Head-Tail Instability of a Super-bunch

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Abstract. Super-bunch acceleration is a key concept in an induction synchrotron. In the induction synchrotron, super-bunches confined in the longitudinal direction by a pair of barrier voltages are accelerated with long induction step voltage pulses. Synchrotron oscillation of the super-bunch is notable, which consists of long drifting between the barriers and quick reflection in the barrier regions. This is apparently distinguished from that of the conventional RF bunch, which is the pendulum oscillation. This property has been supposed to bring about qualitatively different features in the head-tail instability of the super-bunch. Recently the head-tail instability of the super-bunch has been systematically examined. In this paper, the preliminary results of macro-particle simulations is reported.

INTRODUCTION

Super-bunch acceleration is a key feature in an induction synchrotron [1]. In the induction synchrotron, super-bunches confined in the longitudinal direction by a pair of barrier voltages are accelerated with long induction step-voltage pulses (see Fig. 1).

![FIGURE 1. Schematic view of the induction synchrotron. $V_c$ is the barrier voltage for the longitudinal confinement and $V_a$ is the accelerating voltage.](image)

Experiments for proof-of-principle (POP) of the induction synchrotron are going to be performed step by step using the KEK 12-GeV PS [2]. In the first step, a single bunch that is captured in the RF bucket has been accelerated with the induced step-voltage alone. As the second step, an induction barrier experiment is planned, where 1-9 booster RF bunches are injected into the main ring, immediately captured in the induction barrier bucket, and then merged into a single super-bunch. In the last step, a super-bunch will be accelerated up to the flat-top energy with the induction voltage.

A super-bunch confined in the barrier bucket has a notable feature: extremely slow synchrotron oscillation with drifting between the barriers and the quick reflection in the barrier region. This is distinguished from that of the conventional RF bunch. Following beam dynamics issues associated with this property have been addressed; 1) emittance blow-up due to stochastic motion caused by the steep barrier, 2) asymmetric beam acceleration by a droop voltage, which is indispensable in the induction accelerator system, 3) delicate reaction of super-bunch against various perturbations in the accelerating voltage, and 4) effects of slow mixing on various coherent instabilities. The results of study on issue 1) - 3) were presented in Ref. [3]. In this paper, the preliminary study results of issue 4) is presented.

Before proceeding, assumptions concerning the preliminary study discussed here are noted as follows. (a) A half turn constant wake, $W_0 = 10^{12} V/(mC)$ [4] for the half turn and 0V/(mC) for another half turn, was assumed. (b) The bunch length of a super-bunch was supposed to be shorter than the half of the circumference of the KEK-PS MR. The assumptions...
of (a) and (b) mean that the tail-head effects for a super-bunch was not included. Air-bag and water-bag distribution were taken into account in the longitudinal direction. The energy of test particles was fixed at the injection energy 500MeV. The space charge effect was not considered.

**BARRIER BUCKET CONFINEMENT AND BETATRON OSCILLATION**

A barrier voltage for the super-bunch beam is defined by using the step-function,

\[ u_x(t) = \begin{cases} 0 & (t < x) \\ 1 & (t \geq x) \end{cases} \]

where \( V(z) = V_{step} f(z) \left[ u_{\pm z_0}(z) + u_{z_0}(z) - 1 \right] \),

where \( z \) is the fractional time-difference from a synchronous phase, \( 2z_0 \) is the core-length of the super-bunch, and \( V_{step} \) is the peak voltage. \( f(z) \) represents a rising or falling profile of the barrier voltage. In this paper, \( f(z) = \left( |z| - z_0 \right) / \tau \) \( \left( |z| < z_0 + \tau \right) \) and \( 1 \left( |z| \geq z_0 + \tau \right) \) were used, where \( \tau \) is the rise and fall time of the barrier voltage. The sign of the voltage must be changed beyond the transition to maintain the phase stability, just as in a conventional RF synchrotron.

The temporal evolution of the momentum spread \( \delta \) and \( z \) for the particle of interest are given by the following difference equations:

\[ \delta_{n+1} = \delta_n + \frac{eV(z_n)}{\beta^2 E_n}, \quad (1) \]

\[ z_{n+1} = z_n + \eta \gamma_0 T_0 \delta_{n+1}, \quad (2) \]

where \( T_0 = 2\pi / \omega_0 = C_0 / (\beta c) \) is the revolution period of the synchronous particle, \( C_0 \) is the circumference of a ring, \( \beta \) is the relativistic beta of the design particle, \( c \) is the velocity of light and \( \eta \) is the slippage factor.

The betatron motion for test particles was calculated by using a transfer matrix, where the perturbation caused by a wake field was included as delta-function-like kicks in orbit tracking:

\[ \begin{pmatrix} x_{n+1} \\ \dot{x}_{n+1} \end{pmatrix} = M \begin{pmatrix} x_n \\ \dot{x}_n \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{F}{\gamma_0 m} T_0 \end{pmatrix}, \quad (3) \]

\[ F_s = e^2 W_a \int \left| \rho(z) (s(z')) \right| dz', \quad (4) \]

\[ \omega_\beta = \frac{2\pi}{T_0} Q_s (1 + \xi \delta), \quad (5) \]

where \( m \) is the rest mass, \( Q_s \) is the bare tune, \( \xi \) is unnormalized chromaticity and \( \rho \) is the beam distribution in the longitudinal. \( <x> \) means the center of beam distribution in the horizontal.

The multi-particle simulation for the head-tail instability was performed by using Eq. (1)-(6), where \( V_{step} \) was set to 85kV for capturing particles with 0.4% by the pulse of ~100nsec, and \( \tau \) was set to 10nsec for avoiding the longitudinal diffusion due to the steep barrier \( [3] \). \( Q_s \) was set to 7.10.

**AIR-BAG DISTRIBUTION**

Fast of all, the head-tail instability for a super-bunch of the air-bag distribution has been examined. The dependences of the growth rate on the beam intensity and the bunch length are shown in Fig. 2 and 3, respectively. In the case of \( \xi = 0 \), the threshold can be seen in Fig. 2. On the other hand, it seems that there is no threshold for \( \xi = -1 \). In both cases, the growth rate becomes larger as the longer bunch length. This means that the super-bunch of the air-bag distribution becomes more unstable as the longer bunch length against the head-tail instability independent of \( \xi \).

**FIGURE 2.** Dependence of growth rate on beam intensity. Air-bag distribution.
WATER-BAG DISTRIBUTION

In the case of the water-bag distribution, the dependences of the growth rate on the beam intensity and the bunch length are shown in Fig. 4 and 5, respectively. It seems that there is the threshold for $\xi = -1$ and no threshold for 0 in contrast to that of the air-bag. However, it is notable that the beam intensity on the threshold for $\xi = -1$ is very huge, so that the longitudinal space charge effects should be serious rather than the head-tail instability.

In the case of $\xi = 0$, the growth rate becomes larger as the longer bunch length in the similar way to the former section. On the other hand, the super-bunch becomes more stable for $\xi = -1$ as the longer.

The theoretical approach is now under studying. So the results will be presented soon.

SUMMARY

The preliminary results for the head-tail instability of a super-bunch have been discussed. In the realistic system ($\xi < 0$, water-bag distribution), longer super-bunch seems to be more stable against Head-Tail instability. However, more realistic study should be needed.

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