Advances of Plasticity Experiments on Metal Sheets and Tubes and Their Applications to Constitutive Modeling

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Abstract. This paper provides a review of experimental techniques which are effective in observing and modeling the anisotropic plastic behavior of metal sheets and tubes under a variety of loading paths. These include biaxial compression tests, biaxial stress tests for metal sheets (cruciform specimens) and tubes using closed-loop electrohydraulic testing machines, an abrupt strain path change method for detecting a yield vertex and subsequent yield loci without unloading, in-plane compression or stress reversal tests for metal sheets and multistage tension tests. Comparison of observed material response with those predicted using phenomenological plasticity models are presented, where possible. Special attention is focus on the verification of the validity of conventional anisotropic yield criteria and its associated flow rule. The effects of the anisotropic yield criteria on the accuracy of forming simulations, such as springback and forming limit strains and stresses, are also discussed.

INTRODUCTION

Product development cycles have shortened recently, ruling out traditional empirical approaches for determining optimum forming conditions. The ultimate goal in manufacturing must be to establish “try-less” manufacturing. Accordingly, there is a need for accurate simulation techniques for metal forming using finite element analysis. For accurate and time-effective finite element simulations, it is vital to use accurate phenomenological plasticity models based on anisotropic yield functions [1]. Material response to variety of modes of loading has been of interest for over a century. Many anisotropic yield criteria and constitutive models have been proposed so far. However, experimental evidence is often lacking and especially so for materials for industrial use.

In sheet or tube forming processes materials are generally subjected to multiaxial loads. Moreover, metal parts are very often manufactured in two or more forming stages. Therefore, multiaxial and/or multistage loading tests are infinitely preferable for the purpose of checking plasticity models to be used in simulations. Servo-controlled testing machines are necessary for these tests. Although not categorized as multiaxial tests, in-plane compression tests or in-plane stress reversal tests are effective in observing and modeling the strength differential effect and the Bauschinger effect. These tests are also important from industrial point of view, because such modes of deformation are very typical in sheet forming processes, such as deep drawing and draw-bending.

This paper provides a review of experimental techniques for measuring anisotropic plastic behavior of metal sheets and tubes under a variety of loading paths. Special attention is focused on the plastic response to multiaxial loading, multistage loading and in-plane compression or reverse loading. Comparison of observed material response with those predicted using phenomenological plasticity models are presented, where possible.

Experimental plasticity is much too broad a subject to review exhaustively in a manuscript such as this. The reader is also referred to the excellent reviews of the early plasticity work by Michno and Findley [2], Hecker [3], Ikegami [4], Szczepiński [5], Stout and Kocks [6] and Mcdowell [7].
BIAXIAL STRESS TESTS ON SHEET METALS AND VERIFICATION OF THE VALIDITY OF NORMALITY RULE

This section provides a review of biaxial stress testing techniques for sheet metals. These include the biaxial compression test on adhesively bonded sheet laminate specimens, the biaxial tension and plane-strain tension tests using cruciform specimens. Examples of measured yield loci (contours of plastic work) and incremental plastic strain vectors for steel alloys are presented and compared with theoretical predictions based on conventional anisotropic yield functions. An effect of material modeling on the accuracy of springback simulation for plane-strain stretch bending is also discussed.

Hereafter, the rolling and transverse directions of the specimen are defined as the \( x \)- and \( y \)-axes, respectively.

Biaxial Compression Test

One of the most appropriate biaxial loading tests for sheet materials is the biaxial compression test [8-10]. Figure 1 shows a schematic diagram of biaxial compression tests using adhesively bonded sheet laminate specimens [9], showing in the \( \pi \)-plane how different stress states can be obtained. One of the disadvantages of this method is the difficulty in obtaining accurate stress-strain relationships at small strains because of the friction between the specimen and the tool. Moreover, when the plastic deformation mechanism of the material is influenced by the hydrostatic component of stress [11, 12], the yield locus shapes obtained from the biaxial compression test may be different from those obtained from the biaxial tension test, as has been demonstrated for a titanium alloy [11].

Biaxial Tension Test on Sheet Metals Using Cruciform Specimens

Another approach for biaxial testing on sheet metals is to use a cruciform specimen and apply biaxial tension. There are two types of cruciform specimen (Fig. 2). One has a smaller thickness in the gage section than at its periphery [13-16], as shown in Figs. 2a-b, and the other is directly made of rolled sheet metals [17-22], as shown in Figs. 2c-f. The former, however, appears to be difficult to realize such geometry using sheet materials approximately 1 mm thick, commonly used in sheet forming operations in the industry. Another shortcoming of this type of specimen is the difficulty in determining an effective cross sectional area for measuring correct stress values.

Kuwabara and coworkers designed and constructed a servo-controlled biaxial tensile testing machine [22] and carried out biaxial tension tests on steel alloys [22-25], aluminum alloys [26, 27] and pure titanium [28] using cruciform specimens shown in Fig. 2f. Each arm of the specimen has seven slits so as to exclude geometric constraint on the deformation of the gage section. Normal strain components, \( \varepsilon_x \) and \( \varepsilon_y \), were measured using biaxial strain gages. Normal stress components, \( \sigma_x \) and \( \sigma_y \), in the gage section were determined by dividing the measured loads, \( F_x \) (rolling direction) and \( F_y \) (transverse direction), by the current cross-sectional area of the gage section, determined from the measured values of plastic strain components, \( \varepsilon_x^p \) and \( \varepsilon_y^p \), under the assumption of constant volume. According to FEM analysis of the cruciform specimen [23], measurement errors of stress are 1.3 % for equibiaxial tension (\( \sigma_x : \sigma_y = 1:1 \)), and 2.2 % (\( \sigma_{11} \)) and 3.2 % (\( \sigma_{33} \)) for plane-strain tension (\( \sigma_x : \sigma_y = 2:1 \)).

