Application of Six Sigma Robust Optimization in Sheet Metal Forming

Y. Q. Li, Z. S. Cui, X. Y. Ruan, D. J. Zhang

National Die and Mold CAD Engineering Research Center, Shanghai Jiao Tong University, Shanghai, 200030, China

Abstract. Numerical simulation technology and optimization method have been applied in sheet metal forming process to improve design quality and shorten design cycle. While the existence of fluctuation in design variables or operation condition has great influence on the quality. In addition to that, iterative solution in numerical simulation and optimization usually take huge computational time or endure expensive experiment cost. In order to eliminate effect of perturbations in design and improve design efficiency, a CAE-based six sigma robust design method is developed in this paper. In the six sigma procedure for sheet metal forming, statistical technology and dual response surface approximate model as well as algorithm of “Design for Six Sigma (DFSS)” are integrated together to perform reliability optimization and robust improvement. A deep drawing process of a rectangular cup is taken as an example to illustrate the method. The optimization solutions show that the proposed optimization procedure not only improves significantly the reliability and robustness of the forming quality, but also increases optimization efficiency with approximate model.

INTRODUCTION

Sheet metal forming is one of the most widely used manufacturing processes in industries. However, in the cases of complicated sheet metal deformation, improper design of process parameter will lead to defects such as fracture, wrinkling and springback, etc. In order to predict defects and avoid time consuming in trial and error tryout procedure, numerical simulation and optimization method have been separately applied for decades. Many researches have been recently reported on integrating numerical simulation and optimization technology [1-4] together. While with the increase of the design variables and complexity of the problem, the optimization will take too long computational time or even impossible to obtain optimized solutions. In addition to that, perturbations existed in material properties, geometry and processing parameters may have significant effects on the forming quality [5]. Traditional deterministic optimization methods can not take into account of perturbations, therefore may lead to unreliable or non-robust solutions. Based on numerical simulation technique and approximation approach of response surface model, this paper develops a six-sigma robust design method for sheet metal stamping optimization. The philosophy of DFSS [6, 7] in quality engineering is applied in this optimization procedure to improve process quality and design efficiency. A deep drawing process of a rectangular cup is taken as an example to illustrate the proposed method.

DFSS ROBUST OPTIMIZATION

In conventional optimization problems, the objective is to minimize/maximize a linear or nonlinear function of many variables subject to a set of constraints. In contrast, robust optimization aims to optimize not only the function value but also its sensitivity to the variation of the design variables. Robust optimization can be formulated as a multi-objective optimization problem shown as following:

Minimize

\[ [\mu_f, \sigma_f] \]

Subject to

\[ h_k(x) = 0 \quad k = 1, 2, ..., K \]
Where \( \mu_j \) and \( \sigma_j \) are the mean and standard deviation of the objective function \( f(x) \), respectively. The mean and the variance of design variables are identified as \( x \) and \( \Delta x \). \( h_k \) and \( g_j \) are constraint function. \( p_j \) is penalty factor.

Six sigma methodology was proposed at Motorola and developed into DFSS at General Electric (GE). The main purpose of DFSS is to prevent defects at design stage instead of fixing them at later stages, and also to improve parts quality up to 6\( \sigma \) level. The performance level of 6\( \sigma \) is equivalent to 3.4 defect parts per million (PPM), while at 3\( \sigma \) level (the average sigma level for most companies) the defect ratio is about 66800PPM. Figure 1 shows a graphical illustration of probability distribution of part quality at 3\( \sigma \) level and 6\( \sigma \) level. It can be seen that at 6\( \sigma \) level the part quality is more steady than at 3\( \sigma \) level.

