are of most interest for $^{209}$Bi and $^{208}$Pb, where, according to the code TALYS, the fission branch of the composite nuclei $^{209}$Pb and $^{210}$Bi remains at more than 75% up to neutron energies of 50 MeV, as well as measurements for the $^{207}$Pb nucleus, where the composite nucleus $^{208}$Pb, with closed neutron ($N=128$) and proton ($Z=82$) shells, forms.

For determining the influence of the shells, differential calculations are necessary, i.e., taking into account a consecutive contribution to the anisotropy of all intermediate compound nuclei with their inherent $\langle I^2 \rangle$, $K^2$, and formation probability. The problem can be facilitated by the fact that at comparatively low proton and neutron energies, below 50 MeV, where the influence of shell effects may be significant, the contribution to fission and, correspondingly, to the anisotropy, is introduced by no more than 4–5 composite nuclei. Probably, with such a differential approach, conclusions can be made about the presence or absence of non-statistical pre-saddle neutrons, caused by a dynamic delay of the fission process.

**CONCLUSION**

It is shown that the magnitude of the angular anisotropy, and its dependence on energy, can be described in the framework of the standard statistical model, taking into account the characteristics of intermediate compound nuclei formed in the process of interaction of intermediate energy protons and neutrons with bismuth and lead nuclei. For studying the influence of shell effects, differential calculations are necessary.

**REFERENCES**