To evaluate the work-hardening behavior of anisotropic sheet metals under biaxial tension, we introduce the notion of the contour of plastic work in stress space [29-31], because, from the viewpoint of metal forming simulation, modeling the flow surface as an average behavior of the material over a deformation range is more important than just determining an initial yield locus of the material. A uniaxial tension test in the rolling direction of the material (\( \sigma_x : \sigma_y = 1:0 \)) was first carried out, and the
uniaxial true stress $\sigma_0$ and plastic work $W$ dissipated per unit volume were determined for particular values of the uniaxial logarithmic plastic strain $\epsilon_0^p$. Also, the uniaxial tension test in the transverse direction of the material ($\sigma_0^T : \sigma_0^L = 0:1$) and biaxial tension tests were carried out with the stress ratios in particular proportions: $\sigma_0^T : \sigma_0^L = 4:1, 2:1, 1:2, 1:4, 2:1, 1:1, 3:4, 1:2$ and $1:4$. Finally, groups of stress points, $(\sigma_x, \sigma_y)$, for which the same amount of plastic work as $W$ was needed, were plotted in principal stress space so as to construct contours of plastic work corresponding to particular values of $\epsilon_0^p$. When $\epsilon_0^p$ is small enough, the corresponding work contour is effectively a yield locus of the material. For the stress ratios $\sigma_0^T : \sigma_0^L = 1:0$ and $0:1$, standard uniaxial tension specimens (JIS 13 B-type) were used.

It is noted that we have not used the offset plastic strain for defining of a yield locus, as has been customarily done by a number of other authors. Since the tested materials are anisotropic, the plastic work definition of yielding, that has a definite physical meaning, appears to be more appropriate than the offset plastic strain definition.

Figure 3 shows contours of plastic work measured for two types of uniphase ferritic steels: an IF steel (average $r$-value $r = 1.87, 0.0026$ mass% C) and a hot-rolled, Al-killed steel ($r = 0.95, 0.037$ mass% C) [25]. The stress components are normalized by $\sigma_0$, corresponding to a specific $\epsilon_0^p$. The IF steel exhibits definite differential hardening; the work contour for $\epsilon_0^p = 0.0005$ is very close to the von Mises yield locus, and is elongated in the equibiaxial tension direction as the material work hardens. On the other hand, the degree of differential hardening of the hot-rolled, Al-killed steel is much less than that of the IF steel. The difference of the degree of differential hardening between these steel alloys was further investigated using a hydraulic bulging test up to a level of 40% plastic strain [25]. It was found that the IF steel exhibited much larger differential hardening than the hot-rolled, Al-killed steel. In [24], Kuwabara et al. also observed that steel alloys having average $r$-values greater than 1.5 appear to exhibit definite differential hardening. Considering that the $r$-value is determined mainly by texture, the occurrence of the differential hardening of steel alloys is possibly attributed to texture.

For comparison, the theoretical yield loci calculated from the equation:

$$\sigma_0^M = \frac{r_{90}\sigma_0^M + r_0\sigma_0^M + r_{90}r_0(\sigma_x - \sigma_y)^M}{r_{90}(1 + r_0)}$$

are also plotted in the figure, where $r_0$ and $r_{90}$ are the $r$-values in the rolling and transverse directions of the material, respectively. When $M = 2$, Eq. (1) represents...
Hill’s quadratic yield criterion [32]. When \( M = 6 \) and 8, Eq. (1) represents Hosford’s yield criterion for b.c.c. and f.c.c. metals, respectively [33, 34]. With \( r_0 = r_{\theta 0} = 1 \) and \( M = 2 \), Eq. (1) coincides with the von Mises yield criterion. The yield loci calculated using Hosford’s yield criterion are in better agreement with the experimental work contours than Hill’s quadratic yield criterion for both steel alloys. Hill’s quadratic yield criterion has a tendency to overestimate the flow stresses in the vicinity of equibiaxial tension for the IF steel.

From the normality flow rule, the directions of plastic strain rate vectors coincide with those of the outward normals to the yield locus. With the yield criterion given by Eq. (1), the directions of the outward normals, \( \theta_{p x}, \theta_{p y}, \) and \( \theta_{p b} \), for the stress ratios \( \sigma_{xy} : \sigma_{\theta} = 2:1, 1:2 \) and 1:1, respectively, are given by the following equations,

\[
\tan \theta_{p x} = \frac{r_0 (1 - r_{\theta 0})}{r_{\theta 0} (2^{M-1} + r_0)}
\]

(2)

\[
\tan \theta_{p b} = \frac{r_0 \{2^{M-1} - (-1)^{M-1} r_{\theta 0}\}}{r_{\theta 0} (1 + (-1)^{M-1} r_0)}
\]

(3)

**FIGURE 3.** Contours of plastic work measured for IF steel (a) and hot-rolled, Al-killed steel (b), compared with several theoretical yield loci [25].

**FIGURE 4.** Directions of plastic strain rate vectors, \( \mathbf{D}^p \), measured at the stress ratios \( \sigma_{\theta} : \sigma_{xy} = 2:1 \) (a) and 1:1 (b), respectively, compared with those calculated using Hill’s quadratic and Hosford’s yield criteria [24].
\[ \tan \theta = \frac{r_2}{r_0} \]  
(4)

where \( \theta \), \( \theta_0 \), and \( \theta_\psi \) are taken to be zero in the \( x \)-axis direction (rolling direction) and positive for anti-clockwise rotation. It is noted that Hill’s and Hosford’s yield criteria give the same prediction for \( \sigma_0: \sigma_y = 1:1 \).

Figure 4 shows the relationship between the \( r \)-value of six different steel alloys and the direction of the plastic strain rate vector, \( \mathbf{D} \), measured for stress ratios \( \sigma_0: \sigma_y = (a) 2:1 \) and \( (b) 1:1 \), as well as the local outward normals with respect to Hill’s quadratic and Hosford’s yield loci. The directions of \( \mathbf{D} \) are generally in good agreement with those of the local outward normals with respect to Hosford’s yield locus. This means that Hosford’s yield criterion can be regarded as an instantaneous plastic potential of the test materials at least under linear loading paths, and is an effective phenomenological model for predicting the plastic deformation behavior of sheet steels.

