Springback prediction in sheet metal forming process based on the hybrid SA

Yuqin Guo*, Fuzhu Li†, Hong Jiang*, Xiaochun Wang*

* School of Mechanical Engineering, Xi’an Jiaotong University, Xi’an 710049, China
† Technology Institute, Xuzhou Normal University, Xuzhou 221011, China

Abstract. In terms of the intensive similarity between the sheet metal forming-springback process and that of annealing of metals, it is suggested that the simulation of the sheet metal forming process is performed with the Nonlinear FEM and the springback prediction is implemented by solving the large-scale combinational optimum problem established on the base of the energy descending and balancing in deformed part. The BFGS-SA hybrid SA approach is proposed to solve this problem and improve the computing efficiency of the traditional SA and its capability of obtaining the global optimum solution. At the same time, the correlative annealing strategies for the SA algorithm are determined in here. By comparing the calculation results of sample part with those of experiment measurement at the specified sections, the rationality of the schedule of springback prediction used and the validity of the BFGS-SA algorithm proposed are verified.

INTRODUCTION

In the simulation technology of the sheet metal forming process, the precise springback prediction for the stamped part has been one of the open problems because it is involved with the material mechanical properties such as the anisotropy behavior, the elastic-plastic constitutive model, the Bauchinger effect and work-hardening model etc., the stamping mode, the process conditions, the loading history, the geometry shape of the stamped part, the theory and approach of springback prediction, and so on. And it has already become an important factor affecting the quality of designing and fabricating of the die/mould. Simultaneously, it also severely restricts the reduction of the production cost and times. In earlier years, there has been considerable efforts aimed at this problem, most of them have concentrated on the cases of the strip bending or the 2-D sheet forming operation[1,2]. However, the springback prediction of complex cover panels has become an open issue, recently[1-5]. In earlier research work, the methods often used consist of the analytic method, FEM, experimental method and their combination[6]. In the present work, for the complex cover panel with the free form surface, the new idea is suggested that the simulation of the forming process is conducted by the dynamic explicit FEM and the springback prediction is performed with the BFGS-SA hybrid SA. That is to say, the factors having an important influence on the springback prediction accuracy are considered in the stage of the sheet forming simulation with the FEM, such as the anisotropy behavior, the elastic-plastic constitution relationship, the work-hardening model, Bauchinger effect, the geometric shape of the die cavity and the opening radius, and drawbeads geometry parameters and layout etc. Whilst the springback process of cover panel after being formed is treated as a similar process with the physical annealing of metal. Hereby, the springback prediction is expressed as a large-scale combination optimum problem, in terms of its characteristics, a BFGS-SA hybrid SA algorithm is put forward to solving this problem. By analyzing and comparing the simulation results with the experiment ones for sample part at the specified sections, the rationality of the model of the springback prediction established and the validity of the “BFGS-SA” hybrid SA algorithm proposed in this paper are verified.
ESTABLISHMENT OF THE OPTIMIZATION MODEL OF SPRINGBACK PREDICTION

Because any error during forming simulation will be accumulated in the stage of springback stage and affects the springback prediction accuracy. Thus, in the present work, the anisotropy yield behavior and the work-hardening model of the material, which is able to reflect the Bauschinger effect, are introduced in the FEM calculation so as to reduce the springback prediction errors. Denote the node coordinate vector, the vector of displacement, stress, strain and the yield stress at the end of forming process are $X_0^i, u_0^i, \sigma_0^i, \varepsilon_0^i$ and $\sigma_y^i$, respectively. Then regard them as the initial state, and solve the springback prediction problem.