![FIGURE 1. 3\( \sigma \) design and 6\( \sigma \) design](image)

Taguchi method is widely used to achieve certain objectives in a mean response while simultaneously minimizing the variance. However, Taguchi method relies heavily on the measure of "signal-to-noise ratio" and encounters difficulties when trying to optimize the object function while satisfying the limitation to the variance. For this reason, a methodology of dual response surface model was proposed by Vining and Myers to realize Taguchi philosophy, in which the response surface for the mean and variance was established simultaneously. Upon the dual response surface, one can directly see which factors primarily influence the mean and which factors primarily influence the variance. It leads a way to minimize the objective function while keeping the variance to a given limit. By integrating the dual response surface model and robust optimization strategy, the six-sigma robust design formulation can be established as follows:

Minimize

\[
F(\mu_j, \sigma_j) = \lambda \mu_j(x) + (1 - \lambda) \sigma_j(x)
\]  

Subject to

\[
h_k(x) = 0 \quad k = 1, 2, ..., K
\]  

\[
g_j(\mu_j(X), \sigma_j(X)) + n \sigma_{\delta_j} \leq 0 \quad j = 1, 2, ..., J
\]  

\[
X_i^l + n \sigma_{\mu_i} \leq \mu_i \leq X_i^u - n \sigma_{\sigma_i} \quad i = 1, 2, ..., N
\]

Where \( \lambda \) is a weight defined by the designer to decrease the sensitivity of quality to the deviation of design variables (controllable variables) or the noise factors (uncontrollable variables). \( n \) denotes the desired sigma level. The six-sigma design is achieved when setting \( n = 6 \).

The dual response surface models for the mean and standard derivation are established following equation (9) and (10),

\[
\hat{\mu} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{l} \gamma_i x_i^2 + \sum_{i=1}^{m} \sum_{j=1}^{n} \beta_{ij} x_i x_j + R_\mu
\]  

\[
\hat{\sigma} = \gamma_0 + \sum_{i=1}^{k} \gamma_i x_i + \sum_{i=1}^{l} \gamma_i x_i^2 + \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{ij} x_i x_j + R_\sigma
\]

Where \( \hat{\mu} \) is the response of the mean \( \mu \) and \( \hat{\sigma} \) is the response of standard derivation \( \sigma \). \( \beta_i \) and \( \gamma_i \) are the unknown coefficients determined by least square method. \( R_\mu \) and \( R_\sigma \) are random errors.

\section*{DFSS PROCEDURE FOR SHEET METAL FORMING}

The main steps to achieve DFSS can be summarized as DMADOV which stands for Define, Measure, Analyze, Design, Optimize and Verify. As shown in figure 2, the steps of DFSS in sheet metal forming can be explained below:

\textbf{Define:} This step is to setup optimization problem, including definition of objective function, constraint conditions, controllable factors and noise factors. In
addition, the statistic distribution of noise factors should also be specified.

Measure: The measure step takes a role to separate the significant factors from all would-be factors that affect the forming quality, and reduce the number of input variables. The common technique for measure step is screening design, which includes design of experiment (DOE), analysis of variance (ANOVA) and significant test. For this sheet metal forming problem, the finite element simulation, instead of physical test, was applied to implement the DOE process.

Analyze: Significant factors include both design variables and noise factors are sampled in this step. The design of experiment upon these significant factors is conducted and the performance in each case study is simulated by finite element. Upon the response of objective function and standard derivation, the approximate dual response surface model is established for the interests including object and constraints.

Design and Optimize: Based on the dual response surface model, the deterministic optimization or six sigma robust optimization is performed by applying optimization method such as sequential quadratic programming (SQP) or genetic algorithm (GA). The solutions in each iteration can be converted to sigma level $n$ to evaluate the robustness and reliability. In fact, by defining different value for $n$, the optimization can be evaluated at different sigma level.

Verify: The optimum result should be verified by experiment or numerical simulation; currently we verify the results by numerical simulation. Since the optimum result is obtained from the response surface model, sometimes it is different from the result given by simulation in the case of same design variables. In this situation, the design work goes back to the measure stage. The simulation result at this pseudo “optimum point” is then added to the response model to make it more approximate to the real performance.

APPLICATION IN DEEP DRAWING

A deep drawing process of a square cup is taken as an example to illustrate the six sigma robust design method. The main defect in this process is wrinkle and fracture. The thickening of the blank will leads to wrinkle while the thinning leads to fracture. To decrease the possibility of wrinkle, the thickening of blank after forming, described using a natural logarithm function, is taken as the objective to be minimized, i.e.

$$\text{Minimize: } \ln(t_{\text{max}}/t_0)$$

(11)
To avoid fracture, the thinning of the blank after drawing should be less than a critical value that obtained from experiment; therefore the constraint for this optimization problem is formulated

$$H = \ln\left(\frac{t_0}{t_{\text{max}}}\right) - T_L < 0$$  \hspace{1cm} (12)

where $t_{\text{max}}$ and $t_{\text{min}}$ are maximum and minimum thicknesses of blank after forming, $t_0$ is initial thickness, and $T_L$ is the upper limit of thinning.