In [24], texture measurements and calculation of the crystallographic orientation distribution function (CODF) with the series expansion method [35] have been carried out for steel alloys with different textures. The theoretical yield loci based on the Taylor-Bishop-Hill model exhibit a large difference between the full constraints (FC) and relaxed constraints (RC) assumptions. Experimental work contours corresponding to \( \varepsilon_0^p = 0.002 \), practically viewed as a yield locus, were closer to the theoretical yield loci based on the TBH-FC model. The yield loci calculated using Hosford’s yield criterion are similar in shape to those based on the TBH-FC, and are in better agreement with the experimental work contours than Hill’s quadratic yield criterion for both materials.

**Plane-Strain Tension Test**

Plane-strain tension is one of the typical modes of deformation of sheet metals in forming automotive body panels [36] and bending wide sheet. Plane-strain properties are, therefore, the crucial factor in the accurate simulation of sheet forming operations and in the reliable prediction of the defects of formed parts, such as split-type failures and springback. Attempts to model the deformation behavior of sheet metals under the plane-strain state have had to rely on plasticity theories based on tensile work hardening data because of the experimental difficulties of plane-strain testing. Direct plane-strain testing eliminates this problem and allows evaluation of both the current plasticity theories and the accuracy of plane-strain modeling based on tensile work hardening properties.

Wagoner and Wang [37] developed a technique for near-plane-strain tension testing. Wagoner [38] performed the near-plane-strain tension test on aluminum-killed steel and dual-phase steel using newly designed specimens as shown in Fig. 5a. He concluded that Hill’s quadratic yield criterion, including only normal anisotropy, adequately predicts the plane-strain behavior of both steel alloys. However, this conclusion contradicts the experimental findings obtained from biaxial tensile tests on several steel alloys by Kuwabara and coworkers [22, 24, 25], in which the flow stresses in near-plane-strain tension in the rolling and transverse directions were significantly lower than those predicted by Hill’s quadratic yield criterion, as shown in Fig. 3.

![Figure 5](image_url)
Figure 5b shows the newly designed cruciform specimen for the plane-strain tension test proposed by Kuwabara et al. [39]. Tensile direction is parallel to the shorter arms of the specimen (axis 1). The total (elastic-plastic) strain component in the direction of the longer arms of the specimen (axis 2) is kept to be almost zero within the resolution of strain measurement, i.e., $0 \pm 22 \mu \varepsilon$, using a computer-controlled, closed-loop hydraulic system [23]. Biaxial strain components in the gage section of the specimen were monitored by uniaxial strain gages mounted at the centerlines of the specimen. The longer arms have nine slits, both ends of which have circular holes of 1 mm diameter made for excluding stress concentration. The number of slits and the diameter of holes have been optimized using finite element calculations. The stress measurement error was estimated to be less than 2% in the plastic strain range, $\epsilon_p \leq 0.09$.

Figure 6 shows the measured stress-strain curve, $\sigma_1 - \sigma_2$, for the plane-strain tension of a SPCEN (IF steel) [39] (a) and a high strength steel with a tensile strength of 340MPa [40] (b), with axis-1 taken parallel to the rolling direction. Also depicted in the figure are the predictions of von Mises’, Hill’s quadratic and Hosford’s yield functions. Take the uniaxial stress-strain curve in the direction of rolling, $\sigma_{\text{U}x} - \epsilon_{\text{U}x}$, as the equivalent stress-equivalent plastic strain curve. Then, by applying the principle of equivalence of plastic work, the true stress $\sigma_{\text{TP}}$ and plastic strain increment $d\epsilon_{\text{TP}}$ for the plane-strain tension with the axis 1 taken to be in the direction of rolling are calculated as

$$\sigma_{\text{TP}} = \sigma_{\text{U}x} / K_1 \text{ and } d\epsilon_{\text{TP}} = K_1 d\epsilon_{\text{U}x}, \quad (5)$$

where

$$K_1 = \left( \frac{r_{00} + r_0 \alpha^M + r_0 r_{90}(1-\alpha)^M}{r_{00}(1+r_0)} \right)^{1/M} \quad (6)$$

and $\alpha$ is the stress ratio, $\sigma_1 / \sigma_{\text{TP}}$, given by

$$\alpha^{-1} = 1 + (r_{00})^{-1/(M-1)} \quad (7)$$

The measured flow stresses agree closely with the predictions of von Mises’ and Hosford’s yield functions in the early work hardening stage, $\epsilon_p \leq 0.01$. At larger strains, $\epsilon_p > 0.01$, the measured stress-strain curve tends to lie above the predicted curve. Hill’s quadratic yield function clearly overestimates the flow stress for the plane-strain tension. These observations are consistent with the results for steel alloys having average $r$-values greater than 1.5 [24]; initially the work contours are very close to the von Mises yield locus, and are elongated in the equibiaxial tension direction to approach Hill’s quadratic yield locus as the material work hardens.

Pijlman et al. developed a novel testing devise which is capable of combining plane-strain tension with simple shear on a sheet specimen [41].

**Effect of Material Modeling on the Accuracy of Springback Simulation**

Springback refers to the elastic recovery of press-formed parts, and is caused by the relief of bending moment retained in the material after press forming. Accordingly, factors related to the generation of stress in the material during the loading and unloading processes necessarily influence the springback behavior of press-formed parts. Such factors include
the stretching forces applied to the material [42], the coefficient of friction at the material/die interfaces, the tool temperature, the elastic deflection of the die, the deformation history of the material, the work-hardening characteristics, and the Bauschinger effect of the material [43-45]. Therefore, when one observes a discrepancy between the magnitudes of springback in simulation and in reality, it is generally difficult to establish its true cause, especially in the case of parts that have complex geometry. The accuracy of the simulation code itself can be in doubt. Therefore, a simple evaluation method for springback is necessary to validate the material model and the accuracy of the code used in its simulation.

