Texture Evolution of FCC Sheet Metal during Deep Drawing Based on Rate Independent FEM Analysis*


School of Mechanical Engineering, Shanghai Jiaotong Uni., Shanghai 200030, P. R. China

Abstract. It is well known that the macroscopic anisotropy and formability of a sheet metal are significantly affected by the crystallographic texture. So it is very important to find texture evolution rule in metal forming processes. In this paper, a rate-independent polycrystalline plasticity model is developed and introduced into dynamic explicit element method. Metal flow is assumed to occur by crystallographic slip on given slip systems within each crystal. Every integration point is a single crystal. Then the stability of ideal orientations and texture evolution of polycrystal are investigated.

INTRODUCTION

It is well known that the macroscopic anisotropy and formability of a sheet metal are significantly affected by the crystallographic texture. As a result, it is necessary to introduce individual or combined effects of these microstructure properties (such as crystallographic textures, texture distributions and texture evolutions) into metal forming simulation.

The polycrystalline texture in plastic deformation has been experimentally investigated and theoretically predicted both in microscopic and macroscopic scales. In macroscopic scales, the plastic behavior of polycrystalline materials is assumed to be well described by analytical stress and strain rate potentials[4-5] or macro anisotropic yield functions[6-7]. The anisotropy coefficients of the potentials or yield functions are determined using initial texture or experimental mechanical behavior data. These functions are easy to implement into FEM procedure that simulate forming processes. However, sometimes the anisotropy coefficients are not sufficient to describe the macroscopic anisotropy of polycrystalline materials.

In micro scales, Savoie et. al[8] experimentally investigated the texture evolution of aluminum alloys in deep drawing by considering non-orthorhombic sample symmetry. Zhou et. al[9] also investigated the stability with strain of initial texture components of FCC sheet metals during deep drawing. However, these investigations were performed under the plane strain condition. Recently, finite element analysis was used to analyze the texture evolution during deformation. Chang-hee et al.[10] used the rate sensitive constitutive model to develop a finite element analysis which can simulate the evolution of the crystallographic texture in the bulk forming process. Sheila[11] and Choi[12,13] predicted the evolution of the texture in FCC metals with initial texture using rate sensitive model.

In this paper, a rate-independent polycrystalline plasticity model is developed and introduced into dynamic explicit element method. The material is assumed to be a polycrystal consisting of many grains. Metal flow is assumed to occur by crystallographic slip on given slip systems within each crystal. Every integration point is a single crystal. This method can be used to take crystallographic texture evolution into account during metal forming processes. Self and latent hardening is also taken into account. Then cup drawing of aluminum sheet is simulated with the developed rate-independent polycrystalline plasticity FEM program. Texture evolutions with different initial orientations are calculated.

* Supported by the Ph. D. Programs Foundation of Ministry of Education of China, No. 20030248029
1. THE RATE-INDEPENDENT 
CRYSTALLINE PLASTICITY FEM 
MODEL

1.1 Crystalline plasticity constitutive 
equation

The evolution of texture has been calculated by the following procedures. The deformation mechanism of FCC sheet metal is assumed to be slip mechanism on the twelve 
\{111\} <110> slip systems.

The deformation gradient \( F \) can be written as in Lee \cite{14} in the form:

\[
F = F^c F^p 
\]  
(1)

where \( F^c \) is the deformation caused by the aberrance and rotation of the crystal lattice, \( F^p \) is the deformation due to plastic shearing on crystallographic slip systems.

As the crystal deforms, vectors connecting lattice sites are rotated according to \( F^c \). For a slip system \( s \), the initial normal vector of the slip plane is defined by \( m^{(s)} \), the unit slip direction vector by \( n^{(s)} \). During the plastic deformation, the crystalline lattice rotates. Then these vectors become \( m^{*(s)} \) and \( n^{*(s)} \):

\[
m^{*(s)} = F^c m^{(s)} 
\]

\[
n^{*(s)} = (F^c)^T n^{(s)} 
\]  
(2)

Then the velocity gradient tensor \( \dot{L} \), the rate of deformation tensor \( D \) and material spin tensor \( \omega \) can be decomposed as follows:

\[
\dot{L} = \dot{F} = D + \dot{\omega} 
\]

\[
L^p = D^p + \omega^p = \sum_{s=1}^{M} \dot{\gamma}^{(s)(s)} n^{(s)} m^{*(s)} 
\]  
(3)

where \( \dot{\gamma}^{(s)} \) is the shear strain rate on the slip system \( s \), \( M \) is the total number of slip systems. The plastic rate of deformation tensor \( D^p \) and plastic rotation tensor \( \omega^p \) are

\[
D^p = \sum_{s=1}^{M} L^p_{ij} \dot{\gamma}^{(s)} \quad \omega^p = \sum_{s=1}^{M} W^p_{ij} \dot{\gamma}^{(s)} 
\]  
(4)

where

\[
L^{(s)}_{ij} = \frac{1}{2}(m^{*(s)}_{i} n^{*(s)}_{j} + n^{*(s)}_{i} m^{*(s)}_{j}) 
\]

\[
W^{(s)}_{ij} = \frac{1}{2}(m^{*(s)}_{i} n^{*(s)}_{j} - n^{*(s)}_{i} m^{*(s)}_{j}) 
\]  
(5)

