On The Influence Of The Yield Locus Shape In The Simulation Of Sheet Stretch Forming

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Abstract. In the present paper results from an ongoing project at Volvo Cars and Chalmers University will be presented. The object of this project is to reduce the gap between the research frontier and the industrial practice concerning material modeling. One of the targets of the project is to identify a yield function, which can fulfill the special industrial demands concerning accuracy, easy parameter identification, and computational efficiency. Lately, some new yield functions have been presented, which seem to satisfy these demands. These yield functions belong to a group of non-quadratic yield criteria, sometimes referred to as "the Hosford family". These criteria are characterized by a stress exponent, which has been shown to have a strong coupling to the crystallographic structure of the material. The present paper addresses the issue of how the main parameters controlling the shape of the yield locus are influencing the material flow in sheet forming simulations. Since it is a well established fact that the shape of the yield locus has its major influence in pure stretch forming, i.e. on the right hand side of a FLD, we have chosen stretch forming with a hemispherical punch as a simple demonstration example. The M-K approach will also be used as a means to try to explain some of the observed phenomena.

INTRODUCTION

In the present paper results from an ongoing project at Volvo Cars and Chalmers University will be presented. Improved material modeling and characterization have been identified as key issues for taking industrial sheet forming simulations one step forward in terms of accuracy and reliability. The subject of material modeling involves several sub-tasks of which can be mentioned

- A yield condition
- A material hardening curve (up to high levels of effective plastic strain)
- A mixed hardening law
- Strain rate dependence

Each of these items requires a separate and careful consideration. It should also be mentioned that the subject of necking failure prediction is closely related to the subject of material modeling. The present paper deals only with the first of these topics, the yield function, and especially how the shape of the yield locus will influence the material flow in sheet forming simulations.

Previous Results

Results from the current project have previously been reported in Mattiasson and Sigvant [1]. In that report it was described how the Miyahuchi shear test was used to determine effective stress-strain curves up to much higher levels of effective plastic strains than what is possible to achieve with ordinary uniaxial tensile tests. Furthermore, in that study the yield condition by Barlat and Lian [2] (Yld89) was employed. It was considered as a natural, first step in advancing from the use of the quadratic Hill '48 yield condition towards more advanced ones. The stress exponent m, involved in the Yld89 condition, was determined by an inverse method from hemispherical stretch forming tests.
NEW YIELD CRITERIA FOR ANISOTROPIC METAL SHEETS

The Hosford Family Of Yield Criteria

One of the main tasks of the current project has been the identification of a suitable yield condition, which can fulfill the industrial demands regarding accuracy, easy parameter identification, and computational efficiency. Some newly presented yield conditions seem to fulfill these demand. These belong to a group of yield conditions, sometimes referred to as “the Hosford family”.

Since we assume that the forming simulations are performed by means of shell finite elements, we will in the following focus attention on yield functions expressed in the plane stress space. In 1972 Hosford [3] presented a non-quadratic, isotropic yield function, which in plane stress has the following form

\[ f = \left( \frac{1}{2} \left( |\sigma_1| + |\sigma_2| + |\sigma_1 - \sigma_2| \right) \right)^{\frac{m}{2}} - \sigma_Y = 0 \]  

The exponent m is here assumed to be a real number in the range \( 2 \leq m < \infty \). In the limits the current yield condition is identical to the von Mises and the Tresca yield criteria, respectively. Some authors prefer instead to write the exponent as \( 2k \), where \( k \) is a positive integer. We have, however, preferred the current interpretation for greater generality.

The development of this yield criterion was motivated by the observation that most experimental results lie between the yield criteria by von Mises and Tresca. Later, in 1980, Logan and Hosford [4] (see also Hosford [5]) compared the phenomenological model in Eq. (1) with yield loci calculated with polycrystalline models. It was found that the model represented by Eq. (1) was able to closely represent the yield locus for a body centered cubic (BCC) material using the exponent \( m=6 \), and the yield locus of a face centered cubic (FCC) material with \( m=8 \).

Hosford’s yield criterion has been further developed by many authors to include normal and planar anisotropy. One of the more well-known yield criteria within this family is the one by Barlat and Lian [2] (Yld89) for plane stress and planar anisotropy. This model involves four independent parameters, which have to be determined from equally many experimental tests.