The idea of treating the springback prediction problem with the SA algorithm, derives from the intensive resemblance of the physical annealing process with the forming-springback process of cover panels. From the perspective of the energy changing, i.e., the process of “forming-keeping pressure-springback” of cover panel corresponds respectively to the process of “heating-heat preservation-cooling” of metals. For the cooling, their similarity is that the energy of a substance system is gradually decreasing as the decreasing of the annealing temperature (corresponding to the punch stroke) or the increasing of the annealing time (corresponding to the time of the springback). And when the temperature drops to the room temperature (corresponding that the punch stroke is reduced to zero or the springback process is over), the system arrives at a new balance state with the minimum energy (corresponding that the discrepancy of the residual strain energy of all elements in the deformed part reaches the minimum). Hereby, in terms of the particularities of the springback prediction problem, it is treated as a combinational optimization problem with quite a few variables. The establishment of the optimization model can be depicted as follow:

Suppose that the volume of cover panel is incompressible during forming, and the FEM model is meshed with the 3 or 4-node shell elements, then the interpolation function $N_i(\xi, \eta)$ of element node $i$ is known. Also assume, during springback, for the displacement, strain and stress increment $\Delta u$, $\Delta \varepsilon$ and $\Delta \sigma$, the geometric and physical equations of the linear-elastic material are satisfied. So the formulation $\Delta \varepsilon = B \Delta u$ and $\Delta \sigma = D \Delta \varepsilon$ are held, where $\Delta u = \sum_{i=1}^{n_u} N_i(\xi, \eta) \Delta u_i$, $\Delta u_i$ is the displacement increment of the element node $i$, $n_u$ is the node total number of element, $B$ and $D$ are the element strain matrix and material elastic matrix, respectively. Thereby, the vectors of the residual stress ($\sigma'$) and strain ($\varepsilon'$) can be expressed as:

$$\sigma' = \sigma^0 - \Delta \sigma \quad (1)$$

$$\varepsilon' = \varepsilon^0 - \Delta \varepsilon \quad (2)$$

The strain energy per unit volume after springback is formulated as:

$$E'(\Delta u) = \frac{1}{2} (\varepsilon')^T (\sigma')$$

Denote the average value of the strain energy per unit volume all of elements as $\overline{E}(\Delta u)$. Describe the state of the minimum energy in the deformed part as a function $F(\Delta u)$, expressing the most uniform strain energy. By taking the displacement increment $\Delta u$ during springback as the design variable $x$, taking the displacement constraints exerted on the specified points of the deformed part as the constraint conditions, the objective function can be written as:

$$F(x) = \sum_{\varepsilon=1}^{N} (E'(x) - \overline{E}(x))^2 \to \min \quad (4)$$

Where, $N$ is the total number of elements.

MATERIAL MODEL USED IN THE FEM SIMULATION OF COVER PANEL

The Anisotropy Yield Function

During the forming simulation of cover panel, the following assumptions are given: 1) The material volume is uncompress; 2) The thickness stress component $\sigma_z = 0$; 3) The sheet metal undergoes a monotonically or cyclic loading process without any intermediate unloading happening during forming. Then the anisotropy behavior of material is described by the generalization of the Hill’s 1979 anisotropy yield function[7] as:
\[ f = C \sigma_x + B \sigma_y^2 + H (\sigma_y - B \sigma_x)^2 + (2D \sigma_y)^2 - \bar{\sigma}^2 = 0 \quad (5) \]

Where \( \sigma_x \), \( \sigma_y \) and \( \sigma_{xy} \) are the plane normal stress and the shear stress, respectively. \( \bar{\sigma} \) is the equivalent stress. \( B, C, H, D \) are the constants determined by the uniaxial anisotropy parameters \( R_0 \), \( R_{44} \) and \( R_{90} \).