The rate of deformation tensor \( D^p \) can be decomposed as

\[
D^p = D^p + \dot{D}^p 
\]  
(6)

Then the crystalline plasticity constitutive equation can be expressed as

\[
\sigma_{ij} = C^{\varepsilon e}_{ijkl} (D_{kl} - \sum_{s=1}^{M} L^{(s)} \dot{\gamma}^{(s)}) 
\]  
(7)

where \( \sigma_{ij} \) is the Jaumann rate of Kirchhoff stress tensor, \( C^{\varepsilon e}_{ijkl} \) is the tensor of the elastic modulus.

1.2 The calculation of the plastic strain

The resolved shear stress in the slip system \( s \) can be calculated with

\[
\tau^{(s)} = L^{(s)}_{ij} \sigma_{ij} 
\]  
(8)

The Schmid law is given by

\[
sign(\tau^{(s)}) \dot{\gamma}^{(s)} = k_s 
\]  
(9)

where \( k_s \) is the critical yielding stress on the slip system \( s \).

If the total strain increment \( \dot{\varepsilon}_{ij} \) is given, the shear increment on the active slip systems can be determined by equation (8) and equation (9). The active slip system is judged from the following conditions:

\[
sign(\tau^{(s)}) \dot{\gamma}^{(s)} \geq 0 \quad \text{for active slip systems} 
\]

\[
sign(\tau^{(s)}) \dot{\gamma}^{(s)} < k_s \quad \text{for inactive slip systems} 
\]

Equation can be rewritten as:

\[
X_s > 0, \quad Y_s = 0 \quad \text{for active slip system}
\]
\[ X_s = 0, \ Y_s < 0 \quad \text{for inactive slip system} \]

where:

\[ X_s = \text{sgn}(\varepsilon^{(s)}) \dot{\gamma}^{(s)} \]
\[ Y_s = \text{sgn}(\varepsilon^{(s)}) L^{(s)} \sigma_{ij} - k_s \]
\[ (10) \]

The key problem of the rate independent formulation of crystal plasticity is the possible occurrence of redundant constraints due to linear dependent active slip systems. In order to overcome this problem, Takahashi proposed a quite simple method, called the “successive integration method”[15]. Consider the following equation for all slip systems:

\[ 2G \cdot (dX_s / d\rho) = Y_s \]
\[ (11) \]

where \( \rho \) (\( \geq 0 \)) is a monotonously increasing parameter such as time, \( G \) is shear modulus. Then each slip system must belong to one of the following two groups:

a) \( X_s > 0 \) and \( dX_s / d\rho = 0 \), for active systems;

b) \( X_s = 0 \) and \( dX_s / d\rho < 0 \), for inactive systems.

The algorithm for the numerical computation is as follows.

1. Set \( \dot{\gamma}^{(s)} = 0 \) and \( D^{p}_{ij} = 0 \)

2. Calculate trial stress with eq.(7);

3. Calculate the trial resolve shear stress in each slip system with eq. (8)

4. Determine the potentially active systems that satisfy: \( Y_s \geq 0 \)

5. Obtain \( dX_s \) from (11) and we have

\[ \dot{\gamma}^{(s)} + d\dot{\gamma} = \dot{\gamma}^{(s)} + \dot{\gamma} \]

6. Amend \( D^{p}_{ij} \) by equation (4) and update \( k_s \) by hardening law.

7. If the convergence of \( D^{p}_{ij} \) is not attained, return to step 2

### 1.3 Orientation assignment to finite elements

In this study, crystals are represented with FE integration points. The orientations, determined from the measured ODF (orientation distribution function) data and expressed in term of Euler angles, should be assigned to FE integration points.