New Yield Criteria Within The Hosford Family

Recently, Banabic et al. have in a series of papers [6, 7, 8] presented different versions of a yield criterion, which is an extension of the Yld89 criterion by including more anisotropy parameters. This yield criterion will in the following be called BBC2000. It involves a maximum of nine independent parameters. By putting some of the parameters equal to each other, or putting them equal to one, the number of independent parameters can be reduced. The parameters must be determined from a number of experimental tests, which is equal to, or higher than, the number of independent parameters.

Barlat et al. [9] have recently presented a yield criterion (Yld2000) for plane stress, in which the anisotropy is introduced by means of two linear transformations of the Cauchy stress tensor. There is a maximum number of eight independent anisotropy parameters in this model. The BBC2000 and Yld2000 models are very similar. They are, however, not identical, except in some special cases. It should finally be mentioned that Yld2000 can be viewed as a special case of a general method for deriving anisotropic yield criteria based on linear stress transformations proposed by Barlat et al. [10].

Some Comments On The Practical Applicability Of The New Yield Criteria

The new yield criteria BBC2000 and Yld2000 seem to be very well suited for industrial application from many points of view. First of all, it is well documented from numerous experiments and polycrystalline models that the shapes of the yield loci for steel and aluminum alloys can be found somewhere between the ones of the von Mises and Tresca yield loci, respectively. This is the case for all the yield conditions within the Hosford family. Secondly, the number of anisotropy parameters involved in these new yield conditions (7-8) seems to be well adapted to the number of test data available for sheet metals used for automotive applications. The standard test is the uniaxial tensile test in three directions, which yields six parameters (three yield stresses and three R-values). The tensile tests have to be supplemented by a bi-axial test, e.g. a hydraulic bulging test, or a shear test, in order to determine the hardening curve for large values of plastic strain. This yields one or two additional conditions for the determination of the parameters involved in the yield condition. Finally, the complexity of these yield conditions...
functions is not so big so that it deteriorates the computational efficiency.

**The Influence Of The Yield Surface Shape**

Within the current study we have found it important to attain a thorough understanding of how the yield surface shape, and how the basic parameters, which are controlling the shape, are influencing the material flow during a sheet forming operation. In the sheet forming society there seems to exist a number of "truths" or "myths" within the current area, whose veracity some times can be questioned. Examples of such "truths" are:

- The Hill'48 model is well suited for mild steel.
- The risk for a necking failure is increasing with an increasing value of the exponent $m$ in any of the material models of the "Hosford family".
- A high $R$-value is beneficial for any sheet forming operation.

The above statements have formed the basis of the current investigation, and will be discussed further on in this report.

**HEMISPHERICAL PUNCH STRETCH FORMING**

It is a well known fact that the yield surface shape has its major influence on the material flow in stretch forming, i.e. on the right hand side of a FLD. We have therefore chosen hemispherical punch stretch forming as a simple demonstration problem in the current study.

Of course all anisotropy parameters involved in a yield condition will influence the shape of the yield locus. However, the most influencing parameters are the exponent $m$, the average anisotropy parameter $R$, and for the new yield criteria, the equibiaxial yield stress $\sigma_b$. We will, thus, assume normal anisotropy, i.e. isotropic properties in the plane of the sheet. The two material models Yld89 and Yld2000 will be used in this study.

**Finite Element Model**

*Geometrical And Finite Element Data*

Punch radius: 125 mm  
Die corner radius: 10 mm  
Blank radius: 138 mm  
Sheet thickness: 1.00 mm  

Number of elements: 3150 triangular shells

**Material Data**

The hardening curve is given in the form

$$\sigma = K \left( \varepsilon_0 + \varepsilon^p \right)^n$$  \hspace{1cm} (2)

with the following material parameters

- $K = 600 \text{ MPa}$  
- $n = 0.24$  
- $\varepsilon_0 = 0.02605$  
- $\sigma_b/\sigma_u = 1.2086$

$\sigma_b/\sigma_u$ is the ratio between equibiaxial and uniaxial yield stress. This is given as input to the Yld2000 material model. The rather high value given here is typical for mild steel.