Kuwabara et al. [40] proposed a method for evaluating the springback characteristics of sheet metals based on the plane-strain stretch bending test, as shown in Fig. 7. A sheet specimen is fixed into position on the blank holder, and a blank holding force is applied. The radius of the punch is 100mm. The draw height $h$ is measured with a displacement gage, and the punch force, $P$, with a load cell. The nominal tensile stress, $T$, applied to the specimen were determined using the equilibrium equation as:

$$T = \frac{P}{2bt_0 \sin \theta}$$

where $b$ and $t_0$ respectively denote the initial width and thickness of the specimen. The magnitude of $T$ was varied by changing the blank holding force. The punch head was lubricated using a Teflon sheet of thickness 0.05mm and machine oil #32.

The magnitude of springback, $\eta$, is defined as

$$\eta = \frac{|1/r' - 1/r|}{1/r} = \frac{r' - r}{r'} = \frac{\Delta r}{r'}$$

where $r$ (=100mm) and $r'$ are respectively the radius of curvature of the inner surface of the specimen with load and after load removal. We measured $r'$ at the center of the bent specimen using a dial gage with the minimum scale value of 1µm. The length of the measurement section was 20mm.

Figure 8 shows the deformed shapes of stretch-bent specimens after springback [40]. Material: high-strength steel sheet, JSC340P, of thickness 0.7mm and tensile strength 340MPa. Larger than the yield stress of the material. The calculated values based on von Mises’ yield function agree better with the observations than those based on Hill’s quadratic yield function. This is consistent with
the observations in Fig. 6b; the von Mises yield function accurately predicted the flow stress of the material under plane-strain tension, but Hill’s quadratic yield function overestimated the flow stress of the material under plane-strain tension. Hill’s quadratic yield function therefore overestimates the bending moment, leading to overestimation of the magnitude of springback. Figure 9 shows that the choice of an appropriate anisotropic yield function is vital for accurate springback simulations.

USE OF ABRUPT STRAIN PATH CHANGE FOR DETECTING YIELD VERTEX AND SUBSEQUENT YIELD SURFACE

Conventionally, yield surfaces are determined by probing in many different stress directions from the elastic region into the plastic range (unloading-reloading method). Regarding the possibility that there is a corner on the subsequent yield surface at the point of loading, as predicted by crystal plasticity [46], it has been argued by Hecker [3] that if a corner exists, it will be erased by the unloading needed to probe the yield surface. But whether or not a vertex has formed on the yield surface is very important for predictions of plastic instabilities [47-49]. To avoid this effect of unloading, some authors have used a zig-zag loading path to study corners, and this has indicated the development of a yield surface vertex for some materials [50].

Kuroda and Tvergaard [51] proposed a new method for determining the shape of the subsequent yield locus in the vicinity of a current loading point. It is suggested that a proportional strain path is prescribed until the loading point of interest has been reached, and then prescribe an abrupt strain path, which will make the stress point move quickly along the yield locus. This can be done without occurrence of unloading, which would be required if the subsequent yield surface was to be determined by probing from the elastic region; it is, therefore, possible that a yield vertex formed at the point of loading can be detected by this method.

Kuwabara et al. [23] applied the abrupt strain path change method to the measurements of subsequent yield loci of an aluminum alloy and an IF steel, using cruciform specimens in a closed-loop, servo-controlled biaxial tensile testing machine. In the first step of straining equibiaxial stretching $D_{11} = D_{22} = \dot{\alpha} > 0$ was prescribed. At a nominal strain $\varepsilon_{11} = \varepsilon_{22} = 0.01$, the prescribed strain rates were abruptly changed to

$$
\begin{align*}
\Delta \varepsilon_{11} &= 0.0017 \\
\Delta \varepsilon_{22} &= 0.001 \\
\Delta \varepsilon_{11} &= 0.0019
\end{align*}
$$

The accumulated equivalent (von Mises) plastic strain $\Delta \varepsilon^p$, which is measured from the strain path change point, is indicated by the ◇ markers in the figure. Also depicted in the figures are the subsequent yield loci determined using the conventional unloading-reloading method: $\varepsilon^p_0 = 100 \mu \varepsilon$ (□), $1000 \mu \varepsilon$ (○) and $5000 \mu \varepsilon$ (△), where the plastic work definitions of yielding is adopted and $\varepsilon^p_0$ is the corresponding uniaxial plastic strain in the rolling direction of the materials. The curve marked with ● is the work contour observed for as-received materials subjected to linear loading paths.

![FIGURE 10](image) Subsequent yield loci observed for an SPCE (IF steel) (a) and an 6000-series aluminum alloy (b) using abrupt strain path change following equibiaxial tension [23]. The accumulated equivalent (von Mises) plastic strain $\Delta \varepsilon^p$, which is measured from the strain path change point, is indicated by the ◇ markers in the figure. Also depicted in the figures are the subsequent yield loci determined using the conventional unloading-reloading method: $\varepsilon^p_0 = 100 \mu \varepsilon$ (□), $1000 \mu \varepsilon$ (○) and $5000 \mu \varepsilon$ (△), where the plastic work definitions of yielding is adopted and $\varepsilon^p_0$ is the corresponding uniaxial plastic strain in the rolling direction of the materials. The curve marked with ● is the work contour observed for as-received materials subjected to linear loading paths.
\[ D_{22} = -D_{11} \quad \text{with} \quad D_{11} = \dot{\alpha} \quad \text{or equivalently with} \quad D_{22} = \dot{\alpha}. \]

Figure 10 shows the observed stress paths for both materials. It is definite that the stress paths for the abrupt strain path change with \( D_{22} = -D_{11} \) cannot be non-yielding stress paths in the elastic region. Therefore, we can conclude that a yield locus vertex has been successfully detected at the point of loading in the figure. By contrast, if we considered only the subsequent yield surfaces determined by the unloading-reloading method with \( \varepsilon_0^p \) ranging from 100 \( \mu \varepsilon \) (\( \square \)), 1000 \( \mu \varepsilon \) (\( \circ \)) and 5000 \( \mu \varepsilon \) (\( \triangle \)), the formation of the vertex would be overlooked. In [52], Kuwabara et al. also applied the abrupt strain path change method to a steel tube using a servo-controlled tension-internal pressure testing machine; a yield vertex and non-normality behavior of the plastic strain rate vector were successfully observed.

