Longitudinal and Transverse Impedances and Shielding Effectiveness of a Resistive Beam Pipe for Arbitrary Energy and Frequency

A.M. Al-khateeb*, O. Boine-Frankenheim†, R.W. Hasse‡ and I. Hofmann‡

*Department of Physics, Faculty of Science, Yarmouk University, Irbid, Jordan
†GSI Darmstadt, Planckstr. 1, D-64291 Darmstadt, Germany

Abstract. The longitudinal coupling impedance of a cylindrical beam pipe for arbitrary relativistic energy and mode frequency is obtained analytically for finite wall conductivity and finite wall thickness. Closed form expressions for the electromagnetic fields excited by a beam perturbation are derived analytically. General expressions for the resistive–wall impedance in the presence of a metallic shield and for the rf shielding effectiveness of the beam pipe have been obtained. The results are applied to the GSI synchrotron SIS, where the thickness of the vacuum chamber in the dipole magnets is much smaller than the skin depth at injection energy.

In addition, the transverse space-charge and resistive-wall impedances have been investigated analytically of a smooth cylindrical beam pipe of finite conductivity. Transverse beam charge distributions of a hollow beam and of a uniform beam are considered yielding different results. Closed form expressions for the excited electromagnetic fields in the beam-pipe-wall regions and for the corresponding total transverse impedance can be derived analytically for high energy beams.

Keywords: Impedances, shielding effectiveness, resistive beam pipe

PACS numbers: 3.35.Ei, 78.60.Mq,29.27.Bd, 29.20.-c,47.27.v,85.30.F

INTRODUCTION

In order to prevent beam instabilities, for the design of accelerators it is desired to reduce the coupling impedance of the beam to its environment in. The longitudinal coupling impedance, which includes the space-charge and resistive-wall impedances, is an important physical quantity for understanding and modeling of the longitudinal dynamics of charged particle beams and of the corresponding longitudinal beam instabilities [1-4].

When the skin depth is larger than the wall thickness the beam induced electromagnetic fields can penetrate through the wall and the impedance depends on the structures outside the pipe. The shielding effectiveness (SE) of a conducting layer is defined in terms of the incident and transmitted electric fields as SE=20log_{10}(E_i/E_t). During the bunch compression foreseen in the proposed heavy ion synchrotron SIS 100 for example, peak currents approaching 100 A should be reached. Assuming 40 dB shielding effectiveness of the beam pipe still 1% of the induced fields or 1 A of the peak image (displacement) current could in principle ‘leak through the pipe’.

The shielding of beam generated rf fields by thin conducting layers was considered in a number of works [5, 6]. The well known ability of a thin layer of thickness d less than the skin depth δs to shield electromagnetic fields [7] produced by a particle beam was considered in refs. [8, 9], where approximate expressions for the impedance and for the shielding effectiveness of thin layers in the limit of low frequencies or high γ0 were found.

The shielding by a beam pipe which is thin as compared to the skin depth is of relevance for the SIS 18 heavy ion synchrotron at GSI as well as for the design of the new SIS 100/300 as part of the FAIR project [10]. The SIS 18 magnets can be ramped with 10 T/s, the superconducting SIS 100 magnets shall be ramped with 4 T/s. In order to reduce Eddy current effects, the stainless steel beam pipe of SIS 18 is only 0.3 mm thick. The skin depth at injection (11.4 MeV/u) is 1 mm. For the stainless steel or titanium beam pipe in the new SIS 100 a thickness of a few 0.1 mm will be required [11].

In this paper we will derive the electromagnetic fields associated with a particle beam moving in a beam pipe of finite wall conductivities. The excited electromagnetic fields and the corresponding coupling impedance will be obtained and discussed in the presence of a metallic shield. Shielding happens via a thin metallic cylindrical layer, where expressions like resistive–wall impedance, shielding effectiveness, and wall losses via attenuation of fields behind a good conducting shielding layer will be calculated. Then we apply our results to the SIS heavy...
ion synchrotron. More details can be found in ref. [12].

At the end we will calculate transverse impedances on the same footing thereby using a hollow or a uniform beam charge distribution yielding different results.

LONGITUDINAL IMPEDANCES AND SHIELDING

Here we calculate the coupling impedance and the shielding effectiveness of a cylindrical pipe. We consider the case of a finite size beam of radius $a$ inside the metallic cylindrical pipe of thickness $d$ and of conductivity $S$ extending from $r = b$ to $r = h = b + d$. Outside the pipe for $r > h$ there is vacuum.

In the region of the beam-pipe which is free of charges, we find the electric field in that region by setting the source term to zero. On the other hand, within a conducting metallic region of conductivity $S$ which includes no bulk free charges, the source term $k_2^2 / \gamma^2$ is included.

