Analytical Formability Model for Elevated Temperature Sheet Metal Forming Processes

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Abstract. A closed form analytical model for the prediction of strain space formability as a function of temperature and strain rate is proposed. The analytical results are compared with experimental observations. For plane strain conditions, good correlation is reported. This work is significant to those aiming to incorporate formability models directly into numerical simulation programs for the purpose of design and analysis of products manufactured through the warm forming process.

INTRODUCTION

In recent years, there has been a continued demand for vehicle weight reductions in the automobile industry. Many studies indicate that although aluminum components are relatively light-weight and structurally sound, they feature poor formability characteristics. In order to circumvent poor formability of aluminum materials, warm forming processes have received much attention. Most of the studies found in literature focus upon experimental observations. Ayres [1], for example, studied the formability of the oil pan with AL5180 at 180°C. Shehata [2] studied the elongation and flow stress of aluminum alloys with 0–6.6% magnesium at a temperature range extending from 20°C to 300°C for both uniaxial tension and the cup draw tests. Naka [3] studied how the strain hardening coefficient, strain rate sensitivity coefficient, strain rate and temperature influenced the AL5083 FLD. For this study, the strain rate range of $10^{-7}$s to $10^{-5}$s and temperature range of 20°C to 300°C was considered. Li and Ghosh studied the mechanical properties and FLD of AL5754, AL5182 and AL6111 at elevated temperatures ranging from 250°C to 350°C tensile testing and the pan drawing (biaxial tension) [4] [5][6]. Siegert and Jager studied AZ31at 350 °C and 0.02/s strain rate by tensile tests and bulging test [7]. All of the studies mentioned above found that as temperature increases, the flow stress decreases, the strain hardening coefficient ($n$) decreases, strain rate sensitivity ($m$) increases, elongation increases and fracture strain increases. Since the influence of the strain rate at the elevated temperatures is amplified, the mentioned studies involved expressing various mechanical properties as functions of strain rate. Trends indicate that as strain rate increases, flow stress increases, elongation increases, strain hardening coefficients increase and failure strain decreases.

Warm forming research has also been carried out at the micro-structure level. For example, Murty [8], who observed flow localization at the micro-level, further verified that a decrease in flow stress coincides with an increase in temperature and a decrease in strain rate.

Marciniak [9] studied the influence of the strain rate and anisotropy on the forming limit diagram (FLD) from a theoretical continuum-level perspective. Limiting dome height (LDH) studies were also carried out by Marciniak. In additional studies, Marciniak quantified changes in strain rate history for as a function of temperature during torsion experiments [10]. Cabera [13] obtained stress-strain curves experimentally. He further verified that the flow stress decreases as temperature increases and that flow stress increases as strain rate increases.

Currently a heavy emphasis upon numerical simulation of warm forming processes exists. Yet, without a reliable formability model used within the simulation program, analysts have less opportunity to advance warm metal forming technology. The objective of this study is to propose a closed form analytical model for the prediction of formability as a function of temperature and strain rate.
THEORY

Assuming Hill’s 48 in-plane isotropic yield model, the effective stress may be expressed as follows:

\[ \sigma = \sigma_1 \left[ 1 + \alpha^2 - c_2 \alpha + c_1 \gamma (\gamma - 1 - \alpha) \right]^{\frac{1}{2}} \]  

where

\[ \alpha = \frac{\sigma_2}{\sigma_1}, \beta = \frac{\varepsilon_2}{\varepsilon_1}, \gamma = \frac{\sigma_3}{\sigma_1}, c_1 = \frac{2}{R + 1}, c_2 = \frac{2R}{R + 1} \]

Figure 1 shows the geometry of the sheet metal.