It is noted that the curvature of the stress paths for the second step of straining observed for the IF steel (Fig. 10a) is much larger than that observed for the aluminum alloy (Fig. 10b). This difference in curvature between both materials is similar to that between b.c.c. and f.c.c. polycrystalline materials predicted by a Taylor model in [51]. Therefore, we conclude that the difference in curvature of the stress paths is basically attributed to the difference in crystallographic structure. Another interesting finding in the figure is that the stress paths observed after an abrupt strain path change lie within the contour of equal plastic work (●). This observation accords with the theoretical prediction by Hill et al. [30] that a yield locus in stress space touches the work contour at the corresponding stress point and otherwise lies within.

Kuroda and Tvergaard [51] suggested that immediately after an abrupt strain path change, elasticity plays an important role in forcing the stress point to quickly move along the current yield surface. Plastic strain contributions play an increasing roll as the direction of the current yield locus normal rotates towards the prescribed strain rate direction, and these additional plastic strains give a small amount of hardening, which moves the stress point slightly outside the actual yield locus. However, the important point here is that the initial part of the yield locus is very well approximated, and that this is not very sensitive to moderate differences in the direction of the second step of straining, as is observed in Fig. 10b.

In Fig. 10, the directions of the plastic strain rate vector, \( \mathbf{D}^p \), are shown by the short lines attached to the stress paths. It is noted that the direction of \( \mathbf{D}^p \) is inclined to the forward direction of the stress paths and deviates a great deal from the direction normal to the stress path, which is regarded as part of the subsequent yield surface in the present study. This tendency of the apparent non-normality is very similar to the Taylor model prediction for b.c.c. and f.c.c. polycrystalline materials [51]. It is concluded by Kuroda and Tvergaard [51] that the apparent non-normality must be a vertex-type effect, which results from a rather blunt vertex moving with the stress point along what appears to be a smooth yield surface.

Recently, Kuroda and Tvergaard [53] have proposed a phenomenological plasticity model, in which a smooth yield surface for an anisotropic solid is combined with a vertex-type plastic flow rule to represent the observed nonlinear material response. They have shown that improved yield surface shapes and vertex-type effects are important prerequisites for predicting the onset of necking in biaxially stretched metal sheets.

### MEASUREMENT OF STRENGTH DIFFERENTIAL EFFECT AND BAUSCHINGER EFFECT OF SHEET METALS

In sheet metal forming processes sheet materials often undergo bending and/or bending-unbending. In bending, material fibers are subjected to in-plane compression as well as in-plane tension through thickness. In bending-unbending they are subjected to stress reversals. It is often observed that the flow stress of some types of metal is larger in compression than in tension; this phenomenon is known as the strength-differential effect (SDE). The yield stress in reverse loading is lower than that in continuous loading; this phenomenon is known as the Bauschinger effect. Accordingly, accurate knowledge of the SDE and the Bauschinger effect of sheet metals is crucial in the predictive calculations of press loads, strain distributions, failure loci and springback.

In this section, testing methods for measuring accurately the stress-strain relations of sheet metals subjected to in-plane compression and stress reversals are reviewed.
In-Plane Compression Test on Sheet Metals

Dietrich and Turski [54] have proposed a unique experimental technique for in-plane compression test on a sheet specimen as shown in Fig. 11; however, the magnitude of strain imposed on a specimen was limited to 0.3 %.

Kuwabara et al. [55] modified the experimental technique developed by Dietrich and Turski [54] (Fig. 11) to enable the in-plane compression of a sheet specimen up to 16% maximum plastic strain, as shown in Fig. 12. A sheet specimen, 45mm wide and 200mm long, was sandwiched between the upper and lower comb-shaped dies. Then, the specimen was compressed between rigid, parallel platens, as the male dies were forced to slide into the female dies. Since a constant blank-holding force is exerted on the dies during the test, the specimen can be compressed in its own plane without buckling. The blank-holding pressure was set to be about 1% of the 0.2% proof stress of the material. In order to reduce the friction forces generated at the die/specimen interfaces as much as possible, the specimen was lubricated on both sides using Teflon sheets (0.05 mm thickness) and machine oil, resulting in the reduction of the coefficient of friction to 0.02.

Kuwabara et al. [55] modified the experimental technique developed by Dietrich and Turski [54] (Fig. 11) to enable the in-plane compression of a sheet specimen up to −16 % maximum plastic strain, as shown in Fig. 12. A sheet specimen, 45mm wide and 200mm long, was sandwiched between the upper and lower comb-shaped dies. Then, the specimen was compressed between rigid, parallel platens, as the male dies were forced to slide into the female dies. Since a constant blank-holding force is exerted on the dies during the test, the specimen can be compressed in its own plane without buckling. The blank-holding pressure was set to be about 1% of the 0.2% proof stress of the material. In order to reduce the friction forces generated at the die/specimen interfaces as much as possible, the specimen was lubricated on both sides using Teflon sheets (0.05 mm thickness) and machine oil, resulting in the reduction of the coefficient of friction to 0.02.

Figure 12b shows shapes of specimens before and after compression tests. We have confirmed uniform strain distributions over the central 100mm length of the 200mm long specimens. It is observed that the deformation of both edges of the specimen is slightly constraint in the width direction because of the friction between the edges of the specimen and the rigid platens which exert a compression force on the specimen. We have confirmed using FEM analyses, however, that even a coefficient of friction of 0.3 there does not affect the accuracy of the observed values of the compressive flow stress.

