Forming Limit Stresses of Sheet Metal under Proportional and Combined Loadings

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Abstract. The effects of changing strain paths on forming limit stresses of sheet metals are investigated using the Marciniak-Kuczyński model. Forming limits are analyzed for proportional loading and two types of combined loadings: combined loading which includes unloading between the first and second loadings and that which includes an abrupt strain path change without unloading between the first and second loadings. The forming limit stress curves in stress space calculated for the combined loading with unloading are in good agreement with those calculated for the proportional loading, while the forming limit curves in strain space are strongly dependent on the strain paths. The forming limit stresses calculated for combined loading with an abrupt strain path change, however, do not coincide with those calculated for proportional loading. The strain path dependence of the forming limit stresses is discussed in detail.

INTRODUCTION

Forming Limit Curve (FLC), which defines maximum allowable strain levels during sheet metal forming, is commonly used for assessing formability by die engineers. The FLCs are usually determined experimentally or theoretically for proportional strain paths. However during actual forming operations, material elements may undergo the strain paths that deviate significantly from proportional strain paths. Experimental and numerical studies both show that the strain path change affects on the limit strains significantly [1-5]. Thus the FLC method is valid only for the cases where a sheet element subjected to proportional loading.

On the other hand, some researchers reported that Forming Limit Stress Curve (FLSC), which is constructed by plotting the state of stress at the onset of localized necking in stress space, is almost path-independent [5-9]. The limits to localized necking would be predicted accurately by using a combination of the FLSC and FE simulation not only for proportional loading but also for the cases where a sheet element undergo complex strain history. Knowledge of path-independence of the FLSC is not widespread nor is its significance appreciated.

To validate the FLSC concept Yoshida et al. [9] investigated the forming limit stresses of a 5000 series aluminum alloy tube using internal pressure-axial load type testing machine [10] for proportional and combined loadings, and found that the FLSCs are almost path-independent. Wu et al. [8] showed the path independence of the FLSC by numerical simulations using polycrystalline plasticity with the M-K model [11] and discussed the reason why the FLC is dependent on loading paths by observing the work hardening behavior of the polycrystalline material and using the concept of the path independence of FLSC.

In this study, the effects of changing stress/strain paths on FLSC and the mechanism which cause the path independence of FLSC are analyzed in detail for proportional loading and two types of combined loadings: combined loading which includes unloading between the first and second loadings and that which includes an abrupt strain path change without unloading between the first and second loadings. The effect of the material parameters on the path-independence of the FLSC is also investigated.
CONSTITUTIVE MODELS

Assuming small elastic and finite plastic deformation, we can write the kinematics in the rate form, 
\[
\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p = \mathbf{D}^e + \Phi \mathbf{N}^p ,
\]

\[
\mathbf{W} = \mathbf{\omega} + \mathbf{W}^p = \mathbf{\omega} + \Phi \mathbf{\Omega}^p ,
\]

where \( \mathbf{D} \) is the rate of deformation tensor, \( \mathbf{W} \) is the continuum spin tensor, the superscripts e and p denote the elastic and plastic parts, \( \mathbf{\omega} \) is the spin of material substructure, and \( \mathbf{N}^p \) and \( \mathbf{\Omega}^p \) define the direction of \( \mathbf{D}^p \) and \( \mathbf{W}^p \), respectively. \( \Phi \) is a non-negative overstress function for rate-dependent cases. The rate-constitutive relation for the elastic part is given by 
\[
\dot{\sigma} = \sigma - \mathbf{\omega} \cdot \sigma + \sigma \cdot \mathbf{\omega} = \mathbf{C}^e \cdot \mathbf{D}^e + \mathbf{C}^p \cdot \mathbf{D}^p + \Phi \mathbf{C}^e : \mathbf{N}^p ,
\]

where (\( \hat{\cdot} \)) denotes the objective rate of the Cauchy stress tensor with respect to the substructure spin and \( \mathbf{C}^e \) is the forth order elastic moduli tensor. In this analysis we use a yield function proposed by Barlat-Lian [12],
\[
J = J(\sigma, \mathbf{n}, \mathbf{m}) = g(\Phi) = 0 ,
\]

where \( J \) is an equivalent stress, \( \mathbf{n} \) are orthonormal basis specifying the orthotropic axes, \( \mathbf{m} \) is an equivalent plastic strain, \( g \) is a strain hardening function, \( \mathbf{m} \) is a reference value of the overstress function and \( m \) is a rate sensitivity parameter. We use Swift’s power law, \( g = k(\varepsilon^p + \varepsilon)\alpha \). 