Here $\delta_i = \sqrt{2/\mu_0 S \omega}$ is the skin depth for the frequency $\omega$. $\gamma$ is the relativistic factor, $\gamma = k_z / \beta$, $\sigma = k_z / \gamma$ and $\gamma^2 = \gamma_0^2 - \mu_0 S \omega k_z^2$. The presence of a conducting finite thickness cylindrical pipe results in the following contribution to the longitudinal impedance, namely,

$$Z_2(\omega) = (1 - i) \frac{n Z_0 \beta \delta_i^*}{2 \sqrt{n b}} \frac{4 I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)} \times$$

$$\left[ 1 + \eta \frac{K_1(\sigma_0 b)}{K_0(\sigma_0 b)} \tanh \frac{\sigma d}{n} \right] + \left[ 1 + \eta \frac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)} \frac{I_1(\sigma_0 h)}{I_0(\sigma_0 h)} \tanh \frac{\sigma d}{n} \right].$$

For a cylindrical pipe the impedance $Z_2(\omega)$ in eq. (1) represents the resistive-wall impedance, which is the total impedance minus the impedance $Z_1$ of a perfectly conducting beam pipe. The longitudinal coupling impedance without cylindrical pipe ($d \rightarrow 0$) is purely imaginary. We see from eq. (1) that the impedance becomes also independent of $b$. According to the beam-pipe structure treated here and for vanishing thickness of the pipe, the interface at $r = b$ becomes a virtual one so that it can be moved to infinity without affecting the calculation results. This result is consistent with that obtained in the case of a smooth perfectly conducting beam-pipe with $b$ being moved to infinity (‘vacuum space charge impedance’).

In the following we apply our results to the synchrotron SIS at GSI. In Fig. 1 we plot the real part of the resistive wall impedance as a function of $d$ for SIS parameters and the conductivity of stainless steel $S = 10^6 \Omega^{-1} m^{-1}$. As expected, the thick wall limit of Eq. (1) can be used for $d > \delta_i$. For $\gamma = 2$ and $n = 1$ the skin depth is slightly larger than the wall thickness (0.3 mm) in the dipoles. For $d < \delta_i$ the real part of the impedance is proportional to $1/d$. For very small $d$ below 1 $\mu m$ the exact impedance decreases towards zero. For SIS parameters the resistive wall impedance divided by the harmonic number remains well below 10 $\Omega$. The total (space charge and resistive wall) imaginary part of the wall impedance is plotted in Fig. 2. For $d \gtrsim 1 $ $\mu m$ the imaginary part is dominated by the space charge impedance. For smaller $d$ the imaginary part of the total wall impedance tends towards the vacuum result.

FIGURE 1. Real part (solid line) of the resistive wall impedance for $n = 1$ and $\gamma_0 = 2$ as a function of the wall thickness $d$ (SIS 18).

The transmission coefficient through a beam pipe is obtained from the electric fields as

$$\tau^{-1} = \sqrt{\frac{h}{b} \frac{\sigma_0 b K_1(\sigma_0 h) I_0(\sigma_0 b)}{K_0(\sigma_0 h) I_1(\sigma_0 h)}}$$

$$\times \left[ \tanh \frac{\gamma d}{n} + \frac{K_0(\sigma_0 h)}{\eta K_1(\sigma_0 h)} \sinh \frac{\sigma d}{n} \right]$$

$$+ \frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)} \left( \eta \sinh \frac{\gamma d}{n} + \frac{K_0(\sigma_0 h)}{K_1(\sigma_0 h)} \cosh \frac{\sigma d}{n} \right).$$

FIGURE 2. Absolute value of the imaginary part (solid line) of the total (space charge and resistive wall) impedance for $n = 1$ and $\gamma_0 = 2$ as a function of the wall thickness $d$ (SIS 18).
Equation (2) is valid for any pipe thickness $d$ and for arbitrary beam velocity. In the limiting case $\sigma_c b = kb/\gamma_0 \ll 1$ and $\sigma_c h = kh/\gamma_0 \ll 1$ corresponding to not too high frequencies or to ultrarelativistic beam energies and for $b \approx h$, eq. (2) becomes

$$\tau^{-1} \approx 1 + \frac{\beta^2 k^2 b}{2} + i \frac{2bd}{\beta^2 \gamma_0^2 \delta_s^2} \ln \frac{kb}{\gamma_0}.$$  

Note that in the equivalent formula of ref. [8] the first sign in the denominator is a minus sign.

![FIGURE 3](image3.png)

**FIGURE 3.** Transmission coefficient vs. $d$ for $\gamma_0 = 2$ and $n = 1$, circumference $C=216$ m, $b=10$ cm. The dashed line represents the transmission coefficient obtained from eq. (2).

![FIGURE 4](image4.png)

**FIGURE 4.** Transmission coefficient vs. $d$ for $\gamma_0 = 2$ and $n = 1$, SIS 100) with $C=1080$ m, $b=5$ cm. The dashed line represents the transmission coefficient obtained from eq. (2).

The shielding of the beam generated rf fields by the beam pipe is an important issue for the SIS high current operation as well as for the design of the vacuum chambers for the proposed SIS 100/300 synchrotrons. In the SIS the thickness of the stainless steel vacuum chamber in the dipoles is $d = 0.3$ mm. During the planned generation of intense, short (50 ns) $^3$He bunches peak currents exceeding 10 A will be reached in the SIS at a maximum energy of 1 GeV/u, i.e. $\gamma_0 \approx 2.0$. A corresponding peak image current will flow through the beam pipe. For insufficient shielding part of this image current can flow through structures outside the beam pipe, resulting in an undefined longitudinal impedance and possibly in perturbations of the accelerator hardware.