Figure 13 shows the comparison of stress-strain curve between uniaxial tension and compression tests on four different materials [28, 56]. Strain rate was about 0.001/s for both tension and compression tests. The aluminum alloy AA6022-T4 (1mm thickness) exhibits no SDE (Fig. 13a), while the IF steel (1.2mm thickness, Fig. 13b) and the dual-phase steel (1.2mm thickness, Fig. 13c) exhibits the definite SDE; the compressive flow stresses are clearly larger in compression than in tension in the strain range larger than 0.04 for the IF steel and in the entire strain range

![Diagram of in-plane compression test on sheet metals](image)

**FIGURE 11.** Tool for in-plane compression of a sheet specimen [54].

**FIGURE 12.** (a) A schematic of in-plane compression test on sheet metals. (b) Specimens after the compression test [55]. The specimen is 45mm wide and 200mm long.
for the dual-phase steel. For the pure titanium sheet shown in Fig. 13d, the SDE is more complex. In the rolling direction, the flow stress is smaller in compression than in tension for $\varepsilon^* \leq 0.07$, while compressive flow stress becomes larger than the tensile one for $\varepsilon^* > 0.07$. The difference is 120MPa at $\varepsilon^* = 0.15$ (36% larger in compression than in tension). In the 45° and 90° directions, the compressive flow stresses are (8-12)% and (10-19)% larger than the tensile ones, respectively, for $\varepsilon^* \geq 0.02$.

In the studies by Spitzig et al. [57-59], the SDE was interpreted as a direct consequence of the dependence of the flow stress on the hydrostatic stress in the specimen. In addition, the iron-based high-strength metals undergoing uniaxial tension and compression showed a permanent volume expansion (dilatancy) which is insensitive to the sign of the hydrostatic pressure. Kuroda [60, 61] numerically simulated increase in flow stress under superimposed hydrostatic pressure, as well as the SDE, using a strain-rate dependent crystal plasticity theory with non-Schmid effects. The single crystal model for each grain is substantially identical to the model proposed by Asaro and Needleman [62], except for the relation for the slip rates $\dot{\gamma}^{(\alpha)}$, where superscript ' (\alpha)' represents a quantity with respect to the \(\alpha\)th slip system. In his computations, $\dot{\gamma}^{(\alpha)}$ is taken to be

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \text{sgn}(\varepsilon^{(\alpha)}) \left[ \frac{\varepsilon^{(\alpha)}}{\alpha^{(\alpha)} + b\sigma_k} \right]^{1/m},$$

(10)

where $\dot{\gamma}_0$ is a reference slip rate, $\varepsilon^{(\alpha)}$ is the resolved shear stress, $\sigma_k/3$ is the hydrostatic stress (\(\sigma_0\) is Cauchy stress), $m$ is a rate sensitivity parameter, and $a$ and $b$ are non-negative material parameters that characterize the dependence of ‘yielding’ on the normal stress $\sigma^{(\alpha)}$ and $\sigma_k$, respectively, and $g^{(\alpha)}$ is the current hardness of the \(\alpha\)th slip system. In the computation, b.c.c. crystals with slip systems of the

![Graphs showing flow stress vs. strain for different materials](image)

**FIGURE 13.** Difference of flow stresses in tension and compression observed for AA6022-T4 (a), IF steel (SSPDX) (b), dual phase steel (SAFC590D) (c) and pure titanium sheet (JIS #1) (d). All tests for (a) to (c) were carried out in the rolling direction of the materials [56]. In (d), +/- $\sigma_\alpha$ represent the flow stresses observed in the tension/compression tests in the $\alpha$° direction from the rolling direction of the titanium sheet [28].
types \{110\}<111> and \{112\}<111> are adopted. The extended Taylor model used in the analysis consists of 816 grains whose orientations were determined based on ODF from pole figures measured using X-ray diffraction equipment.

The results calculated using the polycrystal model (Eq. (10) with $b = 0.013$ and $a = 0$) for IF steel is in good agreement with the observation (Fig. 13b). The polycrystal model applicable to SDE reproduction has been further pursued in [61]. It is concluded in [61] that the SDE is mainly caused by hydrostatic stress acting on crystals rather than by normal stress acting on each slip plane. This supports the assumption of $b > 0$ with $a = 0$, which has been used in the polycrystal model in Fig. 13b.

**In-Plane Stress Reversal Test**

Figure 14 shows examples of stress-strain curves observed for an aluminum alloy and an IF steel subjected to in-plane, tension-compression stress reversals. The experiments were carried out utilizing the in-plane compression testing apparatus shown in Fig. 12. The flow stresses after stress reversals lie below those of virgin materials, $\sigma_T - \varepsilon_T$, showing clearly the Bauschinger effect. A disadvantage of this testing is that it requires time and labor to remake specimens for every tension and compression tests.

Figure 15 shows experimental methods for applying continuously stress reversals to a sheet specimen and observing the Bauschinger effect. The cyclic bending-unbending method [63-65], as shown in Fig. 15a, is useful for evaluating qualitatively the Bauschinger effect of the sheet metal subjected to small strain amplitude. The cyclic simple shear test [66-68], as shown in Fig. 15b and c, is effective for measuring the Bauschinger effect subjected to large strain amplitude. This method, however, may cause some difficulty in modeling the Bauschinger effect of
the material for such a case as the principal axes of stress coincide with those of material anisotropy, as in the case of 2D draw bending. When one hopes to measure directly the stress-strain relations of sheet metals subjected to tension/compression stress reversals, then the methods as shown in Figs. 15d, e and f, are useful [69-71].

Figure 16 [71] shows examples of cyclic stress-strain curves for a 6000-series aluminum alloy (1mm thick) and a low carbon steel alloy (0.7mm thick), observed using the in-plane tension-compression testing apparatus, as shown in Fig. 15f. The calculated results using Ohno’s constitutive model [72, 73] are in fair agreement with the observations.

**FIGURE 15.** Experimental methods for applying continuously stress reversals to a sheet specimen

**FIGURE 16.** Cyclic stress-strain curves for a 6000-series aluminum alloy (1mm thick) (a) and a low carbon steel alloy (0.7mm thick) (b), observed using the in-plane tension-compression testing apparatus, as shown in Fig. 15f [71]. Dashed lines are those based on Ohno’s constitutive model [72, 73].
BIAXIAL STRESS TEST ON TUBULAR MATERIALS

Tube hydroforming is the process of forming tubular components under hydraulic pressure. This process allows reduction in weight and costs and gives greater structural strength and rigidity in automotive body structures. The deformation histories of specimen materials in tube hydroforming are complex, so that it is difficult for die designing engineers to predict defects in formed parts, including fracture and springback. Product development cycles have shortened recently, ruling out traditional empirical approaches for determining the optimum forming conditions. Consequently, there is a need for accurate simulation techniques for tube hydroforming using finite element analysis. For accurate and time-effective finite element simulations, it is vital to use accurate phenomenological plasticity models based on anisotropic yield functions.