In this analysis we use a yield function proposed by Barlat-Lian [12],
\[
J = \left[ \frac{1}{2} a [K_1 + K_2]^M + a [K_1 - K_2]^M + (2 - a) [2K_2]^M \right]^{1/M} ,
\]

\[
K_1 = (\hat{\sigma}_{11} + h \hat{\sigma}_{22}) / 2 ,
\]

\[
K_2 = \sqrt{((\hat{\sigma}_{11} - h \hat{\sigma}_{22}) / 2)^2 + p^2 \hat{\sigma}_{12}^2} ,
\]

where \( \hat{\sigma}_{ij} \) are components of the Cauchy stress tensor in reference to the orthotropic axes \( \mathbf{n} \).

We adopt a non-normality flow rule proposed by Kuroda and Tvergaard [13]. The direction of \( \mathbf{D}^p \) is taken to be
\[
\mathbf{N}^p = \mathbf{n} + \hat{\delta} \mathbf{m} ,
\]

where \( \hat{\delta} \) is a scalar-valued function, \( \mathbf{n} = \partial J / \partial \sigma / || \partial J / \partial \sigma || \) is a unit normal of the dynamic yield locus and \( \mathbf{m} = [\mathbf{D}' - (\mathbf{n} \cdot \mathbf{D}') \mathbf{n}] / || \mathbf{D}' - (\mathbf{n} \cdot \mathbf{D}') \mathbf{n} || \) is perpendicular to \( \mathbf{n} \). \( \hat{\delta} \) is taken to be given by 
\[
\hat{\delta} = \tan \theta^p , \quad \theta^p = \begin{cases} \pi \theta \quad & \text{for } \pi \theta \leq \theta^p_{\text{crit}}, \\ \theta^p_{\text{crit}} \quad & \text{for } \pi \theta > \theta^p_{\text{crit}}. \end{cases}
\]

\[
\theta = \cos^{-1}[(\mathbf{n} \cdot \mathbf{D}') / || \mathbf{D}' ||] ,
\]

where \( \theta \) is the angle between \( \mathbf{n} \) and \( \mathbf{D} \), \( c \) is a coefficient and \( G \) is the elastic shear modulus.

PROBLEM FORMULATION

Occurrence of localized necking is predicted using the Marciniak-Kuczyński model [11]. At undeformed state the orthonormal axes \( \hat{x}_i \) (i.e. \( \mathbf{n}_i \)) are coincide with the fixed Cartesian axes \( x_i \), the orthonormal basis of which is \( e_i \). Quantities inside the band are denoted by (\( \bullet \))^b. The initial thickness inside and outside the band are denoted by \( h^b \) and \( h \). The initial geometric non-uniformity is defined by \( f_0 = h^b / h_0 \). The current unit normal of the band is \( \mathbf{f} = (\cos \psi, \sin \psi) \). \( \psi \) is the angle between \( \mathbf{f} \) and \( e_1 \). The compatibility at the band interface is given in terms of the differences in the velocity gradients inside and outside the band, 
\[
L_{ij}^b = L_{ij} + \dot{c}_i \mathbf{m}_j ,
\]

where \( \dot{c}_i \) are the parameters to be determined. Hereafter, subscripts range from 1 to 2. The current thickness inside and outside the band are denoted by \( h^b \) and \( h \), respectively. Equilibrium conditions at the band interface are given by 
\[
\mathbf{m}_i \sigma_{ij}^b h^b = \mathbf{m}_i \sigma_{ij} h \ .
\]

Substituting the constitutive relation into the rate form of equation (11), with elimination of \( L_{ij}^b \) using equation (10), gives simple algebraic equations having two unknowns, \( \dot{c}_1 \) and \( \dot{c}_2 \). Once \( \dot{c}_1 \) and \( \dot{c}_2 \) are solved, we can calculate all the rate values of the variables to be updated.
The onset of localized necking is defined by the occurrence of a much higher maximum principal logarithmic strain rate inside the band than outside the band, \( \dot{\varepsilon}_{11}^b / \dot{\varepsilon}_{22}^b \geq 10000 \), where \( \dot{\varepsilon}_{11}^b \) and \( \dot{\varepsilon}_{22}^b \) are the maximum principal values of the rate of deformation tensors \( D^b \) and \( D \), respectively. The strain components at the onset of localized necking are defined by \( \varepsilon_{11}^b \) and \( \varepsilon_{22}^b \), for various values of the initial band angle \( \psi \), as determined by the following steps: (i) calculating the strains outside the band at the occurrence of localized necking, \( \varepsilon_{11} \) and \( \varepsilon_{22} \), (ii) finding the minimum value of \( \varepsilon_{11}^b \), and (iii) defining the minimum value of \( \varepsilon_{22}^b \) and the corresponding strain components \( \varepsilon_{11}^b \) and \( \varepsilon_{22}^b \), to be plotted in strain space. The corresponding stress components, \( \sigma_{11}^b \) and \( \sigma_{22}^b \), are defined as the forming limit stresses to be plotted in stress space.