Alternatively, the effective stress may be written as follows:

\[ \sigma = \frac{\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3}{\varepsilon} \]  

Assuming volume preservation and equating (1) and (2) leads to the expression for the effective strain shown below:

\[ \varepsilon = \psi \varepsilon_1 \]  

where

\[ \psi = \frac{1 + \alpha \beta - \gamma (1 + \beta)}{\left[ 1 + \alpha^2 - c_2 \alpha + c_1 \gamma (\gamma - 1 - \alpha) \right]^{\frac{1}{2}}} \]

Assuming that the stress in thickness direction is negligible, \( \psi \) can be simplified as

\[ \psi = \frac{1 + \alpha \beta}{\left(1 + \alpha^2 - c_2 \alpha \right)^{\frac{1}{2}}} \]

From (3) the effective strain rate is given as

\[ \dot{\varepsilon} = \psi \dot{\varepsilon}_1 + \psi \dot{e}_1 \]

\[ \dot{\varepsilon} = \frac{1}{1 + \alpha^2 - c_2 \alpha} \frac{1}{\left( \alpha \beta + \alpha \gamma \right)^{\frac{1}{2}}} \left( \frac{1 - \left(1 + \alpha \gamma - \alpha \beta \right)}{1 + \alpha^2 - c_2 \alpha} \right) \]

where

\[ \dot{\beta} = \frac{d}{dt} \left( \frac{\varepsilon_2}{\varepsilon_1} \right) = \frac{\dot{\varepsilon}_2 - \dot{\varepsilon}_1}{\varepsilon_1^2} \]

Under necking conditions, \( \dot{\varepsilon}_1 >> \dot{\varepsilon}_2 \). Therefore, \( \dot{\beta} \) may be re-written as

\[ \dot{\beta} = -\frac{\dot{\varepsilon}_2 - \dot{\varepsilon}_1}{\varepsilon_1^2} = -\frac{\dot{\varepsilon}_1}{\varepsilon_1} \beta \]

It should be noted that \( \alpha = \frac{2 \beta + c_2}{c_2 \beta + 2} \) when \( \gamma = 0 \).

\[ \dot{\alpha} = \frac{4 - c_2}{\left( c_2 \beta + 2 \right)^2} \dot{\beta} = -\frac{\dot{\varepsilon}_1}{\varepsilon_1} \beta \left( 4 - c_2 \right)^2 \]

Assuming the power law expression for hardening shown below, the effective stress may be written as

\[ \sigma = K \varepsilon^n \dot{\varepsilon}^m = K \psi^{\alpha} \varepsilon_1^{\gamma} \left( \psi \varepsilon_1 + \psi \dot{e}_1 \right)^m \]

At room temperature the effective stress is expressed as follows:

\[ \sigma_0 = K_0 \varepsilon_0^{\alpha} \dot{\varepsilon}_0^m = K_0 \psi_0^{\alpha_0} \varepsilon_1^{\gamma_0} \left( \psi_0 \varepsilon_1 + \psi_0 \dot{e}_1 \right)^{m_0} \]

where: the subscript “0” denotes room temperature
If for uniaxial tension the major stress may be written as $\sigma_1 = K \varepsilon_1^n \dot{\varepsilon}_1^m$, then the stress at necking for the uniaxial tension may be written as $\sigma_{1u} = K \varepsilon_{1u}^n \dot{\varepsilon}_{1u}^m$.

At room temperature the uniaxial necking stress is given as $\sigma_{1u0} = K_0 \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m$.

Recalling that the flow stress decreases as the temperature increases and assuming that the following relationship prevails,

$$\frac{\sigma}{\sigma_0} = \frac{\sigma_{1u}}{\sigma_{1u0}} \quad \text{(8)}$$

the following ratios hold:

$$\frac{\sigma_{1u}}{\sigma_{1u0}} = \frac{\sigma}{\sigma_0} = \frac{K \psi^n \varepsilon_1^n \dot{\varepsilon}_{1u}^m}{K_0 \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \frac{K_0 \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m}{K \psi^n \varepsilon_1^n \dot{\varepsilon}_{1u}^m}$$

$$\frac{\sigma_{1u0}}{\sigma_{1u}} = \frac{\sigma_0}{\sigma} \quad \text{(9)}$$

The ratio of the strength coefficients may be written as

$$\frac{K_{1u}}{K} = \frac{\sigma_{1u0}}{\sigma_{1u}} \cdot \frac{\sigma_{1u}}{\sigma_{1u0}}$$