The skin depth $\delta_s$ at SIS injection energy (11.4 MeV/u) is 1 mm, at 1 GeV/u the skin depth of 0.4 mm will still be larger than the wall thickness in the dipoles. In the proposed SIS 100 synchrotron $\delta_s$ at injection ($\gamma_0 = 1.1$) will be as large as 1.5 mm, with a wall thickness in the dipoles of a few 0.1 mm. Therefore a detailed analysis of the shielding efficiency of the beam pipe in the existing SIS as well as in the new SIS 100 is of great importance. Fig. 3 shows the transmission coefficient for $n = 1$, $\gamma_0 = 2$ and SIS parameters as a function of the wall thickness $d$. Values exceeding 1 % are obtained for $d$ below 0.1 mm. The approximate expression, however, can be used for $d \lesssim \delta_s$. The transmission coefficient as a function of $d$ for $\gamma_0 = 2$, for stainless steel and for the SIS 100 parameters is shown in Fig. 4. We see that for stainless steel a wall thickness of a few 0.1 mm might be sufficient in order to provide good shielding.

It is important to note that for the very small $d$ of the order of a few $\mu$m the situation corresponds e.g. to a ceramic beam pipe that is coated with a thin conducting film. The effect of the ceramic pipe on the total impedance and on the shielding effectiveness is very small. The impedance of rf-shielding wires inside a ceramic pipe was studied by Wang and Kurennoy [13]. They observed a weak dependence on the dielectric constant of the ceramic pipe.

A ceramic beam pipe coated with a thin (few $\mu$m) conducting film of e.g. Cu ($5 \cdot 10^7$ $\Omega m^{-1}$) could provide transmission coefficients of the order of $10^{-3}$ for SIS 18. For SIS 100 parameters we obtain a 'good' shielding coefficients of the order of 1 % for standard operation.

### TRANSVERSE IMPEDANCES

The transverse radial impedance $Z_r$ is defined as,

$$Z_r = \frac{i}{P_r} \int_{-\infty}^{\infty} dz [E_r - \beta c \mu_0 H_\theta],$$

where the field components are generated by moments $P_r$ of the beam charge and current distributions in radial direction,

$$\rho_r(\vec{r}, t) = \sigma(r, \theta) \delta (z - vt)$$  

$$\bar{j}_r(\vec{r}, t) = \sigma(r, \theta) \beta c \delta (z - vt) \hat{z}.$$  

In the following we treat two cases of particle distributions, a 'hollow' one and a uniform one. Assuming the
hollow particle beam in form of a circular lamina of radius \(a\) and a transverse charge distribution \(\sigma(r, \theta)\). Let the beam motion be off axis with a constant longitudinal velocity in a cylindrical pipe of radius \(b\). To first order the beam radial displacement will be the only dipole source. Dipole sources and the corresponding excited electromagnetic fields in the beam pipe will be used to define the transverse coupling impedance. Accordingly, the total beam charge and current distributions to first order in beam transverse displacement is also radial and, the transverse impedance can now be found as

\[
Z_r = \frac{Z_0 C I_1^2(\sigma_0 a)}{\pi a^2 \gamma_0^2} \left[ \frac{K_1(\sigma_0 a)}{I_1(\sigma_0 a)} - \frac{K_1(\sigma_0 b)}{I_1(\sigma_0 b)} \right], \tag{3}
\]

with \(C\) being the circumference and \(Z_0 = 377 \, \Omega\) is the vacuum resistance.

In the ultra relativistic limit such that \(\sigma_0 a \ll 1\) and \(\sigma_0 b \ll 1\), eq. (3) reduces to the following well known expression for the transverse impedance,

\[
Z_r \approx i \frac{Z_0 C}{2\pi \beta \gamma_0^2} \left[ \frac{1}{a^2} - \frac{1}{b^2} \right].
\]

For a uniformly charged lamina, the surface charge density distribution in the transverse direction is \(\sigma_0 = Q/\pi a^2\). The beam motion is off axis and its charge is radially uniform. The zeroth moment of \(\sigma(r, \theta)\) is the beam charge \(Q\). The dipole field associated with the transverse beam displacement is then obtained from the dipole source correction term for \(\ell = 1\). One integration remains in the final expression for the transverse impedance but, analogously, in the ultra relativistic limit the transverse impedance can be obtained analytically,

\[
Z_r \approx i \frac{C Z_0}{2\pi \beta \gamma_0^2} \left[ \frac{3}{2a^2} - \frac{1}{b^2} \right].
\]

Applications to the present SIS 18 and future SIS 100/300 are under way [14].

ACKNOWLEDGMENTS

One of us (A.A.) thanks the High Current Beam Physics Group of GSI Darmstadt for the kind invitation. He also likes to thank the Council of Scientific Research of Yarmouk University, Irbid, Jordan, for supporting this work by the grant 4-2004.

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