Numerical Model of an Internal Pellet Target

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Abstract. The numerical model of an internal pellet target is proposed to study the beam dynamics in storage rings, where nuclear experiments with such targets are planned. In this model the Monte Carlo algorithm is derived to evaluate the particle coordinates and momentum deviation depending on time and parameters of the target. Some limitations of the conventionally applied methods are discussed. A computer subprogram, which can be easily integrated into any code making particle tracking in storage rings, is developed.

INTRODUCTION

We present a numerical model for the interaction of a circulating proton or anti-proton beam with a pellet target. Such pellet target station is integrated in the WASA detector at the CELSIUS storage ring at The Svedberg Laboratory in Uppsala [1-5]. The target is in operation since 1999. A similar pellet target is proposed for the PANDA detector that will find its home at the HESR storage ring [6] at GSI. The interaction of the protons with the pellets is characterized by rare hits of the pellets, which are moderately thick themselves. The pellet diameter is typically from 20 to 50 µm. This must be compared to, for example, gas scattering events, where the circulating beam interacts with individual rest-gas atoms, but not with small solids, as in case of the interaction with the pellet. On the other hand the pellets are not yet really thick solids that allow use, for example, Landau’s theory of energy loss straggling. Hence the beam-pellet interaction is in between several well-established theories, which are used commonly in accelerator physics [7,8]. In our numerical model we try to solve this dilemma by using algorithms applied in detector simulation tools, such as GEANT [9], and existing alternative algorithms. These algorithms are used for development of special computer subprogram [11], which has possibility to be integrated in other codes for simulations of: the particle coordinates and momentum deviation depending on time and parameters of the pellet target; the time behavior and the equilibrium of the beam for both a collection of non-interacting single particles and a collective ensemble.

PELLET TARGET

The pellets are small spheres of frozen hydrogen with a typical diameter d of about 30 µm, although the size can vary by a factor more then 2. There are about $6 \times 10^{14}$ hydrogen atoms in a single pellet. The pellets are vertically shot through the beam at a rate of about 60 kHz such that the time between pellets is 16 µs. The speed of pellets is on the order of 60 m/s such that one pellet follows the other after about 1 mm. The pellets have a small angular divergence (nearly ±0.04°) and cover about ±2 mm horizontally in the point of intersection with beam. Horizontal profile of the pellet beam is fitted satisfactorily by Gaussian function, distance between vacuum injection capillary and skimmer is nearly 0.7 m, distance from skimmer to beam axis is about 1.7 m[10], length between adjacent pellets particles is nearly constant with not very identified spread. These parameters are only approximate since most of them depend on detailed tuning of the source, used vacuum injection nozzle and many other conditions [1-4, 10].
INTERACTION OF PARTICLES WITH PELLETS

Main target effects, which exert influence on beam parameters of storage rings, are small angle Coulomb scattering (multiple scattering) and energy straggling per target traversal.

Scattering angle?

The widely used model for a multiple scattering is Molière's theory with various modifications. For example the simple Gaussian multiple scattering model for GEANT [9] or more sophisticated method is given in [7]. The main condition for applicability of Molière's theory is exceeding of mean number of collisions \( W_0 \) by proton (or antiproton) passing through the target. For the frozen hydrogen pellet with diameter of 30 \( \mu \)m the average number of collisions \( W_0 \) is about 2. This value is outside of the Molière’s theory validity. In this case the “Plural Scattering” model can be used [9]. Here a direct simulation method is used for determination of scattering angles. The number of scatters is distributed according to the Poisson law with average \( \lambda = 1.167 W_0 \). At every scatter the scattering angle is calculated according to the formula

\[
\theta = \sqrt{\chi_0^2 \left( \frac{1}{\eta} - 1 \right)} \cdot \chi_a^2 = \frac{2.25 \cdot 10^{-11}}{\beta^2 \cdot \gamma^2},
\]

where \( \eta \) is random number uniformly distributed in the interval between 0 and 1, \( \beta \) and \( \gamma \) are normalized speed and energy of the incident beam. One needs the projections of the scattering angle into the horizontal and vertical planes. For that random uniformly distributed phase angle \( \phi \) is generated between 0 and 2\( \pi \). Then \( \cos(\phi) \) and \( \sin(\phi) \) multiplying factors for projection \( \theta \) into horizontal and vertical planes are calculated. The comparison of the scattering angle distribution calculated by “Plural Scattering” model with distribution calculated by Molière's theory is shown in fig.1.