The Material Constitution Model Considering The Bauschinger Effect

In order to describe as precisely as possible the work-hardening effect of material, and the multi-cyclic bending and unbending progress during forming, referring to the literature [8], the influence of the Bauschinger effect is considered by introducing the “memory” factor \( \rho \) into the power exponent hardening model of material. It can be formulated as:

\[
\begin{align*}
\bar{\sigma}_j &= E\bar{\varepsilon}_j (\sigma_j < \sigma_{0j}) \\
\bar{\sigma}_j &= K\bar{\varepsilon}_j^n (\sigma_j \geq \sigma_{0j})
\end{align*}
\quad (6)
\]

Where \( E \) denotes the Young’s modulus, \( K \), \( n \) are the stress hardening coefficient and the strain hardening exponent. \( \bar{\sigma}_j \), \( \bar{\varepsilon}_j \) and \( \sigma_{0j} \) mean the equivalent stress, the equivalent strain and the yield stress in the \( j \)th reverse loading step, respectively. And, in this reverse loading step, \( \sigma_{0j} \) is updated by the equation (7) and (8). Where, \( \Delta \bar{\varepsilon}_j \) is the increment of the equivalent strain. \( \rho_j \) is a “memory” factor reflecting the Bauschinger effect. From equation (8), it is known that the influence of the Bauschinger effect is not considered when \( \rho_{j-1} \) is equal to 1. In the same way, the influence of the loading history is not reflected when \( \rho_{j-1} \) is equal to 0. Generally, take \( \rho_0 = 0 \) and \( \rho_j = 0.83 \sim 0.86 \quad (j \neq 0) \) [8].

\[
\sigma_{0j} = K (\rho_j \bar{\varepsilon}_j)^n 
\quad (7)
\]

\[
\bar{\varepsilon}_j = \rho_{j-1} \bar{\varepsilon}_{j-1} + \Delta \bar{\varepsilon}_j
\quad (8)
\]

A HYBRID “BFGS-SA” ALGORITHM

Suggestion Of The “BFGS-SA” Algorithm

Simulation Annealing algorithm is a heuristic and stochastic global optimization approach, which is proposed to deal with the combinational optimization problems according to its similarity with the metal annealing process in crystallography. That is to say, under an initial solution \( x_0 \) and initial control parameter \( t_0 \), with the descending of the control parameter, the global optima solution of the researched problem can be reached by cyclic-iterating of the “generation – judgment – acceptance or rejection” process of a new solution [9]. It is simple to program and its general application is great. But its disadvantage is easy to “forget ” the previously obtained local optima which is better than present so-called “optima” so as to the solution accuracy is unstable. Furthermore, during annealing, it is required that the cooling is executed “slowly” from a higher temperature to avoid happening the quenching effect because of the too fast cooling rate. This requirement makes it become difficult to improve the computation efficiency when the SA algorithm is used to solve the large-scale optimization problem [10].

Because the optimization model established for springback prediction, as shown in equation (4), has a large dimension ( \( N \) ) and a complex objective function, the traditional SA algorithm mustn’t be suitable for solving this problem. In the present work, the hybrid BFGS-SA algorithm is designed by combining the BFGS approach with the SA algorithm, which takes full advantages each of them, such as the great capability of searching the local optima solution, the higher calculating efficiency, the simple programming for the former and the great ability of achieving the global optimum solution for the latter. And it is fitting particularly to the large-scale combinational optimization problem with the dimension \( N \geq 100 \), and improves the accuracy of solutions obtained by putting a “memory” function in its program.

Description Of The Hybrid “BFGS-SA” Algorithm

The hybrid “BFGS-SA” algorithm can be described as follows:
(1) Set the initial value. Including the initial solution $X_0$, the initial temperature $t_0$, the length of the Markov chain $L_d$, the control variables of iteration $k = 0$ and $l = 0$. And, calculate the relevant objective function $F(X_0)$.

(2) Set an initial value for the “memory” function, i.e., $X_{opt} = X_0$ and $F_{opt} = F(X_0)$.

(3) Search the local optimum solution $X^*$ with the traditional BFGS optimum approach, and calculate the objective function $F(X^*)$.