FLCs and FLSCs are calculated for linear loading and combined loadings A and B:

**Combined loading A**: The ratio of \( \dot{\sigma}_{22} / \dot{\sigma}_{11} \) outside the band is defined by the parameter \( \alpha = \dot{\sigma}_{22} / \dot{\sigma}_{11} \), and \( \alpha_I \) and \( \alpha_{II} \) represent the ratio of stress rate in the first loading and second loading, respectively. In the first loading the sheet is deformed until the equivalent plastic strain achieves \( \varepsilon_{11}^b = 0.2 \) and \( 0.4 \) with \( \alpha_I = 0 \), \( \alpha_I = 1 \) and \( \alpha_I = \infty \). After unloading, the sheet is reloaded with several proportional loading (\( 0 \leq \alpha_{II} \leq 1 \)) until the onset of localized necking.

**Combined loading B**: The ratio of \( D_{22}^b / D_{11}^b \) outside the band is defined by the parameter \( \rho = D_{22}^b / D_{11}^b \). We consider three sets of combined strain paths in which \( \rho \) changes as follows: -0.5 (\( \rho > 0 \)) \( \rightarrow \) 1, 1 \( \rightarrow \) -0.5 (\( \rho = 0 \)) and -0.5 (\( \rho < 0 \)) \( \rightarrow \) -0.5 (\( \rho > 0 \)). In this combined loading, unloading is not included between the first and second loadings.

**RESULTS**

The values of the material constants are taken to be \( E = 65 \) GPa, \( \nu = 0.3 \), \( k = 490 \) MPa, \( \varepsilon_0 = 0.00285 \), \( n = 0.35 \), \( m = 0.002 \), \( \Phi_0 = 0.002 \), \( c = 3 \) and \( \theta_{\text{crit}} = 20^\circ \). Here, \( \theta_{\text{crit}} = 20^\circ \) is based on the polycrystal plasticity analysis [4, 14] and the biaxial stress experiments [15]. Initial imperfection is taken to be \( f_0 = 0.995 \).

The FLCs and FLSCs calculated for the proportional loading and the combined loading A are shown in Fig. 1. It is found that the FLCs are apparently dependent on the strain paths. The apparent path dependence of the FLCs observed in Fig. 1a is
consistent with that found in the literature [2, 3] for both experiment and numerical analysis.

The dashed lines in Figs. 1b and c indicate the dynamic yield surface passing the stress point at the end of the first loading. In this analysis the rate sensitivity parameter, \( m \), is taken to be 0.002, so that the dynamic yield surface is almost identical to the yield surface for time-independent plasticity. For the case of \( \varepsilon = 0.2 \) forming limit stresses for the combined loading A almost drop on those for the proportional loading. For the case of \( \varepsilon = 0.4 \) forming limit stresses for the combined loading A almost drop on the FLSC calculated for the proportional loading, provided that the stress points were located outside the dynamic yield surface at the end of the first loading. On the other hand, for the range where the FLSC for the proportional loading is inside the dynamic yield surface, i.e. \( 50 < \sigma < 300 \) MPa in Fig. 1b, localized necking occurred right after that the stress reached the dynamic yield surface in the second loading. The localized necking did not take place when the stress first reached the FLSC, since the material is still in an elastic region. A similar phenomenon was observed in the experiment on a A5154-H112 tube [9].

In order to evaluate the strain path dependence of the FLSC shown Figs. 1b and c quantitatively, the relative differences between the forming limit stresses for the proportional loading and those for the combined loading A are calculated by \( \left( \frac{\sigma_{11}^* - \sigma_{11}^p}{\sigma_{11}^p} \right)^p \), where \( \sigma_{11}^* \) and \( \sigma_{11}^p \) are the forming limit stresses for the proportional loading and the combined loading A, respectively. The results are shown in Fig. 2. For the case of \( \varepsilon = 0.2 \) the relative differences are less than 0.6%. For the case of \( \varepsilon = 0.4 \) with \( \alpha = 0 \) and 1 the relative differences are less than 1% excluding those for the range \( 50 < \sigma < 300 \) MPa. For the case of \( \varepsilon = 0.4 \) and \( \alpha = \infty \) these are less than 2.5%. The predicted forming limit stresses are not sensitive to the strain path; therefore, we conclude that the FLSC is almost path-independent at least for the combined loading A.

The FLCs and FLSCs for the proportional loading and the combined loading B are shown in Fig. 3. The predicted forming limit stresses for the combined loading B are completely different from those for the proportional loading. The relative differences between
predicted forming limit stresses for the proportional loading and those for the combined loading B are 4 to 9% in maximum. Thus, it is clear that the FLSCs depend on the strain path for the combined loading B.