In this work, the finite element simulation instead of physical test was conducted. The simulation work was performed by using commercial software LS-DYNA. The material of the blank is medium steel. The Young’s modulus is $E = 206 \text{MPa}$ and initial yielding stress is $200 \text{MPa}$. The FEM model for this drawing process is shown in Fig 3. The ANOVA and significant test shows that the blank holding force (BHF) and the radius of die (D) are the major design variables, and the friction coefficient $m$ and initial thickness $t_0$ are the major noise factors.

Assume that the design variables are die diameter (D) and blank holding force (BHF), and the design limit for D is 4.7–5.3mm while for BHF is 20–40kN. The noise factors are friction coefficient $m$ and blank thickness $t_0$, where $m$ has a mean value of 0.14 and varies in 0.12–0.16, while $t_0$ has a mean value of 0.78mm and varies in 0.74–0.78mm.

FIGURE 3. FEM model of rectangular cup forming

In the initial design, the variables are $D = 4.7 \text{mm}$ and BHF=35kN. Fig 4 demonstrates the thickness changes after drawing. It can be seen that the thinnest thickness is 0.61235mm and the corresponding $H = 0.0125$, which show a tendency of fracture.

FIGURE 4. Thickness distribution for initial design

FIGURE 5. Thickness distribution for 6 sigma optimum design

In the optimization, the response of thickening as well as the fracture characteristics $H$ is simulated at each design point in the approach of design of experiment. Upon these results, the dual response surface model for thickening and fracture characteristic are established through equation (9) and (10), respectively. In this way the optimization can be performed by calculating the object and constraint value from dual response surface model, instead of running the simulation software in each iteration of design. This is very important to computational time saving for the situation that each simulation will take a long time. Based on the DRSM, deterministic optimization and robust optimization with different sigma level are performed. The comparison of the design results are shown in table 1. In the results of deterministic optimization, the mean value of $H$ which denotes the thinning characteristics is very close to 0, or the allowable upper limit of constraint. When the design variables and noise factors have fluctuation within their normal distributed probability, the probability of exceeding the upper limit of constraint is about 44.13%, which implies a high possibility of
failure. In addition to that, the value of standard deviation in this case is very big, which shows that $H$ is sensitive to the fluctuation of design variables. With the increasing of design sigma level, the possibility of failure decreases rapidly. For example, at 3σ level, the failure probability of the design decreases to 2.28%. When the quality level increases up to 6σ level, nearly zero failure probability can be achieved, and at the same time, the sensitivity of $H$ to the perturbations of design variables and noise factors is reduced. Fig 5 shows the thickness distribution after 6 sigma robust optimization. Fig 6 demonstrates the probability distribution $H$ in different optimization results. It can be seen that in the deterministic optimization, the mean of $H$ is close to the failure boundary and the distribution covers a wide range, while with increasing sigma level, the mean value goes away gradually from the allowable upper limit, and the probability distribution shrinks much thinner. By this way, the optimization increases the robustness of the design.

<table>
<thead>
<tr>
<th>TABLE 1. Solutions of different optimum methods</th>
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<tbody>
<tr>
<td>Optimization method</td>
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<tr>
<td>Deterministic optimization</td>
</tr>
<tr>
<td>3σ level</td>
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<tr>
<td>4σ level</td>
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<tr>
<td>5σ level</td>
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<td>6σ level</td>
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</tbody>
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FIGURE 6. Histogram of deterministic optimization and six sigma robust design

CONCLUSION

A CAE-based six sigma robust design method was proposed in this paper. This method combines six sigma quality philosophy and computer simulation to perform robust design. The optimum solutions of square cup forming illustrated that the proposed method is superior to the traditional optimization method in improving design efficiency and quality. However, as the increasing of sigma level, the design cost and product cost may increase too. The sigma level should be carefully determined by designer according to the importance and cost of products.

REFERENCES