(4) Update the original solution and the “memory” function. If $F(X^*) < F_{opt}$, then let $X_0 = X^*$ and $F(X_0) = F(X^*)$ to update the original solution with the obtained current local optimum solution, and then let $X_{opt} = X_0$, $F_{opt} = F(X_0)$ to update the “memory” function. Otherwise, only the former is executed.

(5) Searching the global optimum solution with the SA algorithm, it is described as:

1) Generate stochastically a new solution $X$ by the equation (10) in the neighborhood structure of the current original solution $X_0$, and calculate the relevant objective function $F(X)$ and the difference of $\Delta F = F(X) - F(X_0)$.

2) Judge, accept or reject a new solution in terms of the new solution accepted function represented by the equation (10). If $\Delta F \leq 0$, then accept the new solution as the current initial solution by setting $X_0 = X$ and $F(X_0) = F(X)$, in this case, if $F(X_0) < F_{opt}$, then update the “memory” function by setting $X_{opt} = X$, $F_{opt} = F(X)$, or else, nothing to be done. And then let $l = 0$ and goto the step (3). Inversely, if $\Delta F > 0$, then accept probabilistically a new solution as the current one by the status accepted function, i.e., when a new solution is accepted, if $l \leq L_d$, then set $l = l + 1$ and goto the step (3), otherwise, continue 3). Whilst, discard the new solution when it is not accepted, and goto 1).

3) If the stop criterions are satisfied, then goto the step (6), or else, continue 4).

4) Update the temperature parameter $t_k$ in terms to the cooling function expressed by the equation (9), and let $k = k + 1$, $l = 0$, then goto 1).

6) Stop iterating and output the approximately optimum solution stored in the “memory” function.

**Annealing Strategies For The Hybrid “BFGS-SA” Algorithm**

In order to solve the combinational optimization problem as represented in the equation (4), the corresponding annealing strategies of the “BFGS-SA” algorithm proposed are described as below:

1) Determining of the initial temperature $t_0$. For the springback prediction problem, according to the equivalence between the diminishing of the punch stroke during springback and the cooling of the metal annealing process, take the maximum travel distance $d_{max}$ of the punch at the end of panel cover forming process as the initial temperature, i.e., have $t_0 = d_{max}$.

2) Determining of the cooling function. Referring to the reference [11], the cooling function can be formulated as

$$t_k = \alpha_k t_{k-1}$$

Where $t_k$ and $\alpha_k$ are the temperature parameter and the cooling coefficient in the $k^{th}$ times iteration, respectively. And $\alpha_i = 1, \alpha_k = \left(1 - \frac{1}{k}\right)^m (k > 1)$, in here, $m$ denotes the cooling rate control constant.

3) The length of the Markov chain $L_d = 10 \times N$ [13]. In here, $N = 3n_{node} - 6$ denotes the number of design variables of the solved problem after removing the rigid mode, and $n_{node}$ is the number of nodes of the FE model.

4) The new solution generated function.

$$x_i = x_{i-1} + (0.5 - \eta) \times 2 \times \beta \times |x_{i-1}|$$

(10)
where \( x_i \) is the relevant component of the \( i^{th} \) trial point, \( \eta = \text{rand}[0,1] \) defines a random number in \([0,1]\) region, \( \beta_i \) takes the same value as \( \alpha_k \) in equation (9).

(5) The new solution accepted function. Take the Metropolis criterion to judge whether the new solution is accepted or not, i.e., calculate the equation of \( P = \min(1, \exp(-\Delta F / T)) \), if \( P \geq \eta \), then the new solution is accepted, otherwise, it is rejected. In here, \( \Delta F \) is the difference of the objective function of the new trial point with ones of the current solution, and \( T = t_k \) is the current temperature parameter.