**Results**

A substantial number of simulations have been performed with the two material models and with various combinations of the parameters $m$ and $R$. The results will be presented in terms of Punch Depth at Failure (PDF). This is a measure, which is strongly coupled to the typical question, which the sheet forming analyst is confronted with every day, namely: can a stamping process be performed to a certain punch depth without failure?

**Detection Of Necking Failure**

Strain localization, or necking, is normally easily detected in the finite element model. A plot of the strain paths of an element, which is strain localizing, and a neighbor element is displayed in Fig. 1. Necking is usually accompanied by an abrupt drop in the punch force curve (Fig. 2, curve A), while in some rare cases necking takes place after the maximum force has been reached (Fig. 2, curve B). This latter phenomenon has never been observed in experiments and must be considered to be an unphysical behavior.

**Punch Depth At Failure**

Results for $R$-values in the range 0.5-2.5 and the exponent $m$ in the range 2-8 are presented in Table 1 and 2, and in Figs. 3 and 4. Note that those figures in the tables, corresponding to a case where necking occurred after the maximum punch force was reached, are enclosed in a dashed line.
Figure 1. Strain paths of two neighboring elements. (a) Before necking. (b) After necking.

Figure 2. Punch force curves exemplifying cases when necking coincides with punch force maximum (curve A), and takes place after punch force maximum (curve B).

Table 1. Punch depth at failure for the Yld89 model

<table>
<thead>
<tr>
<th>$R \setminus m$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>&gt;150</td>
<td>138.2</td>
<td>107.2</td>
<td>93.7</td>
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<tr>
<td>1.0</td>
<td>137.6</td>
<td>137.6</td>
<td>107.1</td>
<td>93.3</td>
</tr>
<tr>
<td>1.5</td>
<td>118.1</td>
<td>137.6</td>
<td>108.2</td>
<td>93.7</td>
</tr>
<tr>
<td>2.0</td>
<td>106.3</td>
<td>136.4</td>
<td>106.3</td>
<td>93.7</td>
</tr>
<tr>
<td>2.5</td>
<td>98.3</td>
<td>136.4</td>
<td>107.2</td>
<td>92.9</td>
</tr>
</tbody>
</table>

Table 2. Punch depth at failure for the Yld2000 model

<table>
<thead>
<tr>
<th>$R \setminus m$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
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</thead>
<tbody>
<tr>
<td>0.5</td>
<td>100.5</td>
<td>86.9</td>
<td>78.6</td>
<td>77.3</td>
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<tr>
<td>1.0</td>
<td>118.6</td>
<td>95.0</td>
<td>82.3</td>
<td>75.5</td>
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<tr>
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<tr>
<td>2.0</td>
<td>105.4</td>
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<td>89.4</td>
<td>80.0</td>
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<tr>
<td>2.5</td>
<td>101.3</td>
<td>121.3</td>
<td>90.3</td>
<td>82.3</td>
</tr>
</tbody>
</table>

Figure 3. Punch depth at failure for the Yld89 model

Figure 4. Punch depth at failure for the Yld2000 model

M-K ANALYSIS

Introduction

The assumption that a higher value of the exponent $m$ would yield a smaller value of the PDF is not always fulfilled. In an attempt to shed light on this matter, we have made use of the Marciniak-Kuczynski (M-K) approach for estimation of limit strains. In this approach a sheet sample with a geometric imperfection in form of a groove perpendicular to the principal 1-direction is studied. The homogeneous region "a" has a thickness $t_a$, and the groove, region "b", has a thickness $t_b$. The initial relation between the thicknesses is determined by the "imperfection factor" $f_0$, defined by $f_0 = t_b/t_a$. The strain is prescribed
incrementally in region "a" along a linear path determined by \( \rho = \Delta \varepsilon_2 / \Delta \varepsilon_1 \). Stresses and the strain increment \( \Delta \varepsilon_{1b} \) are determined so that force equilibrium is maintained in the 1-direction. The limit strain is obtained as the strains in region "a", when e.g. \( \Delta \varepsilon_{1b} > 10 \Delta \varepsilon_{1a} \).