The position \( g \) in the whole orientation space \( V \) can be determined by the three Euler angles \( g(\phi_1, \phi_2, \phi_3) \). During deformation process, the polycrystalline orientations would spread around some stable orientations. And these orientations, on the whole, present a three-dimension normal distribution. Then the orientation distribution function \( f(g) \) can be divided into two parts: normal distributions and random distribution, and the orientation distribution function can be written as[16]:

\[ f(g) = f_s(g) + \sum \sum f_j(g) \]
\[ (12) \]

Where, \( f_s(g) \) is the random distribution, and \( f_j(g) \) is the main texture component with a peak density at the related position of Euler angle coordinate space, \( \text{SUM} \) is the number of main orientations. So, for the \( i \)th main texture component \( g_i \), the volume fraction \( V_i \) of a spreading space around this peak position can be obtained by:

\[ V_i = \frac{1}{2\sqrt{\pi}} Z_i S_{n_i} \psi_i [1 - \exp(-\psi_i^2 / 4)] \]
\[ (13) \]

In which, \( Z_i \) is the multiplicity factor, \( S_{n_i} \) is the peak ODF value of the \( i \)th normal- distribution texture component, and \( \psi_i \) is the disperse breadth. Here, the fraction of the crystalline numbers \( \text{SUM} \) is assumed to be equal to that of volume in each spreading space:

### 1.4 The crystal lattice orientation

To examine the stability of one texture component in orientation space, a parameter to describe orientation change is required. The change of orientation at a given Euler space can be expressed by the lattice rotation rate \( \dot{g}(\phi_1, \phi_2, \phi_3) \), the gradients and divergence of \( \dot{g} \) as:
\[
\begin{align*}
\dot{\phi}_1 &= \dot{\omega}_x - \dot{\phi}_1 \cos \phi \\
\dot{\phi} &= \dot{\omega}_x \cos \phi + \dot{\omega}_1 \sin \phi \\
\dot{\phi}_2 &= \left(\dot{\omega}_y \sin \phi - \dot{\omega}_2 \cos \phi\right) / \sin \phi
\end{align*}
\] (14)

\[
div \dot{\mathbf{g}} = \frac{\partial \dot{\phi}_1}{\partial \phi_1} + \frac{\partial \dot{\phi}}{\partial \phi} + \frac{\partial \dot{\phi}_2}{\partial \phi_2}
\] (15)

Negative implies that more orientations around ideal orientation \(g\) rotate towards \(g\). And the behaviors of the orientation change can be expressed by the orientation distribution function. In order to describe the change of the ODF during deformation, the continuity equation at a fixed point of the Euler space can be derived from earlier studies\(^{(17)}\):

\[
(\dot{f} / f)_x + \dot{\phi} t g \phi + div \dot{\mathbf{g}} = 0
\] (16)

Then the stability of the ideal orientations can be decided by following criteria:

**Mathematic criteria**

An orientation \(g\) remains stable during deformation if and only if the following stability conditions are satisfied:

\[
\begin{align*}
\dot{\mathbf{g}} &= (\dot{\phi}_1, \dot{\phi}, \dot{\phi}_2) = 0 \\
\frac{\partial \dot{\phi}_1}{\partial \phi_1} &\leq 0
\end{align*}
\] (17)

These stable conditions of orientation can determine whether or not the orientations around grain \(g\) rotate away from \(g\) during deformation. However, these conditions cannot determine whether or not orientation density around \(g\) increases during deformation.

**Texture criteria**

In the stability for texture formation, a texture component at a given \(g\) is stable during formation as long as:

\[
\begin{align*}
\dot{\mathbf{g}} &= (\dot{\phi}_1, \dot{\phi}, \dot{\phi}_2) = 0 \\
(\dot{f} / f)_x &> 0
\end{align*}
\] (18)

These stability conditions mean that zero rotation rates occur and orientation density around \(g\) increases during deformation. The above conditions only describe the stability of the texture formation around \(g\), but cannot determine whether or not orientations around \(g\) rotate away from \(g\).

In order to investigate the stability of the initial texture components during deep drawing, the rotation rate, and relative ODF intensity varieties have to be calculated and considered in ideal orientations.

3. ANALYSIS RESULTS OF DEEP DRAWING

3.1. Experiments for the numerical simulation

The above rate independent polycrystalline elastic/plasticity model is introduced into dynamic explicit FEM procedure\(^{(18)}\), and a cup-drawing problem is adopted to investigate the texture effect on the earing.

The schematic view of the cup drawing process is shown in Fig. 1. The annealing pure aluminum sheets are selected to study the relation between the preferred orientation and earing. The experimental ODF data for annealing aluminum sheet is illustrated in Fig.2.

**FIGURE 1.** Geometry and dimensions of the cup drawing

**FIGURE 2.** The ODF data for annealing aluminum sheet

442
Based on above theory, initial orientations are assigned to FE simulations and the cup drawing of these two sheets is simulated. Other material properties for FCC aluminum are assumed as: elastic modulus $E=69$ GPa, Poisson ratio $\nu=0.3$, sheet thickness $t=0.7$ mm.