(6) The stop criterion. A two-fold stop criterion consisting of the rule 1 and rule 2 is applied to the hybrid “BFGS-SA” algorithm. They are expressed as

\[
t_k \leq \varepsilon_1 \quad \text{and} \quad \left| \frac{F^*_k - F^*_{k-1}}{F^*_k} \right| \leq \varepsilon_2 ,
\]

respectively. Rule 1 means that stop the algorithm when the temperature parameter \( t_k \) is less than the specified infinite small positive number \( \varepsilon_1 \). Rule 2 represents that terminate the algorithm when the relative error of the objective functions \( F^*_k \) and \( F^*_k \), which is corresponding to the local optimum solutions obtained under two adjacent temperature parameters \( t_{k-1} \) and \( t_k \), is less than the infinite small positive number \( \varepsilon_2 \). In here, take \( \varepsilon_1 = \varepsilon_2 = 10^{-6} \).

**EXAMPLE AND RESULTS ANALYSIS**

For the drawing part as shown in figure 1, the springback prediction is performed with the proposed springback prediction method in this paper. Material is the stainless sheet steel (1Cr18Ni9Ti), the hardening coefficient \( K \) and the strain hardening exponent \( n \) are equal to 554.3Mpa and 0.195, respectively, and the yield strength is \( \sigma_y = 205 \text{Mpa} \).

In order to analyze and evaluate the springback prediction results, the local coordinate system is established as illustrated in figure 2. Where the sections paralleling to the xoz principle plane, are taken as the measurement planes, such as \( I_{y=15} \), \( I_{y=0} \), \( I_{y=12} \) as shown in figure 2. At the same time, the springback value \( \Delta h \) is defined as the normal component of the node displacement vector at the specified measurement point during the springback. It took about 868s to implement this example in Matlab6.5, and the calculating results obtained in the defined measurement planes are shown in figure 3. Correspondingly, the measurement results are shown in figure 4.

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**FIGURE 1 Example part for springback prediction**

**FIGURE 2 The definition of the measurement planes**

**FIGURE 3 The calculation results of the specified sections**

**FIGURE 4 The measurement results of the specified sections**
springback prediction method in present paper, the springback trends of the sample part can be reflected reasonably in every section. For instance, the springback value is increasing with the increase of the distance between the measurement point and the origin of the local coordinate system, the springback at the left end is more serious than ones at the right on a certain section, etc.

In addition, from the viewpoint of the accuracy of springback prediction, the statistics results indicate that the maximum springback values in the sections \( I_{y=0} \) and \( I_{y=12} \) are 1.2481 and 1.1459mm respectively. Comparing them with ones of the practical measurement, the corresponding maximum errors are less than 19.12%, 25.83%. Whereas, the errors in the section \( I_{y=15} \) are comparative larger, sometimes, almost exceed 35%. The accuracy of the springback prediction would be improved greatly if the reasons of the obvious fluctuation of measurement values at the ends of every section can be discovered. It is also an important research work in the future.

**CONCLUSIONS**

From the perspective of energy decreasing of the substance system so that a new balance status can be reached, according to the similarity of the sheet metal forming-springback process with the metal annealing process in crystalphysics, this schedule is put forward to simulating the sheet metal forming process with the non-linear FEM and predicting the springback value after it being formed with the hybrid “BFGS-SA” algorithm. The hybrid “BFGS-SA” algorithm is proposed to improve the calculating efficiency of the traditional SA algorithm and its capability of arriving the global optimum solution, which combines the SA algorithm with the BFGS approach because of the higher calculating efficiency, the great capability of searching local optimum solutions, the simple programming, and its suitability for the large-scale optimization problem with dimension of variables \( N \geq 100 \). At the same time, the corresponding annealing strategies are determined to deal with the springback prediction problem. By comparing the calculating results with the experiment ones at the specified measurement points, it is shown that it is a new and valid attempt to solve the springback prediction problem with the hybrid “BFGS-SA” algorithm proposed. In addition, problems that need to be researched in depth are pointed out by analyzing the relative errors between the calculating results with the experiment ones.

**REFERENCES**