We have chosen to study one single case, where our assumption regarding the influence of the exponent \( m \) is violated, namely: Yld89, \( R=2.5 \), \( m=2 \) and \( 6 \), respectively.

In Fig. 5 the strain paths for \( \rho=0.96 \) and \( f_0=0.995 \) are displayed. From this figure it is obvious that necking occurs first for \( m=2 \), which supports the observations in the FE-analyses. In Fig. 6 the corresponding stress paths are shown. From these figures it is not obvious at all for which \( m \)-value necking first occurs. In fact, the stresses are much higher at necking for \( m=2 \) than for \( m=6 \), although the strains are not.

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**The P-Parameter**

By just looking at the yield loci in Fig. 6 it is difficult to judge which parameter combination yields the highest disposition to necking. It has, however, been shown that the tendency for necking is closely related to the curvature of the yield locus at the equibiaxial yield point. According to Barlat [11] and Lian et al. [12] the curvature can be well represented by the following ratio

\[
P = \frac{\sigma_p}{\sigma_b}
\]

where \( \sigma_b \) is the equibiaxial yield stress, and \( \sigma_p \) is the major yield stress at plane strain. It should be noted that \( P \) is decreasing for increasing curvature.

If we apply this measure on our current problem, we get for \( m=2 \), \( P=1.0800 \), and for \( m=6 \), \( P=1.1136 \). This indicates a higher tendency for necking for \( m=2 \), which supports our observations in the FE-analyses.

We have finally calculated the P-parameter for all combinations of \( m \) and \( R \) for the Yld89 model. The results are displayed in a graph in Fig. 7. As can be seen, there is an almost exact correspondence with the PDF in Fig. 3.

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**CONCLUSIONS**

Let us now once again consider those statements that formed the basis for the current investigation.

*Is The Hill'48 Model Well Suited For Mild Steel?*

Both experimental observations and polycrystalline analyses indicate that steel should be modeled by some yield criterion from "the Hosford family" using an exponent \( m=6 \). Note that the Hill'48 model
corresponds to \( m = 2 \) for the Yld89 criterion in Table 1 and Fig. 3. Note, furthermore, that the \( R \)-value normally can be found in the range 2.0-2.5 for these steel qualities. In that range the difference in PDF between \( m = 2 \) and \( m = 6 \) is not so pronounced as for smaller values of \( R \). It seems, thus, that the Hill'48 model can give a descent approximation to the real behavior for these steels. It is, however, strongly recommended for optimal accuracy to use a non-quadratic yield condition, even for mild steel. It should finally be pointed out that, although the Hill'48 model has a descent capability to predict the strain distribution, the predicted stresses can be quite inaccurate, which is evident from Fig. 6. This is particularly important for subsequent springback analyses.

**Is The Tendency For Necking Increasing With Increasing Value Of The Exponent \( m \)?**

The answer to this question is obviously not a simple yes or no. As shown above the disposition to necking is strongly coupled to the P-parameter, which in turn is depending on both \( m \) and \( R \), and on the current yield condition. If we, however, limit our discussion to exponents \( m \) in the range 5-10, the answer to the question is an unconditional yes. This limitation is consistent with polycrystalline yield surface predictions.

**Is A High \( R \)-Value Beneficial For The Sheet Forming Process?**

The influence of \( R \) on the forming limit curve on the left hand side of the FLD is known to be small. There is though a beneficial effect of a high \( R \)-value in e.g. a tensile test, since the \( R \)-value influences the strain path. According to the performed stretch forming analyses the Hill'48 model predicts a strong negative effect of a high \( R \)-value. For exponents \( m \) in the range 6-8 the Yld89 model predicts no effect at all, while the Yld2000 model predicts a weak positive effect.

**The Yld89 Model Versus The Yld2000 Model**

The Yld2000 model involves more anisotropy parameters (8) than the Yld89 model (4), and has, thus, a better capability to model in-plane anisotropy. The importance of incorporating the equibiaxial stress point as one of the fitting parameters for the Yld2000 model should, however, be emphasized. For materials with small in-plane anisotropy the, weakness of the Yld89 condition is that it many times predicts the equibiaxial yield stress quite erroneously.

**REFERENCES**