There have been many experimental studies of multiaxial testing of thin-walled tubular specimens, involving thin-walled specimens loaded in combined tension-torsion or tension-internal pressure modes; see extensive reviews of the early experimental studies by Michno and Findley [2] and Hecker [3].

Kuwabara et al. [52] designed and built a servo-controlled tension-internal pressure testing machine to investigate the anisotropic plastic deformation behavior of tubular materials used for hydroforming automotive parts. This machine is capable of applying arbitrary stress or strain paths to a tubular specimen using an electrical, closed-loop control system. Using this biaxial stress testing machine, Kuwabara et al. [78] investigated the anisotropic plastic behavior of an extruded aluminum alloy tube, A5154-H112, for linear and combined stress paths, and verified the validity of conventional anisotropic yield functions by comparing the observed data with theoretical predictions based on the yield functions. Furthermore, we utilized this testing machine to observe that the stress states at the forming limit lie close to a single curve in the principal stress space for a variety of stress paths; the path-dependence of the forming limit strain vanishes when viewed in stress space [79]. In this section the outlines of the experimental observations reported in the literature [78, 79] are given.

Constitutive Modeling of Extruded Aluminum Alloy Tube

Biaxial stress tests were carried out for a 5154-H112 extruded aluminum alloy tube of 76.3mm outer diameter and 3.9mm wall thickness. Figure 17 shows the experimental data points making up contours of plastic work measured with the stress ratios in certain proportions: \( \sigma_\phi : \sigma_\theta = 1:0, 4:1, 2:1, 4:3, 1:1, 20:23, 3:4, 1:2, 1:4 \) and 0:1 (the subscripts \( \phi \) and \( \theta \) respectively denote the axial and circumferential directions of the specimen); here \( \sigma_\phi : \sigma_\theta = 20:23 \) is the stress ratio at which the ratio of the incremental plastic strain components, \( \Delta \varepsilon_\phi : \Delta \varepsilon_\theta \), is close to unity. The stress components in (b) are normalized by \( \sigma_0 \).
corresponding to each $\varepsilon_p^0$, where $\sigma$ is the axial true stress corresponding to the axial logarithmic plastic strain $\varepsilon_p^0$ observed in the uniaxial tensile test on the material in the axial direction. The normalized data points mostly lie in a very narrow region at all stress ratios. This means that successive work contours for different $\varepsilon_p^0$ are similar in shape.

Also depicted in Fig. 17 are the yield loci based on conventional yield functions. The yield loci based on the Yld2000-2d yield function [80] reproduce the bulging of the observed work contours in the directions of $\sigma_\phi : \sigma_\theta = 20:23$ to 3:4, and is in the best agreement with the observed work contours. In particular, the yield loci based on the Yld2000-2d are in perfect agreement with the work contours for $\varepsilon_p^0 = 0.002, 0.01$ and 0.025.

Figure 18 compares directions of the incremental plastic strain vectors, $\beta$, measured at different values of $\varepsilon_p^0$, with the directions of the local outward normals to the theoretical yield loci. The observed values of $\beta$ were $27\pm2^\circ$ for equibiaxial tension, $\varphi = 45^\circ$, implying strong anisotropy of the material tested. Two specimens were tested for each stress ratio, and the maximum difference in the angle $\beta$ observed for the specimens between the different plastic work contours was $6^\circ$ for any given stress ratio. This means that, at least for the measured range of the work contours shown in Fig. 17, the directions of the incremental plastic strain vectors remained almost constant when the stress ratio was constant.

Again, the predictions based on Hosford’s and the Yld2000-2d yield functions are in good agreement with the experimental results. The maximum difference of the directions of the incremental plastic strain vectors from directions of the local outward normals for Hosford’s yield locus was $10^\circ$, and for Yld2000-2d was $7^\circ$. We therefore conclude that the aluminum alloy tubes work-hardens almost isotropically, and that Hosford’s and the Yld2000-2d yield functions can be regarded as instantaneous plastic potentials for the material, at least for linear stress paths.

Since some material elements of tubular specimens are subjected to complex stress/strain paths in tube hydroforming, it is worth checking whether the

![FIGURE 18. Directions of the incremental plastic strain vectors, $\beta$, compared with directions of the local outward normals to theoretical yield loci [78]. The stress ratio is represented as the direction of the stress path, $\varphi$, in stress space. The angles $\beta$ and $\varphi$ are taken to be zero in the axial direction of the specimen and positive for anti-clockwise rotation. The material coefficients of the Yld2000-2d used in the calculation were determined using the true stress components and the ratio of incremental plastic strain components measured at the stress ratio, $\sigma_\phi : \sigma_\theta = 20:23$, as well as $\sigma$, $\sigma_\theta$, $r_\phi$ and $r_\theta$, for $\varepsilon_p^0 = 0.15$.](image)

![FIGURE 19. Strain paths observed for bilinear stress paths, A and B, compared with the paths predicted using several theoretical yield functions [78].](image)
isotropic hardening assumption remains valid for nonlinear stress paths. To verify the assumption, the bilinear stress paths, A and B as shown in Fig. 19a, were applied to specimens, and the resulting strain paths were observed. Fig. 19b shows the observed strain paths for both stress paths, compared with the results calculated using conventional yield functions under the assumption of isotropic hardening. Again, the strain paths predicted using Yld2000-2d are in closest agreement with the observed ones. We therefore conclude that the isotropic hardening assumption is valid for linear and combined stress paths. We also conclude that the Yld2000-2d yield function is an effective phenomenological plasticity model for predicting the anisotropic plastic deformation behavior of the material.