Energy loss straggling

Various theories are used to calculate the particle energy losses and straggling in target [9]. To make choice between different models the ratio \( k = \frac{\xi}{E_{max}} \) should be defined, where \( \xi \) is mean energy transfer, \( E_{max} \) is maximum energy loss in a head-on collision of the projectile with a target electrons. If \( k > 10 \) the Gauss distribution is applied. The Vavilov’s theory is considered under condition \( 0.01 \leq k \leq 10 \) and Landau’s theory is used in the case of \( k < 0.01 \).

There is additional restriction for Landau's method. The mean energy loss \( \xi \) must be much larger than the ionization energy \( I \). But for pellets with diameter of 30 \( \mu \)m the value of \( k \) is much less than 0.01 at one collision and ratio \( \xi/I \) is about 2. Therefore for this range of \( k \) the Urban's approach [9] is preferred over Landau's. Vavilov’s theory at the Bethe-Bloch formula [11]. More sophisticated photoabsorption ionization (PAI) model gives similar results, but with respect to the simulation time the PAI model is slower than Urban's model. In Urban's model the ionization and excitation process are simulated directly by a Monte-Carlo algorithm. The ratio between the energy losses due to ionization and excitation process can be adjusted by a free parameter \( r \). For example, in GEANT this parameter is set to \( r=0.4 \), which means that only 40% of energy losses is accounted by ionizations and 60% by excitations. The ionization losses have long tail at the region of higher energies and parameters of momentum distribution have strong dependence from this tail. In our code the average number of particle collisions in one pellet is calculated for both excitation and ionization processes. A number of these collisions are sampled from a Poisson distribution. Knowing these numbers one can find easily the total excitation energy loss by multiplying number of the excitation collisions on the excitation energies. This value of energy loss is added with ionization loss. The particle energy distribution due to the ionization and the energy loss process is calculated by the formula

\[
g(E) = \frac{(E_{max} + I) \cdot I}{E_{max}^{2}} \cdot \frac{1}{E^2},
\]
where $E$ is energy loss, $E_{\text{max}}$ is maximum energy transfer minus average ionization energy. Using inverse transformation method one obtains the formula for calculation of the ionization energy losses:

$$\Delta E_i = \sum_{j=1}^{n_j} \frac{I}{1-u_j \frac{E_{\text{max}}}{E_{\text{max}} + I}}$$

where $n_j$ is number of events for ionization (sampled from Poisson distribution) and $u_j$ is uniform random number between 0 and 1.

### Numerical results

In order to show the dependence on the pellet diameter we have calculated $dp/p_{\text{rms}}$ and $\sigma_{\text{rms}}$ scattering angle for a typical energies of antiprotons at the HESR and plotted as a function of pellet diameter in figs.2, 3.

**FIGURE 2.** The rms width of relative momentum deviation vs pellet diameter for scattered antiprotons with energies of 1, 3, 8 and 14 GeV.

**FIGURE 3.** The $\sigma_{\text{rms}}$ scattering angle vs pellet diameter for scattered antiprotons with energies of 1, 3, 8 and 14 GeV.

The quantity $dp/p_{\text{rms}}$ is calculated using Urban's model. The quantity $\sigma_{\text{rms}}$ is calculated using “Plural scattering” model.

### CONCLUSIONS

In this paper the numerical model for interaction of the particles with pellet target is discussed. It was shown that for particle traversal through the pellets one should use the "Plural scattering" model to calculate the scattering angles. For determination of the energy loss straggling it is preferable to use so called Urban's model, which is more acceptable in case of small number scatters in one pellet.

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### REFERENCES