$$\text{(10)}$$

Application of (10) to (9) leads to

$$\frac{\sigma_{1u}}{\sigma_{1u0}} = \frac{\sigma}{\sigma_0} \left[ \frac{K \psi^n \varepsilon_1^n \dot{\varepsilon}_{1u}^m}{K_0 \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right) \right]^m$$

$$= \frac{\sigma_{1u}}{\sigma_{1u0}} \left[ \frac{K \psi^n \varepsilon_1^n \dot{\varepsilon}_{1u}^m}{K_0 \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right) \right]^m$$

$$= \left[ \frac{\varepsilon_{1u0} \dot{\varepsilon}_{1u0}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m}{\varepsilon_{1u} \dot{\varepsilon}_{1u}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \right] \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right)^m$$

$$\left[ \frac{\varepsilon_{1u0} \dot{\varepsilon}_{1u0}^m}{\varepsilon_{1u} \dot{\varepsilon}_{1u}^m} \right] \frac{\psi^n}{\psi_0^n} \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right)^m$$

$$= 1$$

$$\text{(11)}$$

Under a plane strain plane strain mode of deformation,

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} = 0, \alpha = \frac{c_2}{2}, \alpha = 0, \beta = 0$$

$$\psi = \frac{1}{1 + \alpha^2 - c_2 \beta}, \psi = 0$$

Thus

$$\left[ \frac{\varepsilon_{1u0} \dot{\varepsilon}_{1u0}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m}{\varepsilon_{1u} \dot{\varepsilon}_{1u}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \right] \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right)^m$$

or

$$\varepsilon_1 = \left[ \frac{\varepsilon_{1u0} \dot{\varepsilon}_{1u0}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m}{\varepsilon_{1u} \dot{\varepsilon}_{1u}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \right] \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right)^m$$

(12)

It is further assumed that changes in the anisotropy ratio, R, with respect to temperature are negligible.

Accordingly, for the same strain state, $\psi = \psi_0$. Thus (12) can be simplified as

$$\varepsilon_1 = \varepsilon_{1u} \left[ \frac{\varepsilon_{1u0} \dot{\varepsilon}_{1u0}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m}{\varepsilon_{1u} \dot{\varepsilon}_{1u}^m \psi_0^n \varepsilon_{1u0}^n \dot{\varepsilon}_{1u0}^m} \right] \left( \dot{\varepsilon}_{1u} + \dot{\varepsilon}_{1u0} \right)^m$$

(13)

The form of the proposed model shown in (13) allows for the prediction of failure strain as a function of room temperature failure strain and temperature sensitive material properties.

**EXPERIMENT VERIFICATION**

In Li’s paper, various experimentally derived FLD are given [4]. To compare the proposed predictions for failure strain to those observed experimentally, plane strain states under different temperatures for AL5754 and AL5182+Mn are studied. The anisotropy ratio, R, at room temperature is assumed to be the same for both materials. The strain hardening coefficient, n, the strain rate sensitivity index, m, and the failure strains are interpolated from the figures provided by Li [5]. While the strain rate for AL5754 is determined to be 0.5/s, that for AL5182+Mn is determined to be 0.8/s. Applying these strain rates to the parameters under the other two temperatures, results in good correlation between the analytical results and the experimental observations for a reasonable range of temperatures. (Table 1 and Table 2).
CONCLUSIONS

By referencing the failure strain at room temperature and making use of experimental data relating to temperature and strain rate sensitive material properties, a new analytical model for the prediction of failure strains in warm metal forming processes appears to show reasonable promise. Comparison with experimental data indicates that the proposed model is capable of predicting, with reasonable accuracy, the warm forming failure strain for AL5754 and AL5182+Mn sheet metal under the plane strain mode of deformation. Future work will revolve around additional experimental verification and sensitivity studies relating to uncertainty in model input parameters.

ACKNOWLEDGMENTS

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REFERENCES


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