Path Independence of Forming Limit Stress Curve

The forming limit of materials is commonly represented by the Forming Limit Curve (FLC) which specifies the limit strains in the principal strain coordinates [81]. The FLC is commonly used for assessing the forming severity of materials in a plane stress state. Experimental results [82-85] and numerical results [86, 87] show that the FLC depends on the strain path. The FLC corresponding to linear strain paths is therefore not a useful concept in multistage forming processes, such as tube hydroforming. Instead, some authors have represented the forming limit using the state of stress rather than the state of strain. The resulting Forming Limit Stress Curve (FLSC) is constructed by plotting the state of stress at the forming limits in stress space; the FLSC is reportedly independent of the strain path [88-93].

The FLSC is a useful idea, but it has not been widely used. One reason is that there is no experimental evidence validating the FLSC concept. Researchers generally measure strains at forming limits under proportional and combined loadings, and the forming limit stresses are calculated using certain yield functions and hardening laws. The resulting FLSC is questionable, because we seldom have an appropriate constitutive model for any given material. To validate the FLSC concept it is necessary to use an experimental method which measures stresses accurately.

Yoshida et al. [79] measured the forming limit strains and forming limit stresses of a 5154-H112 extruded aluminum alloy tube (the same material as used in Fig. 17) in detail for many linear and combined stress paths, using a servo-controlled, combined tension-internal pressure testing machine. They found that the forming limit strains were dependent on stress paths. Figure 20a shows the results of the observed forming limit stresses. The solid line the FLSC fitted to the forming limit stresses observed for linear stress paths. The FLSC is concave at stress ratios

![Figure 20](image-url)
\(\sigma : \sigma = 1:2 \text{ and } 2:1\), and is very sharp in the direction of the stress ratio \(\sigma : \sigma = 20:23\). The forming limit stresses observed for the combined stress paths I, II, and III lie close to a single curve. We therefore conclude that the FLSC is independent of the stress path, at least for the present experimental parameters.

We found, as shown in Fig. 20b, that the total plastic work was almost the same for the linear and combined stress paths when different stress paths reached the same stress point on the FLSC. Therefore, the saturation of the stress-strain curves of the present material is not responsible for the path-independence of the FLSC of the material.

Yoshida et al. [94] investigated the effects of changing strain paths on forming limit stresses of sheet metals using the Marciniak-Kuczyński model and a non-normality flow rule proposed by Kuroda and Tvergaard [53]. They found that the forming limit stresses calculated for combined loadings which include unloading between the first and second loadings were in good agreement with those calculated for proportional loadings. On the other hand, the forming limit stresses for the combined loadings which do not include unloading were dependent on the strain path. Thus, they concluded that whether the FLSC is path independent or not is affected by loading history.

**MULTISTAGE TENSION TEST**

Kim and Yin [95] studied the evolution of anisotropy of an IF steel sheet employing two stage tension tests: sheet specimens are subjected to uniaxial tensile loading at an angle to the orthotropic axes. They observed that the orientations of the orthotropy axes are changed drastically within a few percent of tensile strain. Kuwabara et al. [96] carried out the same test as performed by Kim and Yin [95] on an IF steel (0.8mm thick) in order to check whether the same phenomenon, i.e. the rotation of the axis of anisotropy, can be observed or not.

Figure 21 shows the experimental procedure for the two stage tension test. The first prestrain, \(e_1 = 0\) (no prestrain) or 3 %, was applied to the first specimen in the rolling direction of the sheet (Fig. 21a). Next, second specimens were cut from the first specimen in the directions of \(\psi = 30, 45, 60\) and \(90^\circ\) from the first prestrain direction (rolling direction, i.e., \(\phi = 0^\circ\)), and 0.2 % proof stress, \(\sigma_{0.2}\), of all third specimens were measured (Fig. 21c).

Figure 22 shows the change of \(\sigma_{0.2}\) with the amount of prestrain, \(e_2\), for the case of \(e_1 = 0\) and \(\psi = 45^\circ\). No anisotropy in \(\sigma_{0.2}\) is observed in the as-received state of the sheet; however, \(\sigma_{0.2}\) changes with increasing \(e_2\) to have local minimum values in the tensile and transverse directions and local maximum values in the 45 and 135° directions. Figure 23 shows the change of \(\sigma_{0.2}\) with prestrain, \(e_2\), for the cases of \(e_1 = 3\%\) and \(\psi = 30, 60\) and \(90^\circ\). The first specimen has anisotropy induced by the 3 % prestrain. As \(e_2\) increases, however, the directions of the local minimum and maximum of \(\sigma_{0.2}\) become vague once at about \(e_2 = 1\%\). As \(e_2\) increases further, \(\sigma_{0.2}\) again comes to have local minimum values in the tensile and transverse directions and local maximum values in the vicinity of \(\phi = 45\) and 135° directions of the second prestrain. In all the three cases of Fig. 23 orthotropic symmetry is maintained through the second prestrains.
It is also noted that the intensity of anisotropy in $\sigma_{0.2}$ increases with increasing $\varepsilon_{II}$. These observations completely coincide with those reported by Kim and Yin [95].

Losilla et al. [97] proposed generally anisotropic quadratic models for big offset-strain plastic yield and hardening of rolled sheet steels. Dafalias [98] suggested the importance of plastic spin in determining the rotation of anisotropy.

**CONCLUDING REMARKS**

Experimental techniques which are effective in observing and modeling the anisotropic plastic behavior of metal sheets and tubes under a variety of loading paths are reviewed. It is hoped that this review paper will guide the user of metal forming simulation software towards more realistic material modeling and provide a reference for further experimental work.

We must always bear in mind Mellor and Parmar’s concluding remark stated in [99]:

“...The plastic deformation of metals is a complex subject and it is not to be expected that such deformation can be fully described by a simple yield locus and its associated flow rule. Any constitutive equations that are used in a metal forming problem will be the result of compromise and they may be applicable only for a certain stress system and only following a certain strain path. It is necessary to check the validity of any theory by careful experiments.”

Regrettfully, little has changed in twenty-seven years.

**REFERENCES**