### 3.2 Rotation tendency around the ideal orientations

According to Fig.2, the initial and main orientations during deep drawing can be found at $\phi_2=45^\circ$ section in the Euler space. Then six ideal orientations are mainly calculated during deep drawing simulation: A(90°,0°,45°), B(45°,0°,45°), C(30°,55°,45°), D(90°,55°,45°), E(20°,50°,45°) and F(85°,60°,45°). Fig.3 shows the relative rate of the variety of ODF data for the ideal orientations along the RD. Even at the beginning of deep drawing, these orientation densities might increase or decrease. With the punch stroke become deeper, especially after the half of the final height, the change of the orientation densities become stable. Then we can get the evolution trends of these ideal orientations from the average values.

![FIGURE 3. The relative rate of the variety of ODF data for the ideal orientations along the RD.](image)

Table 1 shows the calculated results for the rates of change $\dot{\phi}$, gradients $\partial \dot{\phi}/\partial \phi$, divergence $\text{div} \; \dot{\phi}$ and relative rate of change of ODF intensity $(\dot{f}/f)_x$ at the flange area. As shown in this table, only D (90°, 55°, 45°) component satisfies the stability conditions of orientation shown in Eq. (17) and Eq. (18). And C (30°, 55°, 45°) component satisfy the stability condition of texture formation. However it exhibits only two-dimensional convergences with negative $\text{div} \; \dot{\phi}$ and is a sub-stable orientation. The other orientations cannot satisfy any stability condition, and will rotate away from these orientations.

The ODF figure after drawing can be drawn according to the calculated orientations at integration points which represent crystals. Fig. 4 shows the $\phi_2=45^\circ$ sections of ODF for annealing/deformed sample. In order to investigate the rotations of the crystal orientations during deep drawing, the initial and deformed textures are presented together. The crystal orientations distributed around the (90°,0°,45°) component and (90°,0°,45°) move away from these components. The crystal orientations distributed around the (20°,50°,45°) component and the (85°,60°,45°) component move toward the (30°,55°,45°) and (90°,55°,45°) components respectively.

![FIGURE 4. $\phi_2=45^\circ$ sections of ODF](image)

**TABLE 1. Rates of change $\dot{\phi}$, gradients $\partial \dot{\phi}/\partial \phi$, divergence $\text{div} \; \dot{\phi}$ and relative rate of change of ODF intensity $(\dot{f}/f)_x$ for the ideal orientations in the deep drawing**

<table>
<thead>
<tr>
<th></th>
<th>$\phi_1$</th>
<th>$\dot{\phi}$</th>
<th>$\phi_2$</th>
<th>$\partial \dot{\phi}/\partial \phi$</th>
<th>$\partial \dot{\phi}/\partial \phi$</th>
<th>$\partial \dot{\phi}/\partial \phi$</th>
<th>$\partial \dot{\phi}/\partial \phi$</th>
<th>$\text{div} ; \dot{\phi}$</th>
<th>$(\dot{f}/f)_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.88</td>
<td>0.00</td>
<td>-0.86</td>
<td>7.26</td>
<td>0.00</td>
<td>-0.05</td>
<td>7.21</td>
<td>-7.21</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-1.01</td>
<td>0.00</td>
<td>1.02</td>
<td>7.03</td>
<td>0.00</td>
<td>-0.06</td>
<td>6.97</td>
<td>-6.97</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-3.27</td>
<td>-0.59</td>
<td>0.55</td>
<td>-3.31</td>
<td>3.31</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-4.39</td>
<td>-1.13</td>
<td>-2.04</td>
<td>-7.56</td>
<td>7.56</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.23</td>
<td>-0.25</td>
<td>-0.16</td>
<td>3.91</td>
<td>-1.10</td>
<td>1.33</td>
<td>4.14</td>
<td>-3.93</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.38</td>
<td>0.00</td>
<td>-0.37</td>
<td>3.06</td>
<td>0.00</td>
<td>-0.36</td>
<td>2.70</td>
<td>-2.70</td>
<td></td>
</tr>
</tbody>
</table>
Comparisons of the final simulated configure line and calculated ODF results of annealing sheet after forming with experimental one are shown in fig.5. Good consistency can be found. The numerical results have little stronger congregation and higher ODF peak values. After deep drawing, the \{001\}<110> texture decreases, and then rotate to \{001\}<100> texture after deformation; the \{124\}<211> texture rotate to \{111\}<112> texture. And by the balance between \{001\}<110> texture and \{124\}<211> texture components, the deformed cup have no strong earring tendency.

**FIGURE 5.** The final configure lines and ODF data of the annealing sheet after forming

**REFERENCES**