Next, we performed parametric calculations to investigate the effect of several parameters on FLC and FLSC calculations. Forming limits are analyzed for additional four conditions as follows: (1) \( f_0 = 0.99 \) and 0.999 in stead of \( f_0 = 0.995 \), (2) \( M = 2 \) (Von Mises yield function) in stead of \( M = 8 \), (3) \( r_0 = 0.36 \) and \( r_{\theta 0} = 0.59 \) in stead of isotropy and (4) \( c = \infty \) or \( \theta_p = 0^\circ \) (associated flow rule) in stead of non-normality flow rule. All values of the remaining parameters are the same as those used in Fig. 1. We found that these parameters affect the shapes of the FLSC and the FLC, but the forming limit stresses are still path-independent for the combined loading A and are depend on the strain path for the combined loading B for all four cases.

**DISCUSSION**

**Effect of the Saturation of Stress-Strain Curve on the FLSC**

It might be argued that the saturation of the stress-strain curve is responsible for the degeneracy of the forming limit stresses as a single curve in stress space. In this analysis we used isotropic strain hardening rule. If the path-independence of the FLSC holds, then the equivalent plastic strains at the onset of the localized necking must be identical for the same stress ratio irrespective of the strain paths. To check this hypothesis, the forming limits analyzed in Fig. 1 are represented by the relation between the equivalent plastic strain \( \varepsilon^* \) and the stress ratio \( \alpha^* \) at the onset of localized necking. The results are shown in Fig. 4. The differences of \( \varepsilon^* \) between those for the proportional loading and the combined loading A are about 0.004 on average for the case of \( \varepsilon^* = 0.2 \), and are about 0.006 on average for the case of \( \varepsilon^* = 0.4 \) and \( \alpha_1 = 0 \) and 1 (except the results for \( 0.2 < \alpha^* < 0.8 \)). From this result we conclude that the saturation of the stress-strain curve is not responsible for the degeneracy of the FLSC as a single curve in stress space.

**Effect of Loading History on the Development of A Neck**

In the previous section it is observed that FLSC for the combined loading A is path-independent, however, FLSC for the combined loading B depends on the strain path. In this section, we discuss the effect of the loading history on the onset of localized necking and the state of stress at that occasion in detail.

We assume that the material is isotropy, and follows isotropic hardening rule, so that the equivalent plastic strain is the only internal variable that is affected by the strain history. If the magnitudes of the equivalent plastic strain in the first loading are the same, the mechanical properties of the material in the second loading must be the same irrespective of the stress ratio in the first loading. Namely, in this analysis the mechanical property of the material is determined by the magnitude of the equivalent strain and the current stress state.

Next we discuss the effect of the loading history on the development of a neck. The development of a neck is represented by the ratio of the equivalent plastic strain outside the band to that inside the band. The development of a neck for the combined loading A with \( \alpha_1 = 1 \) and \( \infty \) (\( \alpha_2 = 0 \) for both cases) are shown in Fig. 5. The development of a neck for the proportional loading with \( \alpha = 0 \) is also depicted in Fig. 5. The stress ratios at the occurrence of localized
necking are zero ($\alpha^* = 0$) for all loading paths. The necking developments for the combined loading A are slower in the first loading than that for the proportional loading. Then, they increase rapidly at the beginning of the second loading as the re-yielding inside the band occurred first, and approach the curve observed for the proportional loading. After that, they follow the curve for the proportional loading. This is because, as mentioned above, the mechanical property of the material depends only on the equivalent plastic strain. Thus, it is natural that forming limit stresses for the proportional loading and combined loading A map on almost the same points in stress space.

Next, the necking developments for the combined loading B in which $\rho$ change -0.5 to 1 at $\varepsilon_1 = 0.1, 0.3$ and 0.48 without unloading are shown in Fig.6. All the stress states at the onset of localized necking in Fig.6 are equi-biaxial tension (except for the case of $\varepsilon_1 = 0.48$). The development of a neck increased dramatically right after the strain path change. This tendency is quite different compare to that for the combined loading A in Fig. 5, and causes the large decrease of thickness in the band. This result in the lower forming limit stresses than those for the proportional loading.

**CONCLUSIONS**

FLSCs for proportional loading and combined loadings are analyzed using the M-K model. The FLSCs for the combined loading A, which includes unloading between the first and second loadings, are almost identical to those calculated for proportional loading; the FLSC is path independent. On the other hand, the FLSCs for the combined loading B, which does not include unloading, are dependent on the strain path. Thus, we conclude that whether or not the FLSC is path independent is affected by loading history.

**REFERENCES**